Exercise 1

Consider the following timed automaton $\mathcal{T}$:

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\begin{align*}
x &:= 0 \\
x \leq 1 & \xrightarrow{\{p\}} l_0 \\
x > 0 & \xrightarrow{x = 1 \text{ reset}(x)} l_1
\end{align*}
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Please perform the TCTL model checking algorithm as presented in the lecture on $\mathcal{T}$ and verify $\mathcal{T} \models \varphi$, where $\varphi = AF \leq 2p$.

a) Construct $\hat{\varphi}$ by eliminating timing parameters from $\varphi$. Use the name $y$ for the auxiliary clock.

b) Construct a RTS $\mathcal{R}$, such that $\mathcal{T} \models_{TCTL} \varphi$ iff $\mathcal{R} \models_{CTL} \hat{\varphi}$. As $\mathcal{R}$ will become big, use the prepared grid below to sketch the RTS (by adding the required transitions) as follows:

- $\bigcirc$ represents a state, where the location is $l_0$.
- $\square$ represents a state, where the location is $l_1$.
- The position of a state in the grid remarks, which clock region the state represents.
- Please draw only the reachable fragment of $\mathcal{R}$. 

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c) Apply CTL model checking to verify $\mathcal{R} \models_{CTL} \phi$. You can color states in your previously created $RTS$ to indicate that a certain subformula holds in the respective state.

Solution:

a) We add an additional clock $y$ to $\mathcal{T}$, such that $\mathcal{T'}$: 
Removing syntactic sugar from $\varphi$ yields $\varphi = A(\text{true} \ U^{\leq 2} \ p)$ and finally removing time parameters yields $\hat{\varphi} = A(\text{true} \ U ((y \leq 2) \land p))$.

b) The RTS $\mathcal{R}$ is specified as follows:

c) Model checking $\mathcal{R} \models_{\text{CTL}} \hat{\varphi}$

Step 1: $\psi_1 = (y \leq 2) \land p$
Model checking $\mathcal{R} \models_{\text{CTL}} \phi$
Step 2: $\psi_2 = A(true \ U \ \psi_1)$
As for all initial states $\sigma = (l, \nu) \in \mathcal{R}$ with $\nu(y) = 0$ it holds that $\sigma \models \hat{\varphi}$, we conclude $\mathcal{R} \models_{CTL} \hat{\varphi}$, and thus $\mathcal{T} \models_{TCTL} \varphi$. 