Theory of Hybrid Systems
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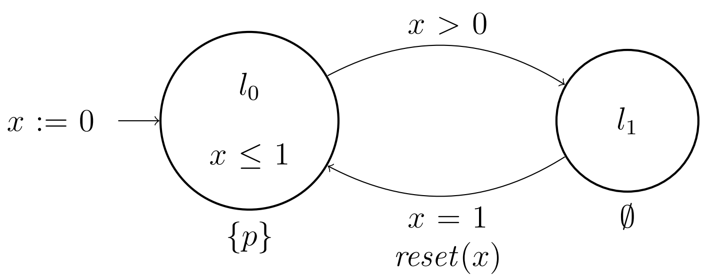
Modeling and Analysis of Hybrid Systems - SS 2015

Series 4

Exercise 1

Consider the following timed automaton $\mathcal{T}$:

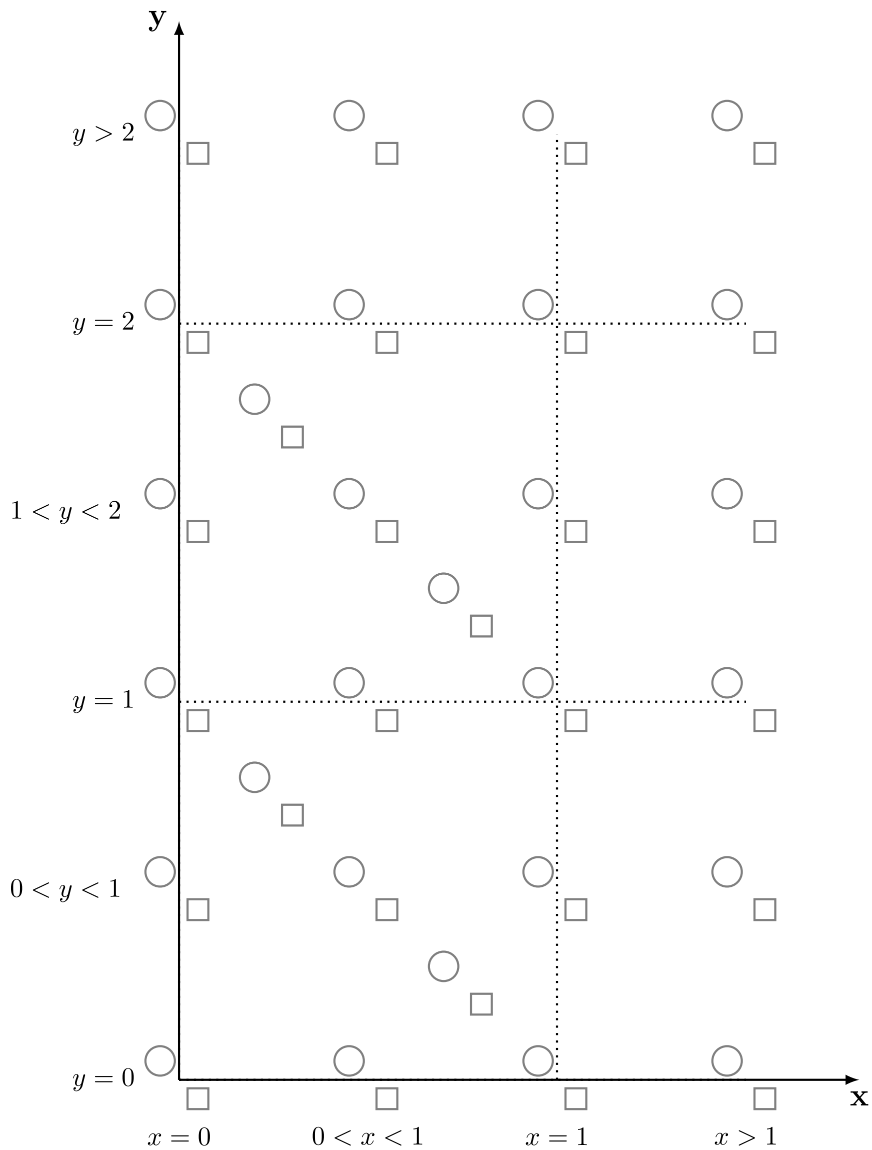


Please perform the TCTL model checking algorithm as presented in the lecture on $\mathcal{T}$ and verify $\mathcal{T} \models \varphi$, where $\varphi = AF^{\leq 2}p$.

a) Construct $\hat{\varphi}$ by eliminating timing parameters from φ. Use the name y for the auxiliary clock.

b) Construct a *RTS* $\mathcal{R}$, such that $\mathcal{T} \models_{TCTL} \varphi$ iff $\mathcal{R} \models_{CTL} \hat{\varphi}$. As $\mathcal{R}$ will become big, use the prepared grid below to sketch the *RTS* (by adding the required transitions) as follows:

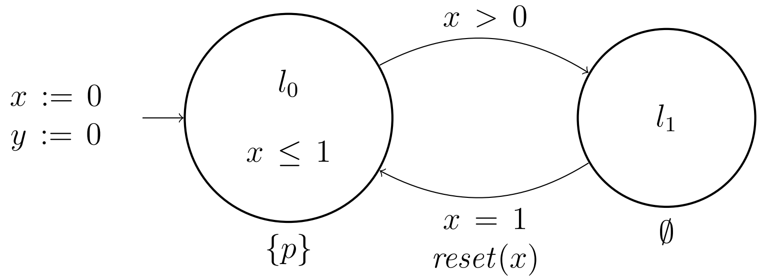
- ○ represents a state, where the location is l_0.
- □ represents a state, where the location is l_1.
- The position of a state in the grid remarks, which clock region the state represents.
- Please draw only the reachable fragment of $\mathcal{R}$.



c) Apply CTL model checking to verify $\mathcal{R} \models_{CTL} \hat{\varphi}$. You can color states in your previously created RTS to indicate that a certain subformula holds in the respective state.

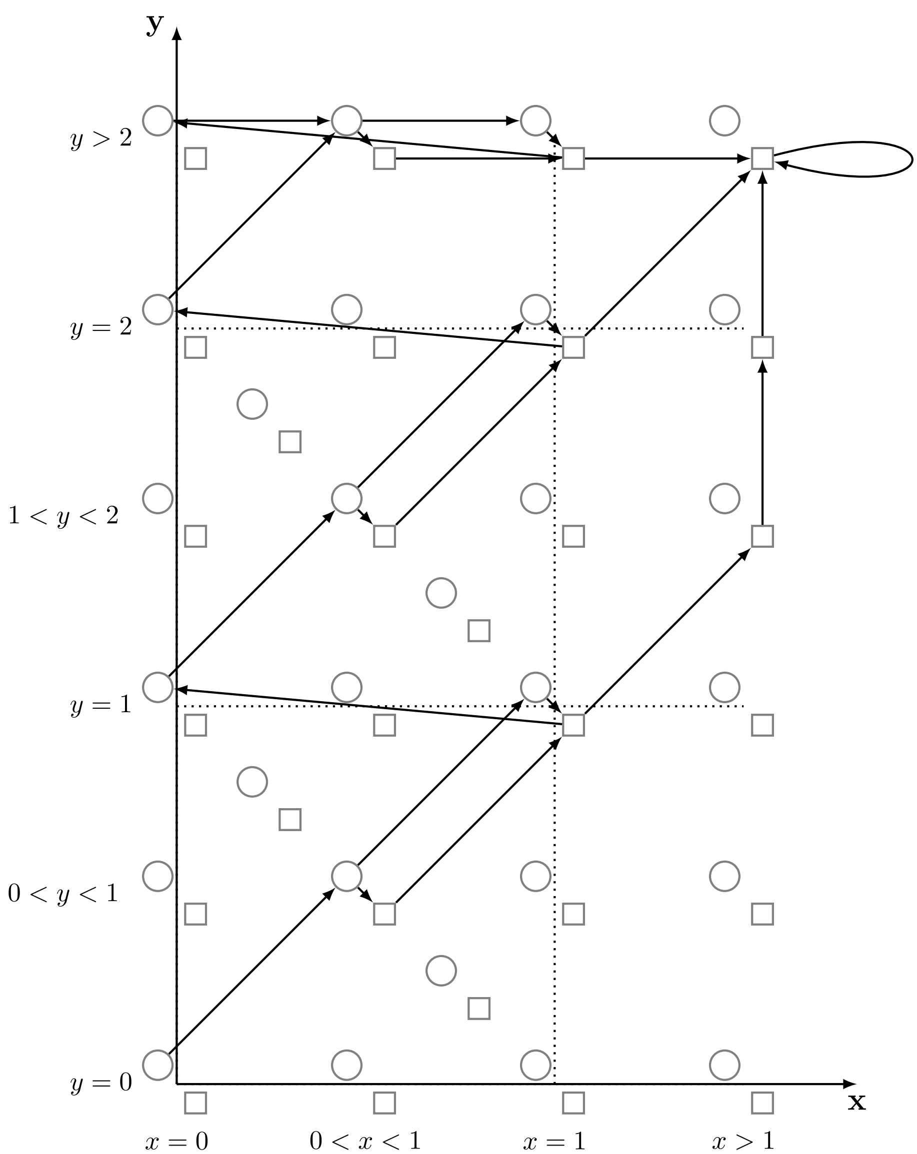
Solution:

a) We add an additional clock y to $\mathcal{T}$, such that $\mathcal{T}'$:

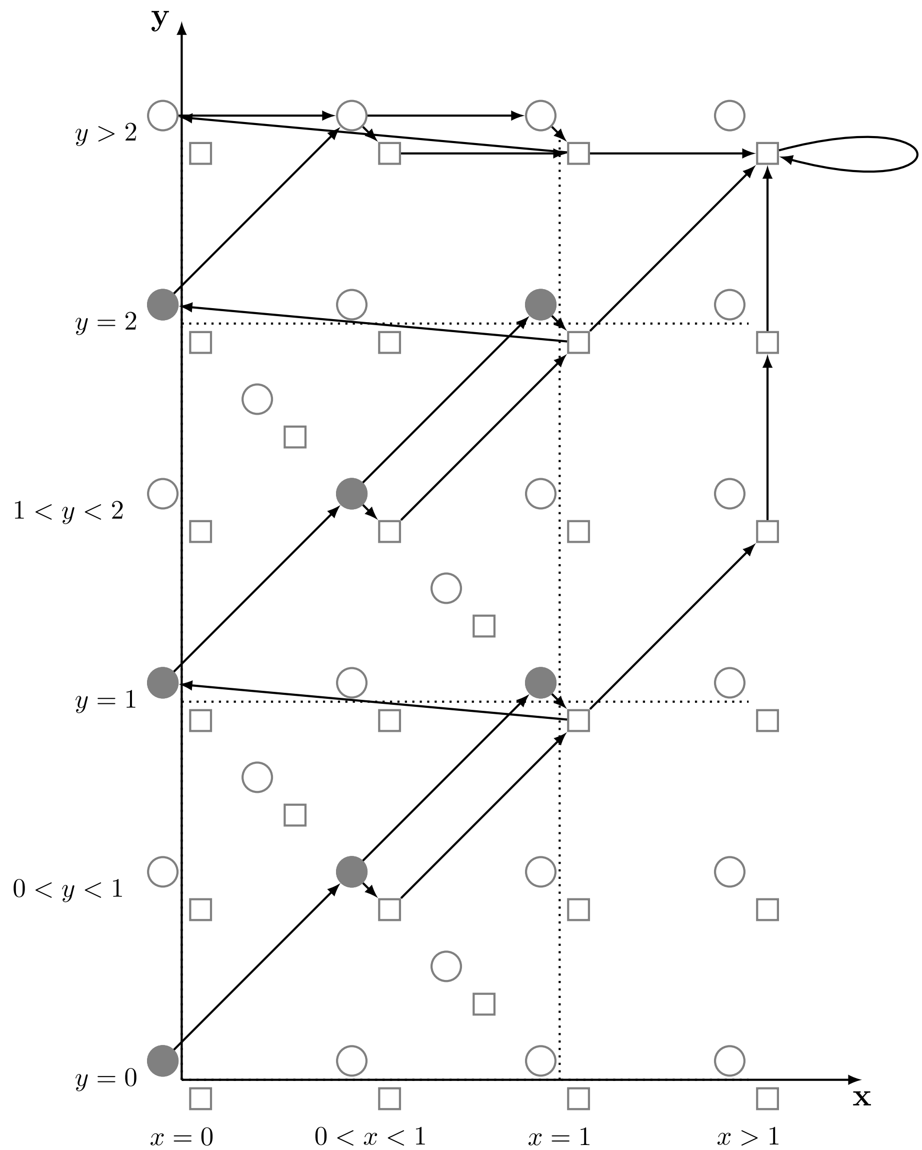


Removing syntactic sugar from φ yields $\varphi = A(true\ U^{\leq 2}\ p)$ and finally removing time parameters yields $\hat{\varphi} = A(true\ U\ ((y \leq 2) \wedge p)))$.

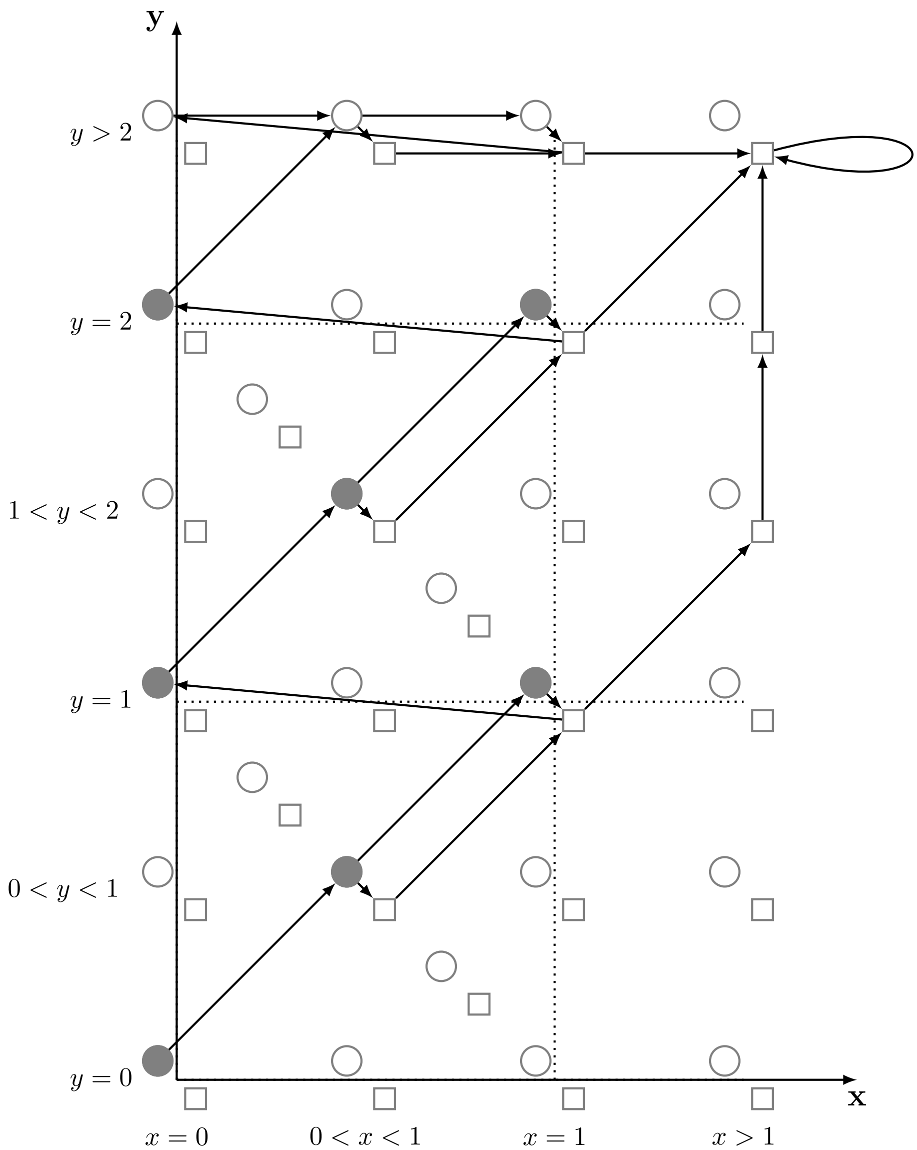
b) The *RTS* $\mathcal{R}$ is specified as follows:



c) Model checking $\mathcal{R} \models_{CTL} \hat{\varphi}$
Step 1: $\psi_1 = (y \leq 2) \wedge p$



Model checking $\mathcal{R} \models_{CTL} \hat{\varphi}$
Step 2: $\psi_2 = A(true\ U\ \psi_1)$



As for all initial states $\sigma = (l, \nu) \in \mathcal{R}$ with $\nu(y) = 0$ it holds that $\sigma \models \hat{\varphi}$, we conclude $\mathcal{R} \models_{CTL} \hat{\varphi}$, and thus $\mathcal{T} \models_{TCTL} \varphi$.