Exercise 1

a) A 2-dimensional polyhedron $P$ is defined by the following linear inequalities.

$$
\begin{align*}
-x & \leq 0 \\
-x - y & \leq -2 \\
x & \leq 3 \\
-x + 2y & \leq 6 \\
x - y & \leq 0
\end{align*}
$$

Please give the vertices of $P$.

b) Given a 2-dimensional rectangle $R$ which is defined by the convex hull of the points $(0,0), (0,1), (1,1), (1,0)$. Please give the vertices of the convex hull of $R$ and $P$ (given in the previous exercise), and the linear inequalities which define it.
Solution:

a) The given inequalities form the following polytope:

![Figure 1: The polyhedron P](image)

b) We use the vertices from the previous exercise and the given vertices to compute the convex hull: The vertices of the convex hull are \((0, 0), (0, 3), (4, 1), (3, 0)\), and

![Figure 2: The convex hull of P and R](image)

\(\text{conv}(P, R)\) can be defined by the inequalities

\[-x \leq 0, -y \leq 0, x + 2y \leq 6, x - y \leq 3\]