

Modeling and Analysis of Hybrid Systems

Reachability analysis for linear hybrid automata I using polyhedra

Prof. Dr. Erika Ábrahám

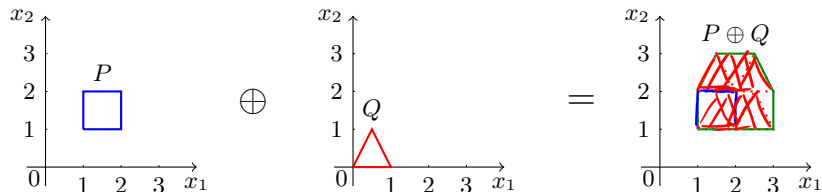
Informatik 2 - Theory of Hybrid Systems
RWTH Aachen University

SS 2015

Some tools for hybrid automata reachability analysis

- Uppaal [Behrmann et al., 2004]
- HyTech [Henzinger et al., 1997]
- PHAVer [Frehse, 2005]
- SpaceEx [Frehse et al., 2011]
- d/dt [Asarin et al., 2002]
- Ellipsoidal toolbox [Kurzhan et al., 2006]
- MATISSE [Girard et al., 2007]
- Multi-Parametric Toolbox [Kvasnica et al., 2004]
- Flow* [Chen et al., 2012]

Reminder: Minkowski sum



$$P \oplus Q = \{p + q \mid p \in P \text{ and } q \in Q\}$$

Most well-known state set representations

Geometric objects:

- hyperrectangles [Moore et al., 2009]
- oriented rectangular hulls [Stursberg et al., 2003]
- convex polyhedra [Ziegler, 1995] [Chen et al., 2011]
- orthogonal polyhedra [Bournez et al., 1999]
- template polyhedra [Sankaranarayanan et al., 2008]
- ellipsoids [Kurzbaniski et al., 2000]
- zonotopes [Girard, 2005]

Other symbolic representations:

- support functions [Le Guernic et al., 2009]
- Taylor models [Berz and Makino, 1998, 2009] [Chen et al., 2012]

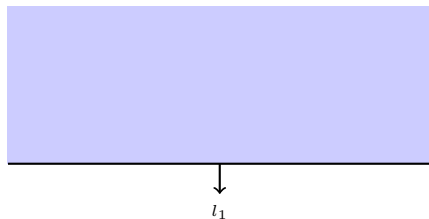
Example: Polytopes

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- **Halfspace:** set of points satisfying $l \cdot x \leq z$

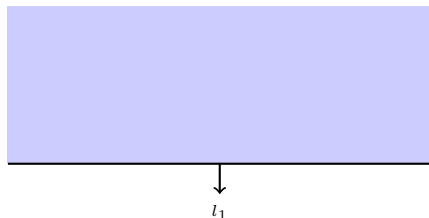
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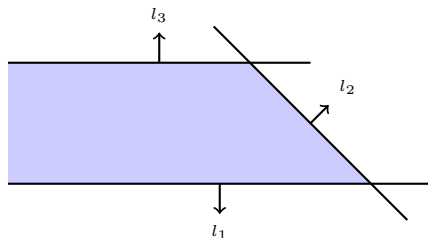
Example: Polytopes

- **Halfspace:** set of points satisfying $l \cdot x \leq z$
- **Polyhedron:** an intersection of finitely many halfspaces



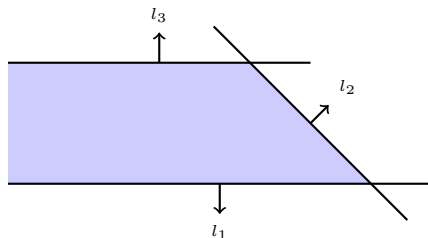
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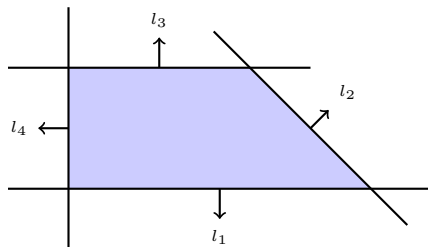
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- **Halfspace:** set of points satisfying $l \cdot x \leq z$
- **Polyhedron:** an intersection of finitely many halfspaces
- **Polytope:** a bounded polyhedron



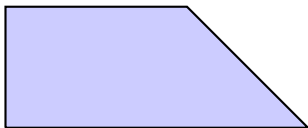
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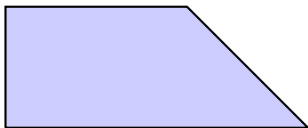
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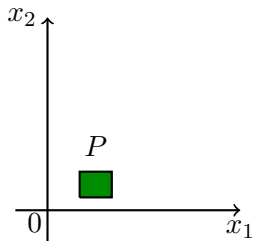
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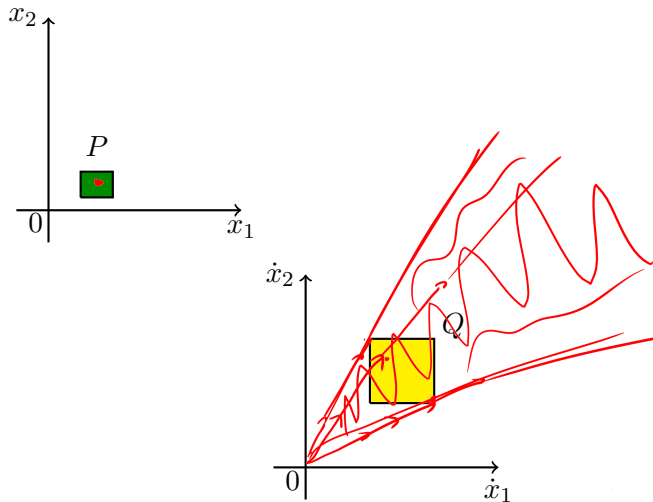
representation	union	intersection	Minkowski sum
\mathcal{V} -representation by vertices	easy	hard	easy
\mathcal{H} -representation by facets	hard	easy	hard

Linear hybrid automata I: Time evolution

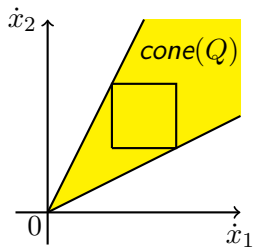
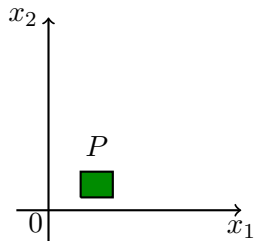
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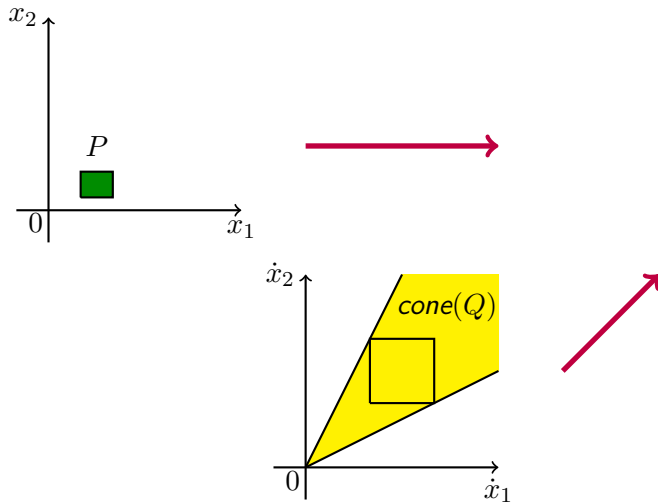
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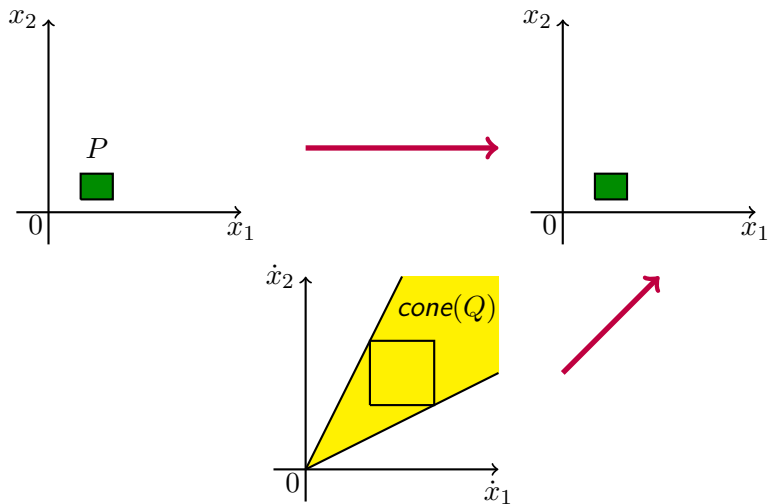
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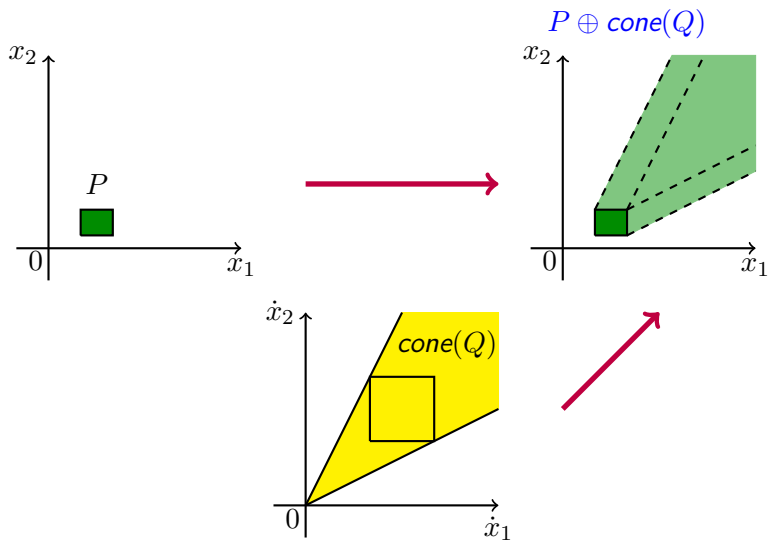
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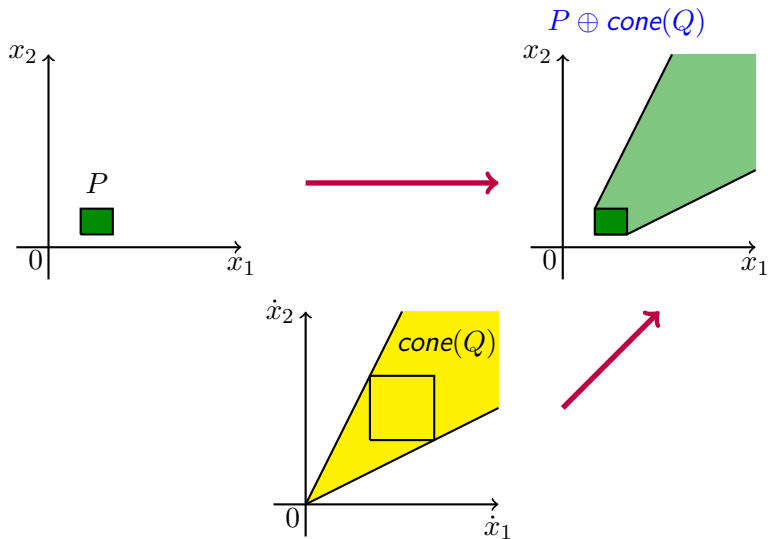
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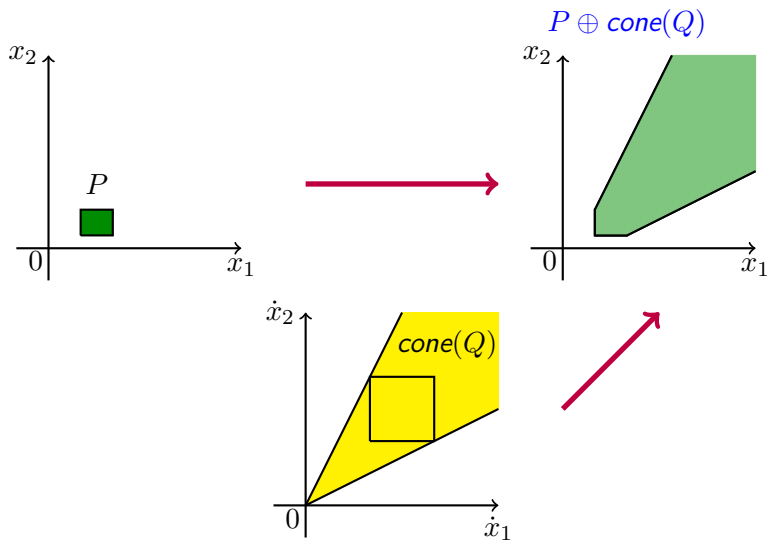
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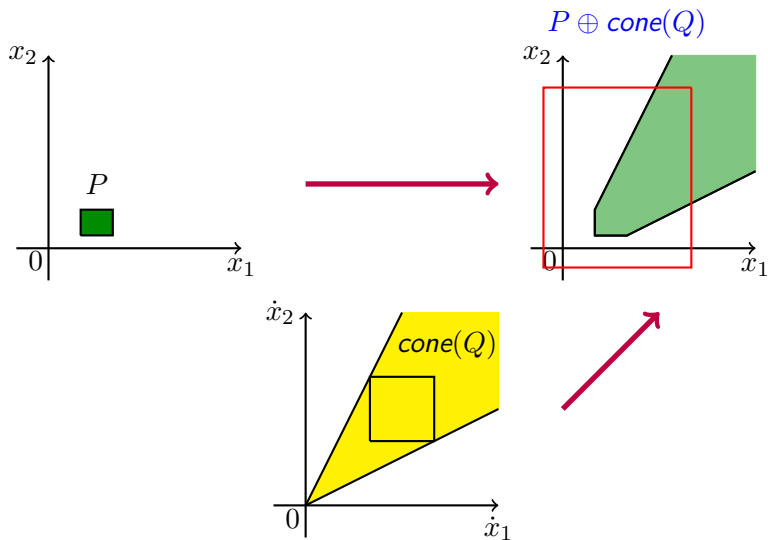
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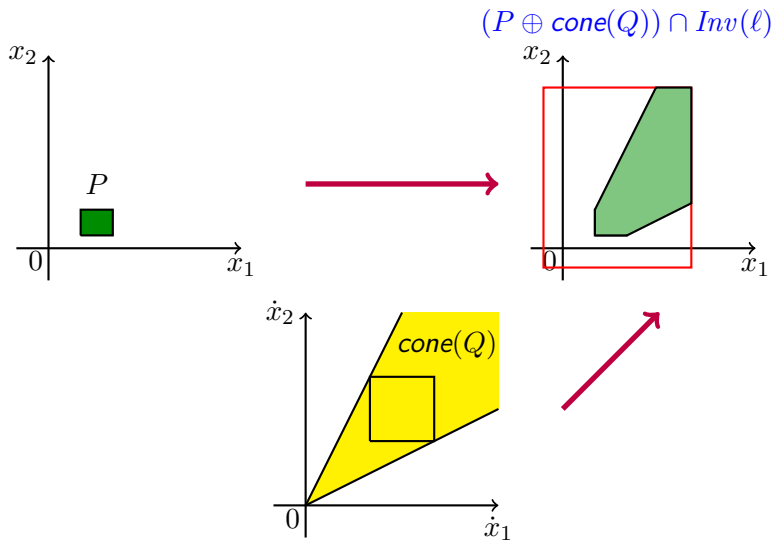
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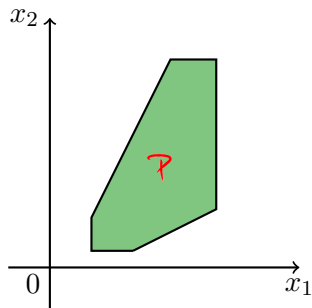
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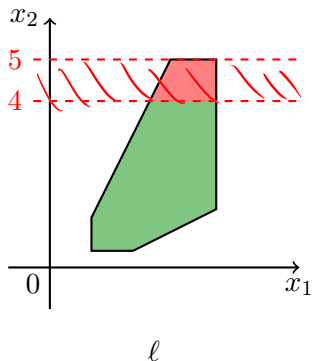
Linear hybrid automata I: Discrete steps (jumps)



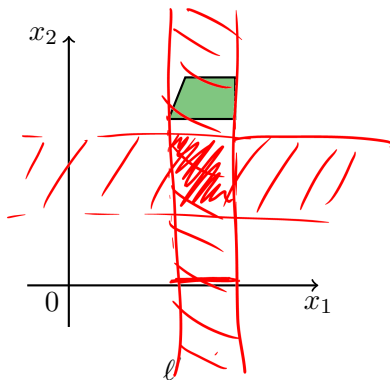
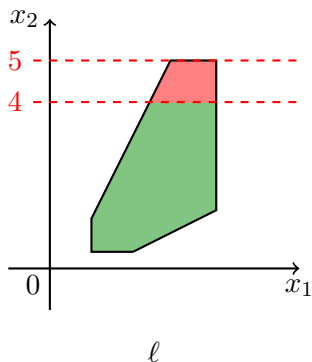
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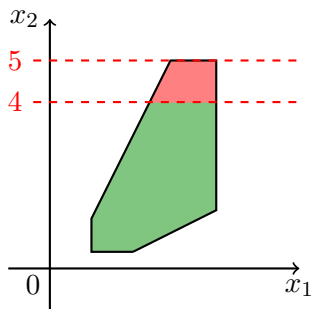
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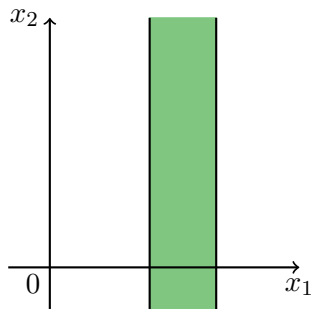
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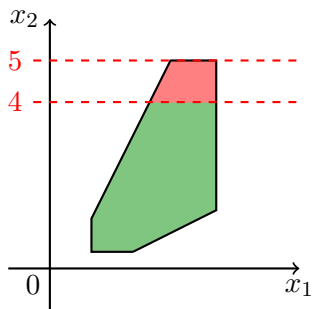


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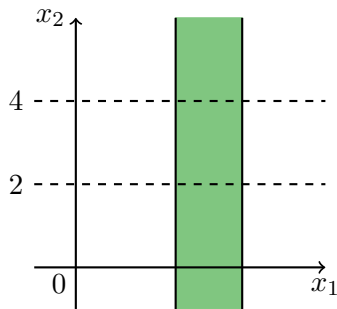


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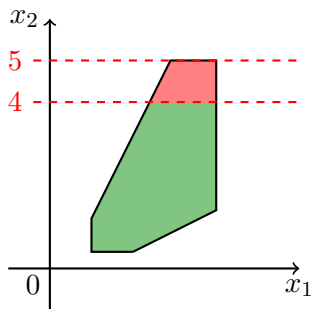


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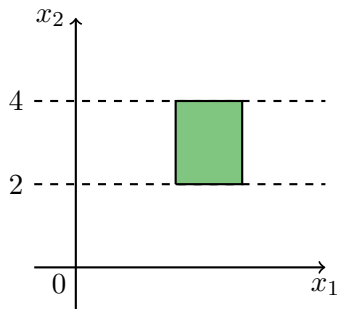


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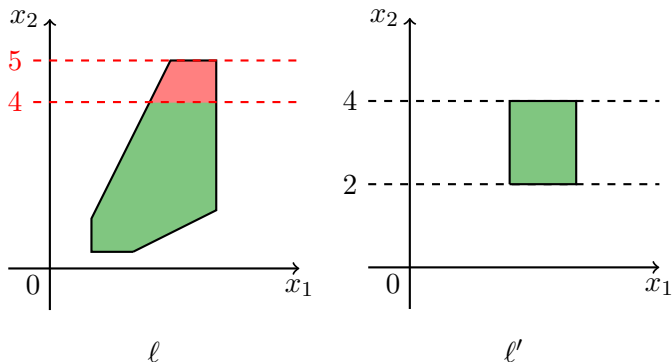


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Linear hybrid automata I: Discrete steps (jumps)



- Computed via [projection](#) and [Minkowski sum](#).