Satisfiability Checking

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RWTH Aachen University
Informatik 2
LuFG Theory of Hybrid Systems

WS 14/15

Literature

- Daniel Kroening and Ofer Strichman.
  Decision Procedures: An Algorithmic Point of View.
  Springer-Verlag, Berlin, 2008.
- Slides
- Video recordings from previous years
- Partially: lecture notes
- Selected papers

Organization

- Language: English or German
- Lecture (V3):

  Monday 14:15-15:45 Room 5056

  Tuesday 14:15-15:00 Room 5056

  Registration in L^2P learning room via Campus required.

  All materials are available in the learning room.
- Exercise (Ü1):

  Tuesday, 15:00-15:45 room 5056, after the lecture

  Exercise sheets are distributed on Tuesday, and are due to Tuesday one week later.
- Exam: either oral ($\leq$ 30 registrations) or written ($>$ 30 registrations)

  Exercise solutions are no entrance requirement, but they are strongly recommended.
- Assistants: Florian Corzilius, Gereon Kremer

This Tuesday: no exercise (exercise sheets will be distributed in the lecture)

What is satisfiability checking?

Problem we want to solve: develop algorithms for the automated check of the satisfiability (or validity) of formulae in different logics.

What is logic?

- Logic is the

  > study of the principles of valid inference and correct reasoning.

- Studied in, e.g., philosophy, mathematics, computer science.
- A logical system defines
  - the logical form of sentences (syntax) and
  - a set of axioms and inference rules (semantics).
- What is the value of a logical sentence?
  - A structure for a logical system gives meaning to the sentences.
  - The logical system allows to derive the meaning of sentences.
- Important properties of logical systems:
  - consistency
  - soundness
  - completeness

Historical view on logic

Historical development goes from

informal logic (natural language arguments) to

formal logic (formal language arguments)

- Philosophical logic
  - 500 BC to 19th century
- Symbolic logic
  - Mid to late 19th century
- Mathematical logic
  - Late 19th to mid 20th century
- Logic in computer science

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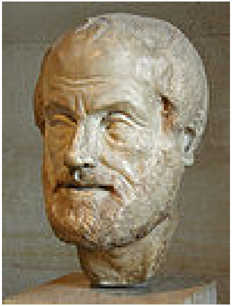
Philosophical logic

- 500 B.C - 19th century
- Logic dealing with sentences in the natural language used by humans.
- Example
  - All men are mortal.
  - Socrates is a man.
  - Therefore, Socrates is mortal.

Philosophical logic

- Natural languages are very ambiguous.
- Aristotle (384 BC – 322 BC) identified 13 types of fallacies in his *Sophistical Refutations*.



Fallacies (1)

Amphibology is an ambiguous grammatical structure in a sentence.

- An example:

  Teenagers shouldn't be allowed to drive. It's getting too dangerous on the streets.

  These sentences could be taken to mean the teenagers will be in danger, or that they will cause the danger.

- Professor to student, on receiving a fifty-page term paper:

  I shall waste no time reading it.

  Implies either that the professor avoids all delays to reading, or that he will not misspend his time by reading.

- And a last one:

  No food is better than our food.

  Implies that ours is best, or that ours is so poor that having none is the better choice.

Fallacies (2)

The fallacy of composition arises when one infers that something is true of the whole from the fact that it is true of some part of the whole.

1. Human cells are invisible to the naked eye.
2. Humans are made up of human cells.
3. Therefore, humans are invisible to the naked eye.

Fallacies (3)

A fallacy of division occurs when one reasons logically that something true of a thing must also be true of all or some of its parts.

Famously and controversially, in the Greek philosophy it was assumed that the atoms constituting a substance must themselves have the properties of that substance: so atoms of water would be wet, atoms of iron would be hard, atoms of wool would be soft, etc.

Fallacies (4)

A figure of speech is the use of a word or words diverging from its usual meaning.

I had butterflies in my stomach.

Fallacies (5)

Affirming the consequent is a formal fallacy, committed by reasoning in the form:

1. If P, then Q.
2. Q.
3. Therefore, P.

> 1. If I have the flu, then I have a sore throat.
> 2. I have a sore throat.
> 3. Therefore, I have the flu.

Fallacies (6)

Irrelevant conclusion is the informal fallacy of presenting an argument that may in itself be valid, but does not address the issue in question.

> I think that we should make the academic requirements stricter for students. I recommend that you support this because we are in a budget crisis and we do not want our salaries affected.

This kind of irrelevant conclusion (red herring) is a debating tactic that seeks to divert an opponent.

Fallacies (7)

Fallacies of questionable cause are informal fallacies where a cause is incorrectly identified.

> HDL ("good") cholesterol is negatively correlated with incidence of heart attack. Therefore, taking medication to raise HDL will decrease the chance of having a heart attack.

Genes affect both HDL levels and the likelihood of having a heart attack; it is possible that medicines may affect the directly measurable factor, HDL levels, without affecting the chance of heart attack.

> With a decrease in the number of pirates, there has been an increase in global warming over the same period. Therefore, global warming is caused by a lack of pirates.

> More and more young people are attending high schools and colleges today than ever before. Yet the rate of youth crime is also much higher then in earier times. This makes it clear that these young people are being corrupted by their education.

Other natural language issues

Besides these fallacies, natural languages allow to argue about the language itself.

This sentence is a lie. *(The liar's paradox)*

$\rightarrow$ inconsistency

Rules for connecting language constructs are not working the expected way:

This sectence has five words.

This sentence has five words and this sectence has five words.

$\rightarrow$ The conjunction of two true sentences is not always true.

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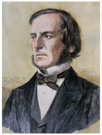
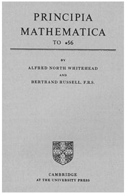
Symbolic and mathematical logic

- 1854: George Boole introduced symbolic logic and the principles of what is now known as Boolean logic.
- 1879: Gottlob Frege created with his *Begriffsschrift* the basis of modern logic with the invention of quantifier notation.
- 1910-1913: Alfred Whitehead and Bertrand Russell published *Principia Mathematica* on the foundations of mathematics, attempting to derive mathematical truths from axioms and inference rules in symbolic logic.
- 1931: Gödel's and Turing's undecidability results (we will deal with them later).



George Boole Gottlob Frege Bertrand Rus-

Alfred Whitehead

Prinzipia Mathematica

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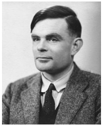
Kurt Gödel
(1906-1978)



Alan Turing
(1912-1954)

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Logic in computer science

Logic has a profound impact on computer science. Some examples:

- Propositional logic - the foundation of computers and circuits
- Databases - Query languages
- Programming languages (e.g. Prolog)
- Specification and verification
- ...

Logic in computer science

- Propositional logic
- First order logic
- Higher order logic
- Temporal logic
- ...

Overview

Propositional logic	$(x \vee y) \wedge (\neg x \vee y)$
Equality	$(x = y \wedge y \neq z) \rightarrow (x \neq z)$
Uninterpreted functions	$(F(x) = F(y) \wedge y = z) \rightarrow F(x) = F(z)$
Linear real/integer arithmetic	$2x + y > 0 \wedge x + y \leq 0$ $2x = 1$
Real algebra	$x^2 + 2xy + y^2 < 0$