Theory of Hybrid Systems
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Modeling and Analysis of Hybrid Systems - SS 2015

Series 2

Exercise 1

Please match each following LTL formulae φ_i to one of the given execution paths π_j, such that $\pi_j \models \varphi_i$ for all $i \leq i, j \leq 6$ and such that each φ_i is assigned a different path. (*Note: You can assume that the paths continue infinitely in the pattern of the last 2 nodes.*)

φ_1 : $true\,\mathcal{U}\mathcal{X}a$

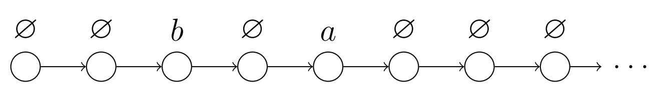
φ_2 : $\mathcal{G}\mathcal{X}a$

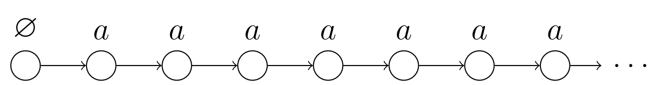
φ_3 : $a\,\mathcal{U}b$

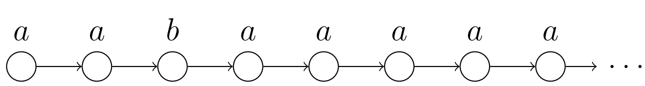
φ_4 : $a \wedge \mathcal{X}b$

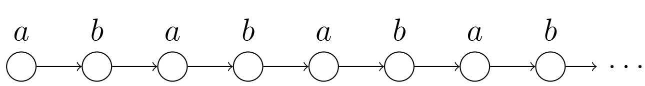
φ_5 : $\mathcal{F}\mathcal{G}a$

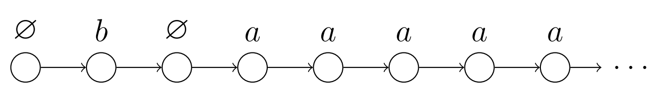
φ_6 : $(\mathcal{X}b)\mathcal{U}a$

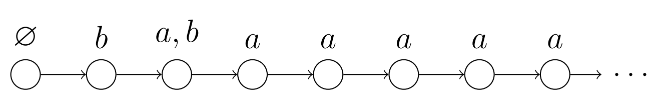
π_1 :


π_2 :


π_3 :


π_4 :


π_5 :


π_6 :


Solution:

$\varphi_1 \models \pi_1, \ldots, \pi_6$

$\varphi_2 \models \pi_2$

$\varphi_3 \models \pi_3, \pi_4$

$\varphi_4 \models \pi_4$

$\varphi_5 \models \pi_2, \pi_3, \pi_5, \pi_6$

$\varphi_6 \models \pi_3, \pi_4, \pi_6$

$\Rightarrow \varphi_i \models \pi_i, i \in \{1, \ldots, 6\}$.

Exercise 2

The LTL formulae $\mathcal{XF}p$ and $\mathcal{FX}p$ are equivalent, since we have the following formal proof: For any path $\pi : s_0 s_1 \cdots$ of an $\mathcal{LSTS}$ $\mathcal{A}$,

$$\begin{aligned} & \mathcal{A}, \pi \models \mathcal{XF}p \\ \Leftrightarrow\ & \pi^1 = s_1 s_2 \cdots \models \mathcal{F}p \\ \Leftrightarrow\ & \exists i \geq 1. s_i \models p \\ \Leftrightarrow\ & \exists i \geq 1. s_{i-1} \models \mathcal{X}p \\ \Leftrightarrow\ & \exists i \geq 0. s_i \models \mathcal{X}p \\ \Leftrightarrow\ & \pi \models \mathcal{FX}p \end{aligned}$$

Is it also the case for the CTL formulae $A\mathcal{X}A\mathcal{F}p$ and $A\mathcal{F}A\mathcal{X}p$? If so, please give a formal proof. Otherwise please present a counterexample.

Solution: The CTL formulae $A\mathcal{X}A\mathcal{F}p$ and $A\mathcal{F}A\mathcal{X}p$ are not equivalent. We give the following counterexample. All of the paths start from s_0 have the second state which is either s_1 or s_2, and any path from these two states satisfies $A\mathcal{F}p$. Hence $TS \models A\mathcal{X}A\mathcal{F}p$. On the other hand, there is a path $\pi : s_0 s_1 s_1 s_1 \cdots$ which does not satisfy $A\mathcal{X}p$, therefore $TS \not\models A\mathcal{F}A\mathcal{X}p$.

Exercise 3

We only consider $\mathcal{LSTS}$s with infinite runs. assume $p, q \in$ AP. Are the CTL formula φ_{CTL} : $A\mathcal{G}(p \rightarrow A\mathcal{F}q)$ and the LTL formula φ_{LTL} : $\mathcal{G}(p \rightarrow \mathcal{F}q)$ equivalent (i.e., $\mathcal{LSTS}, \sigma \models \varphi_{CTL} \Leftrightarrow \sigma \models \varphi_{LTL}$ for all states σ of $\mathcal{LSTS}$)?
(Note: LTL formulae can also be used to describe the properties of states.)

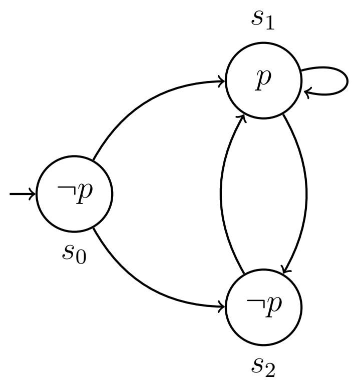


Figure 1: The transition system TS

<u>Solution:</u> Let $\pi(s)$ contain those infinite paths of $\mathcal{LSTS}$ that start in s and $\pi(s, s')$ contain those finite paths starting in s and ending in s'.

The CTL formula $A\mathcal{G}(p \to A\mathcal{F}q)$ is equivalent to the LTL formula $\mathcal{G}(p \to \mathcal{F}q)$, since

$$
\begin{aligned}
&\mathcal{LSTS}, s_0 \models_{LTL} \mathcal{G}(p \to \mathcal{F}q)\\
\Leftrightarrow &\forall \pi \in \pi(s_0). \mathcal{LSTS}, \pi \models_{LTL} \mathcal{G}(p \to \mathcal{F}q)\\
\Leftrightarrow &\forall \pi \in \pi(s_0). \forall i \geq 0. \mathcal{LSTS}, \pi(i) \models p \Rightarrow \exists j \geq i. \mathcal{LSTS}, \pi(j) \models q\\
\Leftrightarrow &\forall \pi \in \pi(s_0, s). \mathcal{LSTS}, s \models p \Rightarrow \forall \pi' \in \pi(s) \exists j \geq 0. \mathcal{LSTS}, \pi'(j) \models q\\
\Leftrightarrow &\mathcal{LSTS}, s_0 \models AG(p \to AFq).
\end{aligned}
$$

Exercise 4

A transition system TS is given in Figure **??**. Decide whether $TS \models \Phi$ where $\Phi = E\mathcal{F}A\mathcal{G}c$. Please sketch the main steps of the CTL model-checking algorithm.

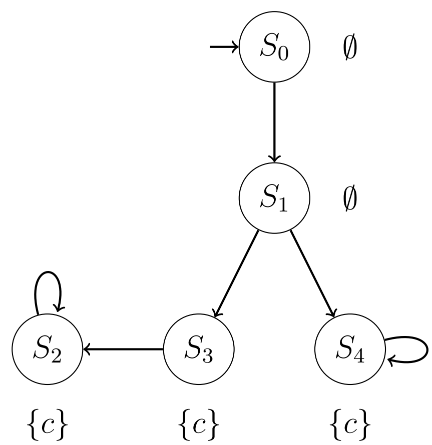
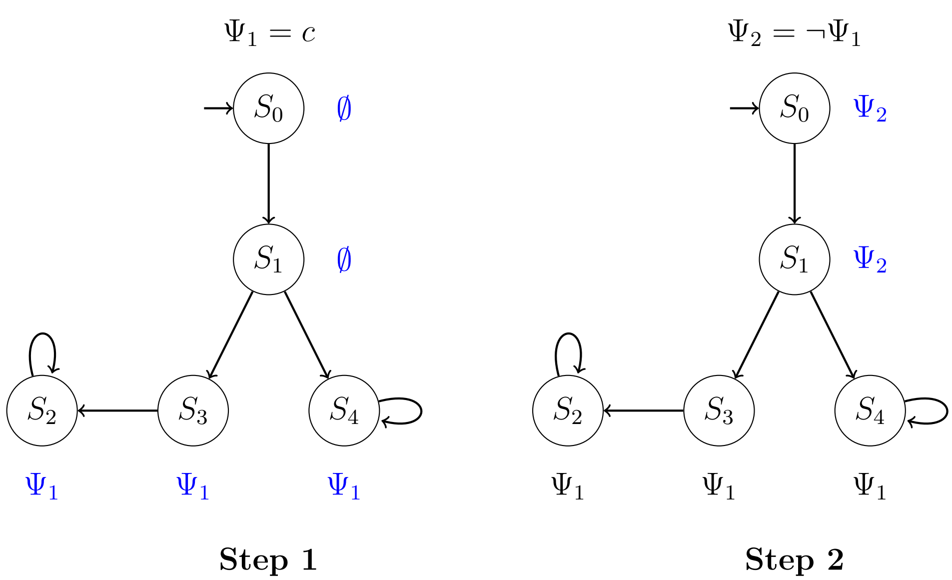


Figure 2: The transition system TS

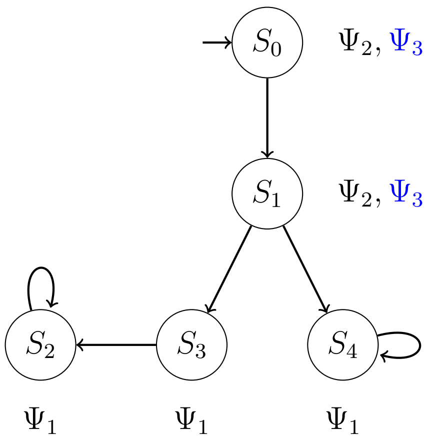
Solution: In the lecture, we only taught the model-checking algorithm for the operators $\neg$, $\wedge$, $E(\cdot\,\mathcal{U}\,\cdot)$ and $A(\cdot\,\mathcal{U}\,\cdot)$. Therefore, we need to rewrite the formula Φ as follows:

$$\Phi = E\mathcal{F}\,A\mathcal{G}\,c = E(true\,\mathcal{U}\,(A\mathcal{G}\,c)) = E(true\,\mathcal{U}\,(\neg\,E\mathcal{F}\,\neg c)) = E(true\,\mathcal{U}\,(\neg\,E(true\,\mathcal{U}\,\neg c)))$$

We present the main steps of checking $TS \models \Phi$.

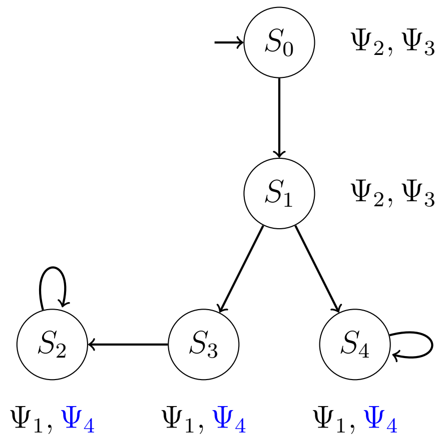


$\Psi_3 = E(true\ \mathcal{U}\ \Psi_2)$



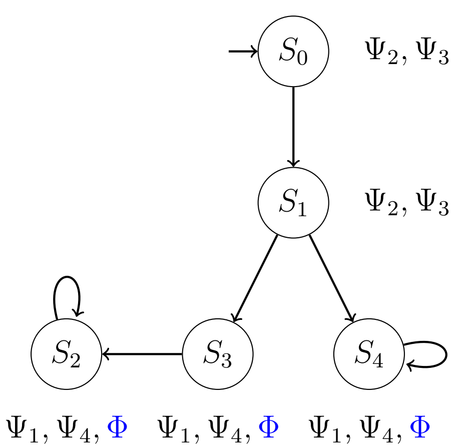
Step 3

$\Psi_4 = \neg\Psi_3$



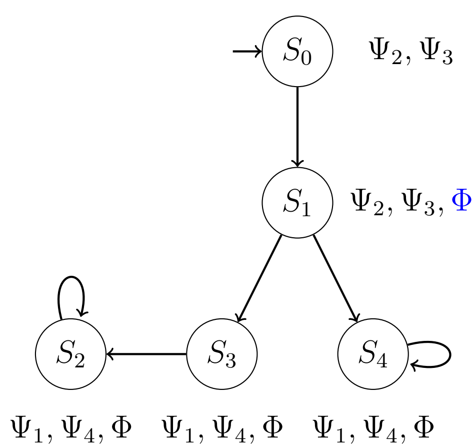
Step 4

$\Phi = E(true\ \mathcal{U}\ \Psi_4)$



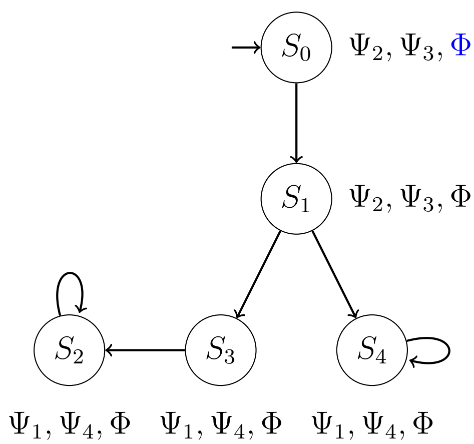
Step 5

$\Phi = E(true\ \mathcal{U}\ \Psi_4)$



Step 6

$$\Phi = E(true\, \mathcal{U}\ \Psi_4)$$



Step 7