Theory of Hybrid Systems
Prof. Dr. Erika Ábrahám
RWTH Aachen University, Informatik 2 · D-52056 Aachen · GERMANY
http://ths.rwth-aachen.de/

Modeling and Analysis of Hybrid Systems - SS 2015

Series 2

Exercise 1

Please match each following LTL formulae φ_i to one of the given execution paths π_j, such that $\pi_j \models \varphi_i$ for all $i \leq i, j \leq 6$ and such that each φ_i is assigned a different path. (*Note: You can assume that the paths continue infinitely in the pattern of the last 2 nodes.*)

$\varphi_1 :\ true\, \mathcal{U} \mathcal{X} a$

$\varphi_2 :\ \mathcal{G} \mathcal{X} a$

$\varphi_3 :\ a\, \mathcal{U} b$

$\varphi_4 :\ a \wedge \mathcal{X} b$

$\varphi_5 :\ \mathcal{F} \mathcal{G} a$

$\varphi_6 :\ (\mathcal{X} b) \mathcal{U} a$

$\pi_1 :\ \varnothing \to \varnothing \to b \to \varnothing \to a \to \varnothing \to \varnothing \to \varnothing \to \cdots$

$\pi_2 :\ \varnothing \to a \to a \to a \to a \to a \to a \to a \to \cdots$

$\pi_3 :\ a \to a \to b \to a \to a \to a \to a \to a \to \cdots$

$\pi_4 :\ a \to b \to a \to b \to a \to b \to a \to b \to \cdots$

$\pi_5 :\ \varnothing \to b \to \varnothing \to a \to a \to a \to a \to a \to \cdots$

$\pi_6 :\ \varnothing \to b \to a, b \to a \to a \to a \to a \to a \to \cdots$

Exercise 2

The LTL formulae $\mathcal{X}\mathcal{F}p$ and $\mathcal{F}\mathcal{X}p$ are equivalent, since we have the following formal proof: For any path $\pi : s_0 s_1 \cdots$ of an $\mathcal{LSTS}$ $\mathcal{A}$,

$$
\begin{aligned}
&\mathcal{A}, \pi \models \mathcal{X}\mathcal{F}p \\
\Leftrightarrow\ &\pi^1 = s_1 s_2 \cdots \models \mathcal{F}p \\
\Leftrightarrow\ &\exists i \geq 1. s_i \models p \\
\Leftrightarrow\ &\exists i \geq 1. s_{i-1} \models \mathcal{X}p \\
\Leftrightarrow\ &\exists i \geq 0. s_i \models \mathcal{X}p \\
\Leftrightarrow\ &\pi \models \mathcal{F}\mathcal{X}p
\end{aligned}
$$

Is it also the case for the CTL formulae $A\mathcal{X}A\mathcal{F}p$ and $A\mathcal{F}A\mathcal{X}p$? If so, please give a formal proof. Otherwise please present a counterexample.

Exercise 3

We only consider $\mathcal{LSTS}$s with infinite runs. assume $p, q \in$ AP. Are the CTL formula $\varphi_{CTL} : A\mathcal{G}(p \to A\mathcal{F}q)$ and the LTL formula $\varphi_{LTL} : \mathcal{G}(p \to \mathcal{F}q)$ equivalent (i.e., $\mathcal{LSTS}, \sigma \models \varphi_{CTL} \Leftrightarrow \sigma \models \varphi_{LTL}$ for all states σ of $\mathcal{LSTS}$)?
(Note: LTL formulae can also be used to describe the properties of states.)

Exercise 4

A transition system TS is given in Figure 1. Decide whether $TS \models \Phi$ where $\Phi = E\mathcal{F}A\mathcal{G}c$. Please sketch the main steps of the CTL model-checking algorithm.

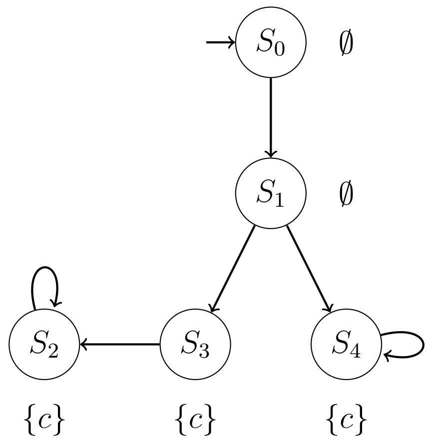

Figure 1: The transition system TS