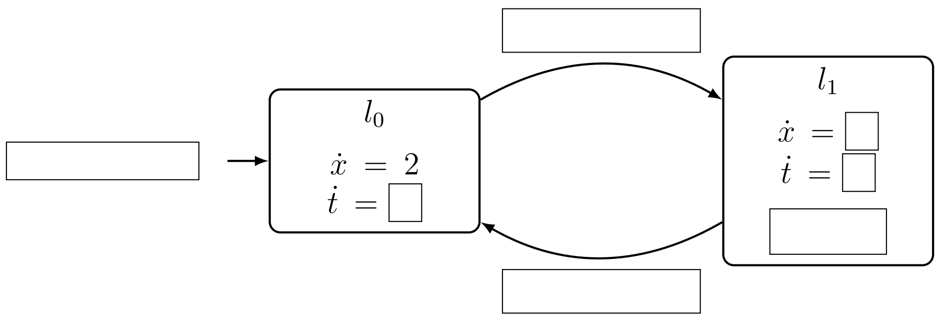
Modeling and Analysis of Hybrid Systems - SS 2015

Series 10

Exercise 1

Below we have given an incomplete graphical representation of a hybrid automaton $\mathcal{H}$:



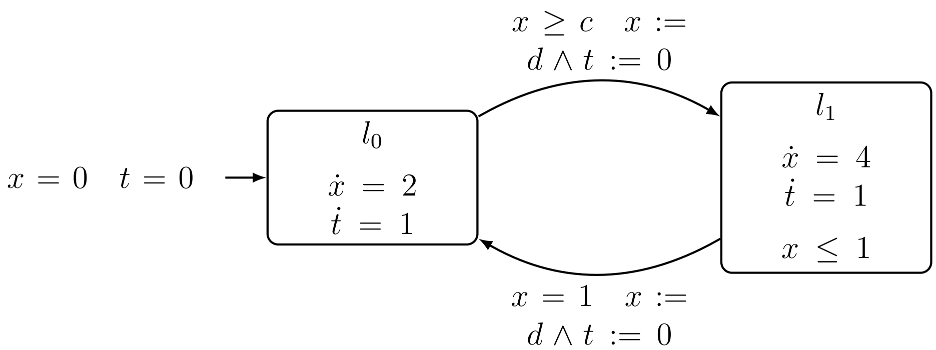
a) Please complete $\mathcal{H}$ by filling in the missing information according to the following specification:

- Both variables x, t are initialized with 0.
- t is used as a clock to measure the time spent in each location.
- In location l_1 x evolves twice as fast as in location l_0.
- Due to inertia the system rests in location l_1 for exactly 1 time unit.
- Whenever x has reached at least value $c, c > 0$, the system may switch from l_0 to l_1.
- Upon each switch, x is set to $d, d \geq 0$.

b) Please give a formal specification of $\mathcal{H}$.

Solution:

a) By the specification we obtain:

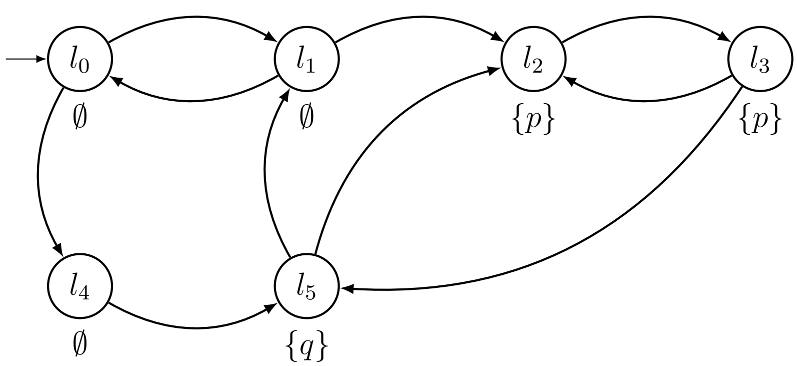


b) We can specify $\mathcal{H}$ as follows:

- $Loc = \{l_0, l_1\}$
- $Var = \{x, t\}$
- $Lab = \{\}$
- $Edge = \{(l_0, x \geq c, x = d \wedge t = 0, l_1), (l_1, x = 1, x = d \wedge t = 0, l_0)\}$
- $Act = \{(l_0, \{(x,t) | \exists x'.\exists t'.\ x = x' + 2\tau \wedge t = t' + \tau\}), (l_1, \{(x,t) | \exists x'.\exists t'.\ x = x' + 4\tau \wedge x \leq 1 \wedge t = t' + \tau\})\}$
- $Inv = \{(l_0, true), (l_1, x \leq 1)\}$
- $Init = \{(l_0, x = 0 \wedge t = 0)\}$

Exercise 2

A transition system TS is given below. Decide whether $TS \models \Phi$ where $\Phi = EFAG(p \vee q)$. Please sketch the main steps of the CTL model-checking algorithm.

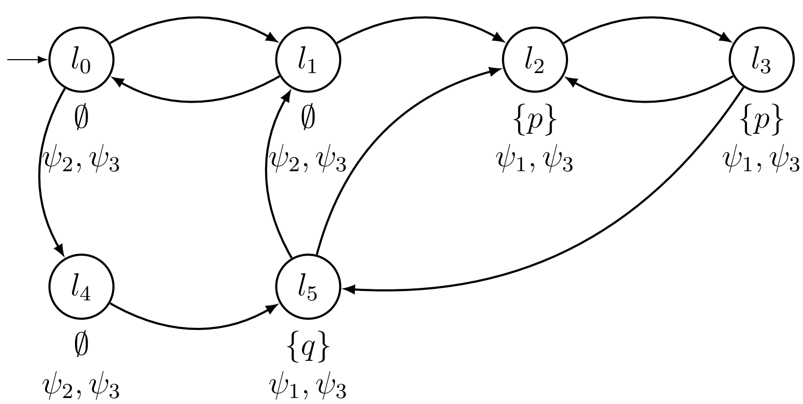


Solution:

Obtain $\hat{\Phi}$ by removing syntactic sugar:

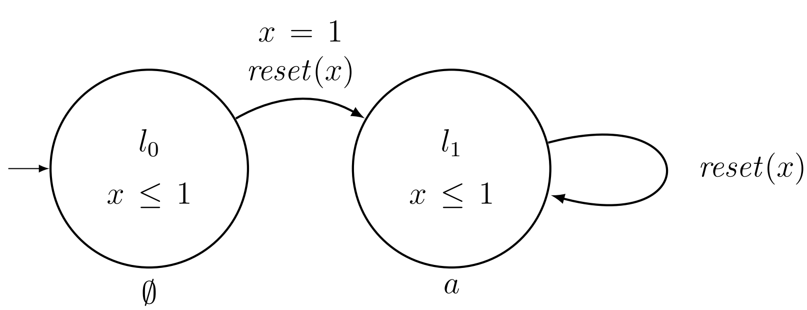
$$
\begin{aligned}
\hat{\Phi} =& EFAG(p \vee q) \\
=& EF(\neg A \neg F \neg (p \vee q)) \\
=& EF(\neg A \neg (true\ U \neg (p \vee q)) \\
=& EF(\neg E \neg\neg (true\ U \neg (p \vee q)) \\
=& EF(\neg E (true\ U \neg (p \vee q)) \\
=& E(true\ U(\neg E(true\ U \neg (p \vee q)))
\end{aligned}
$$

Perform CTL model-checking on $\hat{\Phi} = \underbrace{E(true\ U(\underbrace{\neg\, \underbrace{E(true\ U\, \underbrace{\neg\, \underbrace{(p \vee q)}_{\psi_1}}_{\psi_2})}_{\psi_3}}_{\psi_4}))}_{\psi_5}$



Exercise 3

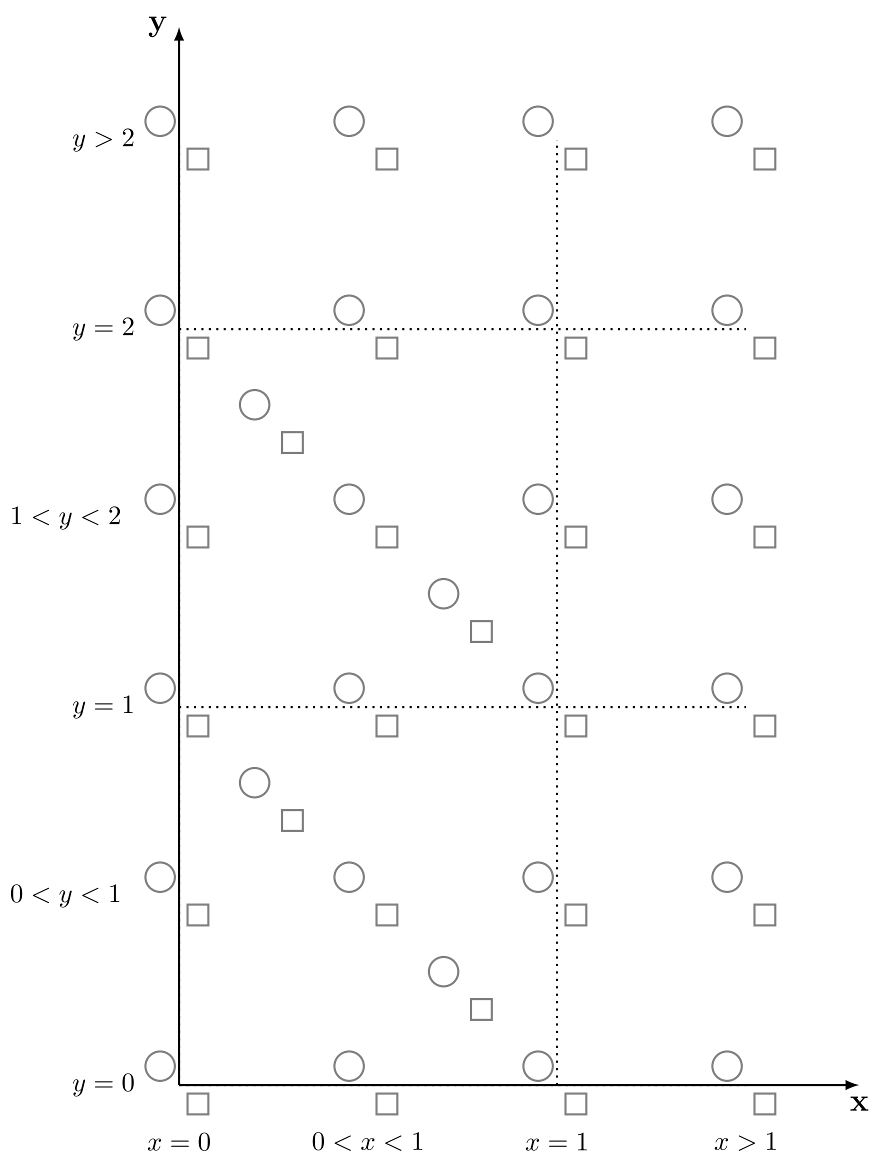
Consider the following timed automaton $\mathcal{T}$ and the TCTL formula $\varphi = AF^{\geq 2}a$:



a) Construct $\hat{\varphi}$ by eliminating timing parameters from φ. Use the name y for the auxiliary clock.

b) Construct a *RTS* $\mathcal{R}$, such that $\mathcal{T} \models_{TCTL} \varphi$ iff $\mathcal{R} \models_{CTL} \hat{\varphi}$. As $\mathcal{R}$ will become big, use the prepared grid below to sketch the *RTS* (by adding the required transitions) as follows:

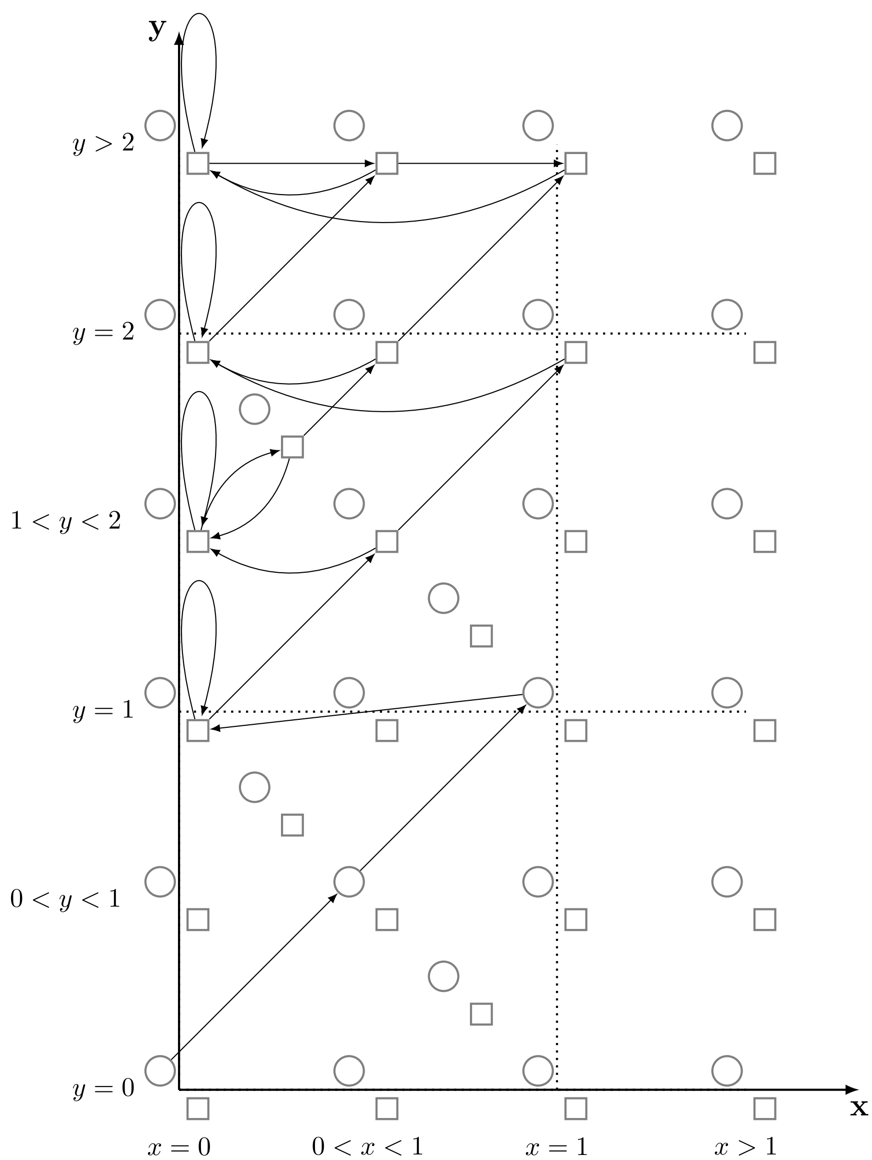
- ◯ represents a state, where the location is l_0.
- □ represents a state, where the location is l_1.
- The position of a state in the grid remarks, which clock region the state represents.
- Please draw only the reachable fragment of $\mathcal{R}$.



Solution:

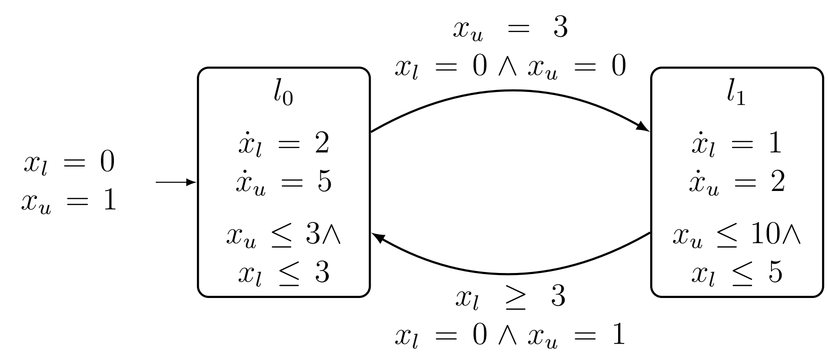
a) $\hat{\varphi} = \neg E \neg (true\ U (y \geq 2 \wedge a))$

b) We can construct $\mathcal{R}$ as follows:



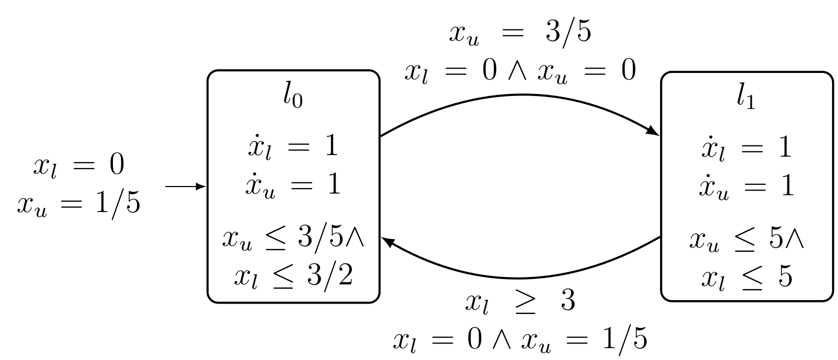
Exercise 4

Consider the following initialized singular automaton $\mathcal{S}$:

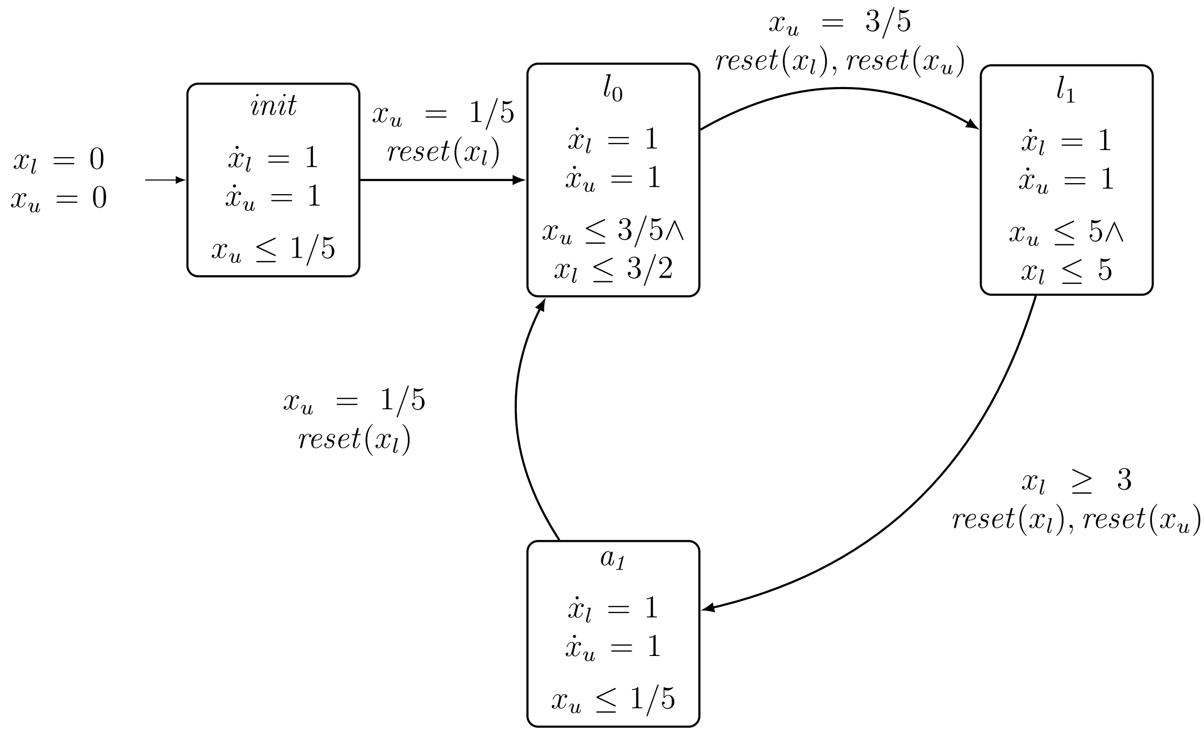


Please reduce $\mathcal{S}$ to a timed automaton $\mathcal{T}$.

Solution: Step 1: Scale clock constraints and clocks:

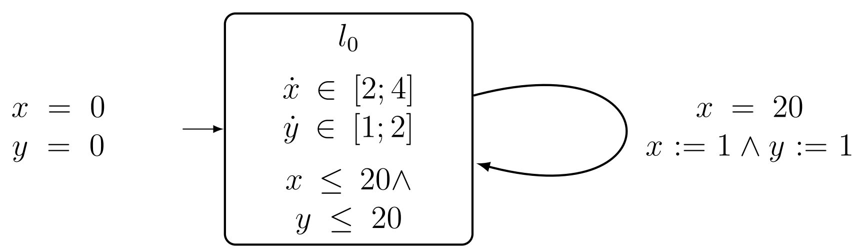


Step 2: Add locations to cope with reset functions:



Exercise 5

Consider the following linear hybrid automaton $\mathcal{H}$:



Please use the forward reachability algorithm presented in the lecture to compute the set of reachable states **after taking one transition** in the model.

Solution:

$$
\begin{aligned}
P =& T^+(\texttt{Init}\hat{\bigcap}\texttt{Inv}) \\
=& T^+(\{(l_0, x = 0 \wedge y = 0 \wedge x \leq 20 \wedge y \leq 20)\}) \\
=& T^+(\{(l_0, x = 0 \wedge y = 0)\}) \\
=& \{(l_0, T^+_{l_0}(x = 0 \wedge y = 0))\} \\
=& \{(l_0, \exists t. \exists x^{pre}. \exists y^{pre}.\ t \geq 0 \wedge x^{pre} = 0 \wedge y^{pre} = 0 \wedge \\
& x \geq x^{pre} + 2t \wedge x \leq x^{pre} + 4t \wedge \\
& y \geq y^{pre} + t \wedge y \leq y^{pre} + 2t \wedge \\
& x \leq 20 \wedge y \leq 20)\} \\
=& \{(l_0, \exists t.\ t \geq 0 \wedge \\
& x \geq 2t \wedge x \leq 4t \wedge \\
& y \geq t \wedge y \leq 2t \wedge \\
& x \leq 20 \wedge y \leq 20)\} \\
=& \{(l_0, x \geq 0 \wedge y \geq 0 \wedge x/4 \leq y \wedge y \leq x \wedge \\
& x \leq 20 \wedge y \leq 20)\} \\
R =& \{(l_0, x \geq 0 \wedge y \geq 0 \wedge x/4 \leq y \wedge y \leq x \wedge \\
& x \leq 20 \wedge y \leq 20)\} \\
R' :=& D^+_e(\varphi^R) \\
=& (l_1, {D_e}^+(\varphi^R)) \\
=& (\exists y^{pre}. \\
& x^{pre} \geq 0 \wedge y^{pre} \geq 0 \wedge x^{pre}/4 \leq y^{pre} \wedge y^{pre} \leq x^{pre} \wedge x^{pre} \leq 20 \wedge y^{pre} \leq 20 \wedge \\
& x^{pre} = 20 \wedge x = 1 \wedge y = 1) \\
=& (l_1, \exists y^{pre}. \\
& y^{pre} \geq 5 \wedge y^{pre} \leq 20 \wedge \\
& x = 1 \wedge y = 1) \\
=& (l_1, x = 1 \wedge y = 1)
\end{aligned}
$$