Modeling and Analysis of Hybrid Systems - SS 2015

Series 6

Exercise 1

Charlie Brown walks his dog Snoopy every day the same way:

- Both leave the house next to each other and start their walk.
- As Charlie (C) is thinking about important things (the girl with the red hair), he walks with continuous pace $v_c$.
- Curious Snoopy (S) is less steady and thus changes his pace between $-v_s$ and $v_s$, while $0 < v_c < v_s$ holds.
- The leash has only a length of 2 meters. Whenever Snoopy is left behind 2 meters, Charlie waits until Snoopy closes up to him and both continue the walk.

Please give a linear hybrid automaton, which models the position $S$ and $C$ of Snoopy and Charlie respectively.

Exercise 2

We consider a vehicle platoon, where two cars are driving with speeds $\dot{x}_i \in [l, u], i \in \{1, 2\}, 0 < l < u$ on a road, such that the 1st car is in front of the 2nd car. The goal is to keep the distance between two cars above some constant $d_0 > 0$. When the distance is at its boundary $d_0$, the rear car brakes, which limits its speed to the interval $\dot{x}_2 \in [l_{\text{min}}, l], 0 < l_{\text{min}} < l$. Additionally we utilize a second constant $d_1 > d_0 > 0$ to prolong the braking process until this target distance $d_1$ is reached. Initially the goal condition is satisfied.

Note that both transitions are urgent transitions, which means that they are taken as soon as they are enabled.

A linear hybrid automaton of the above system is given as follows:
\[ x_1 - x_2 \geq d_0 \]

\[ \dot{x}_1 \in [l, u] \]
\[ \dot{x}_2 \in [l, u] \]
\[ x_1 - x_2 \geq d_0 \]

\[ \dot{x}_1 \in [l, u] \]
\[ \dot{x}_2 \in [l_{\text{min}}, l] \]
\[ x_1 - x_2 \leq d_1 \]

a) Please calculate the forward time closure as presented in the lecture.