Modeling and Analysis of Hybrid Systems - SS 2015

Series 2

Exercise 1

Please match each following LTL formulae \( \varphi_i \) to one of the given execution paths \( \pi_j \), such that \( \pi_j \models \varphi_i \) for all \( i \leq j \leq 6 \) and such that each \( \varphi_i \) is assigned a different path.

(Note: You can assume that the paths continue infinitely in the pattern of the last 2 nodes.)

\[ \varphi_1 : \text{true} \mathcal{U} \mathcal{X}a \]
\[ \varphi_2 : \mathcal{G}\mathcal{X}a \]
\[ \varphi_3 : a \mathcal{U}b \]
\[ \varphi_4 : a \land \mathcal{X}b \]
\[ \varphi_5 : \mathcal{F}\mathcal{G}a \]
\[ \varphi_6 : (\mathcal{X}b)\mathcal{U}a \]

\begin{align*}
\varphi_1 &: \text{true} \mathcal{U} \mathcal{X}a \\
\varphi_2 &: \mathcal{G}\mathcal{X}a \\
\varphi_3 &: a \mathcal{U}b \\
\varphi_4 &: a \land \mathcal{X}b \\
\varphi_5 &: \mathcal{F}\mathcal{G}a \\
\varphi_6 &: (\mathcal{X}b)\mathcal{U}a
\end{align*}
Exercise 2

The LTL formulae $\mathcal{X}Fp$ and $F\mathcal{X}p$ are equivalent, since we have the following formal proof: For any path $\pi : s_0s_1 \cdots$ of an $LSTS$ $A$,

$$A, \pi \models \mathcal{X}Fp$$
$$\iff \pi^1 = s_1s_2 \cdots \models Fp$$
$$\iff \exists i \geq 1. s_i \models p$$
$$\iff \exists i \geq 1. s_{i-1} \models \mathcal{X}p$$
$$\iff \exists i \geq 0. s_i \models \mathcal{X}p$$
$$\iff \pi \models F\mathcal{X}p$$

Is it also the case for the CTL formulae $A\mathcal{X}A\mathcal{F}p$ and $A\mathcal{F}A\mathcal{X}p$? If so, please give a formal proof. Otherwise please present a counterexample.

Exercise 3

We only consider $LSTS$s with infinite runs. Assume $p, q \in AP$. Are the CTL formula $\varphi_{\text{CTL}} : AG(p \rightarrow AFq)$ and the LTL formula $\varphi_{\text{LTL}} : G(p \rightarrow Fq)$ equivalent (i.e., $LSTS, \sigma \models \varphi_{\text{CTL}} \iff \sigma \models \varphi_{\text{LTL}}$ for all states $\sigma$ of $LSTS$)?

(Note: LTL formulae can also be used to describe the properties of states.)
Exercise 4

A transition system $TS$ is given in Figure 1. Decide whether $TS \models \Phi$ where $\Phi = EFAGc$. Please sketch the main steps of the CTL model-checking algorithm.

Figure 1: The transition system $TS$