Modeling and Analysis of Hybrid Systems

Reachability analysis for hybrid automata

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General forward reachability computation

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Input: Set Init of initial states.
Output: Set R of reachable states.

Algorithm:

$$
\begin{array}{l}
R^{\text{new}} := \text{Init}; \\
R := \emptyset; \\
\text{while } (R^{\text{new}} \neq \emptyset)\{ \\
\qquad R \qquad := R \cup R^{\text{new}}; \\
\qquad R^{\text{new}} \quad := \text{Reach}(R^{\text{new}}) \backslash R; \\
\}; \\
\text{return } R
\end{array}
$$

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  How to represent a state set P?
  How to compute the operations on sets?
  Especially, how to compute Reach(P) for a set P?

Computing reachability

Hybrid systems evolve using two types of transitions. To compute Reach(P), we need to solve the following problems:

Continuous dynamics

Given a dynamical system defined by $\dot{x} = f(x)$, where x takes values from $\mathbb{R}^d$, and given $P \subseteq \mathbb{R}^d$, calculate (or over-approximate) the set of points in $\mathbb{R}^d$ reached by trajectories (solutions of $\dot{x} = f(x)$) starting in P.

Discrete steps

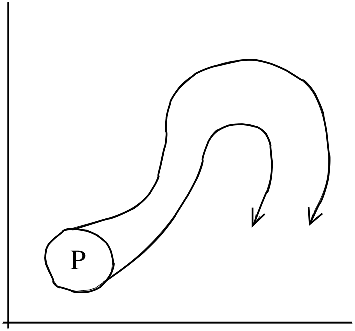
Given a discrete transition of a hybrid system with state space $\mathbb{R}^d$, and given $P \subseteq \mathbb{R}^d$, calculate (or approximate) the set of points in $\mathbb{R}^d$ reachable by taking a discrete transition starting in P.

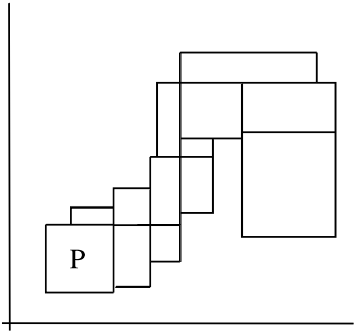
Reachability computation

- In general, the reachability problem for hybrid automata is undecidable.
- For more expressive classes, even one-step reachability is undecidable.
- In such cases, we compute an approximation of reachability.
- Underapproximation for providing counterexamples.
- Overapproximation for proving safety.
- We do only the latter. Thus our task is:

Compute (an over-approximation of) the set of reachable states iteratively using the forward reachability procedure.

Reachability approximation for hybrid automata





State set representation

- The geometry chosen to represent reachable sets has a crucial effect on the efficiency of the whole procedure.
- Usually, the more complex the geometry,
  1. the more costly is the storage of the sets,
  2. the more difficult it is to perform operations like union and intersection, and
  3. the more elaborate is the computation of new reachable sets, but
  4. the better the approximation of the set of reachable states.
- Choosing the geometry has to be a compromise between these impacts.

Representation requirements

The geometry should allow efficient computation of the operations for

- membership relation,
- union,
- intersection,
- subtraction,
- test for emptiness.

State set representation

Approaches:

- Convex polyhedra
- Orthogonal polyhedra
- Oriented rectangular hulls
- Zonotopes
- Support functions
- Ellipsoids
- ...