Modeling and Analysis of Hybrid Systems

Timed automata

Prof. Dr. Erika Ábrahám

Informatik 2 - Theory of Hybrid Systems
RWTH Aachen University

SS 2015

Literature

Christel Baier and Joost-Pieter Katoen:
Principles of Model Checking

Contents

Motivation

Correctness in time-critical systems not only depends on the logical result of the computation but also on the time at which the results are produced.

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Thus if we model such systems, we also need to model the time.
The first choice in modelling: discrete or continuous time?

Discrete-time systems

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- action α lasts $k > 0$ time units $\rightsquigarrow$ $k - 1$ ticks followed by α

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$\mathcal{X}\varphi$: φ holds after one tick

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$\mathcal{F}^{\leq k}\varphi$: φ occurs within k ticks

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We deal in this lecture with continuous-time models.

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Timed automata

- Measure time: finite set $\mathcal{C}$ of clocks $x, y, z, \ldots$
- Clocks increase their value implicitly as time progresses
- All clocks proceed at rate 1

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- Measure time: finite set $\mathcal{C}$ of clocks $x, y, z, \ldots$
- Clocks increase their value implicitly as time progresses
- All clocks proceed at rate 1
- Limited clock access

Read access:

Atomic clock constraints:

$$acc \quad ::= \quad x < c \quad | \quad x \leq c \quad | \quad x > c \quad | \quad x \geq c$$

with $c \in \mathbb{N}$ ($c \in \mathbb{Q}$) and $x \in \mathcal{C}$.

Clock constraints:

$$g \quad ::= \quad acc \quad | \quad g \wedge g$$

Syntactic sugar: *true*, $x \in [c_1, c_2)$, $c_1 \leq x < c_2$, $x = c, \ldots$

$ACC(\mathcal{C})$: set of atomic clock constraints over $\mathcal{C}$

$CC(\mathcal{C})$: set of clock constraints over $\mathcal{C}$

Write access: Clock reset sets clock value to 0

Semantics of clock constraints

Given a set $\mathcal{C}$ of clocks, a clock valuation

Semantics of clock constraints

$x \sim c$

Given a set $\mathcal{C}$ of clocks, a clock valuation $\nu : \mathcal{C} \to \mathbb{R}_{\geq 0}$ assigns a non-negative value to each clock. We use $V_{\mathcal{C}}$ to denote the set of clock valuations for the clock set $\mathcal{C}$.

Definition (Semantics of clock constraints)

$\nu \models x \leq c$ iff $\nu(x) \leq c$

$\models \subseteq V_C \times CC(C)$

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Definition (Semantics of clock constraints)

For a set $\mathcal{C}$ of clocks, $x \in \mathcal{C}$, $\nu \in V_{\mathcal{C}}$, $c \in \mathbb{N}$, and $g, g' \in CC(\mathcal{C})$, let $\models \quad \subseteq \quad V_{\mathcal{C}} \times CC(\mathcal{C})$ be defined by

$$
\begin{array}{lll}
\nu \models x < c & \text{iff} & \nu(x) < c \\
\nu \models x \leq c & \text{iff} & \nu(x) \leq c \\
\nu \models x > c & \text{iff} & \nu(x) > c \\
\nu \models x \geq c & \text{iff} & \nu(x) \geq c \\
\nu \models g \wedge g' & \text{iff} & \nu \models g \text{ and } \nu \models g'
\end{array}
$$

Semantics of clock access

Definition (Time delay, clock reset)

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Definition (Time delay, clock reset)

- For a set $\mathcal{C}$ of clocks, $\nu \in V_{\mathcal{C}}$, and $c \in \mathbb{N}$ we denote by $\nu + c$ the valuation with

$$(\nu + c)(x) = \nu(x) + c$$

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$$(\text{reset } R \text{ in } \nu)(x) = \begin{cases} \nu(x) & \text{falls } x \notin R \\ 0 & \text{sonst} \end{cases}$$

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For a single clock $x \in \mathcal{C}$ we write *reset x in ν*.

valuation for $\mathcal{C} = \{x, y\}$	value of x	value of y
ν	5	1
$\nu + 9$		
reset x *in* $(\nu + 9)$		
(*reset* x *in* $\nu) + 9$		
reset $\{x, y\}$ *in* ν		

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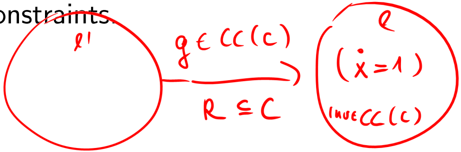
reset $\{x, y\}$ *in* ν	0	0

$\mu \subseteq V_C \times V_C = \{(\nu, \nu') \mid \nu \models g \wedge \nu' = \text{reset } R \text{ in } \nu\}$

A timed automaton is a special hybrid automaton:

- All variables are clocks.
- States $\sigma \in \Sigma$ are pairs of a location and a clock valuation. (ℓ, ν)
- Edges are defined by
  - source and target locations,
  - a label,
  - a guard: clock constraint specifying enabling,
  - a set of clocks to be reset.
- Invariants are clock constraints.



Timed automaton

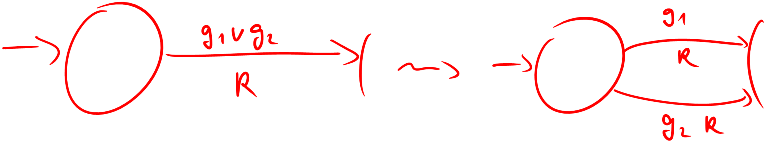
Definition (Syntax of timed automata)

A timed automaton $\mathcal{T} = (Loc, \mathcal{C}, Lab, Edge, Inv, Init)$ is a tuple with

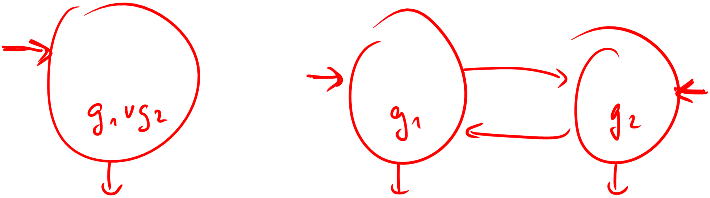
- Loc is a finite set of locations,
- $\mathcal{C}$ is a finite set of clocks,
- Lab is a finite set of synchronisation labels,
- $Edge \subseteq Loc \times Lab \times (CC(\mathcal{C}) \times 2^{\mathcal{C}}) \times Loc$ is a finite set of edges,
- $Inv : Loc \to CC(\mathcal{C})$ is a function assigning an invariant to each location, and
- $Init \subseteq \Sigma$ with $\nu(x) = 0$ for all $x \in \mathcal{C}$ and all $(l, \nu) \in Init$.

We call the variables in $\mathcal{C}$ clocks. We also use the notation $l \overset{a:g,R}{\hookrightarrow} l'$ to state that there exists an edge $(l, a, (g, R), l') \in Edge$.

Note: (1) no explicit activities given (2) restricted logic for constraints



$$\neg(g_1 \wedge g_2) = \neg g_1 \vee \neg g_2 = g_1' \vee g_2'$$



Timed automaton

Analogously to Kripke structures, we can additionally define

- a set of atomic propositions AP and
- a labelling function $L : Loc \to 2^{AP}$

to model further system properties.

Operational semantics

$$\frac{A_1 \quad A_2 \quad \ldots \quad A_n}{B_1 \ldots B_m} \text{ Rule}$$

$$\frac{(\ell_1 \vee \ldots \vee \ell_n \vee a)\,(\ell_1' \vee \ldots \ell_m' \vee \neg a)}{(\ell_1 \vee \ldots \vee \ell_n \vee \ell_1' \vee \ldots \vee \ell_m')}$$

$$\frac{\nu;\ \nu' \models Inv(\ell) \quad \nu' = \nu + t}{(\ell, \nu) \xrightarrow{t} (\ell, \nu')}$$

$$\frac{(\ell, a, (g, R), \ell') \in Edge \quad \nu \models g \qquad \nu \models Inv(\ell);\ \nu' \models Inv(\ell') \quad \nu' = \text{reset } R \text{ in } \nu}{(\ell, \nu) \xrightarrow{a} (\ell', \nu')}$$

Operational semantics

$$\frac{(l,a,(g,R),l') \in Edge \quad \nu \models g \quad \nu' = \textit{reset } R \textit{ in } \nu \quad \nu' \models Inv(l')}{(l,\nu) \xrightarrow{a} (l',\nu')} \quad \texttt{Rule}_{\texttt{Discrete}}$$

$$\frac{t>0 \quad \nu' = \nu + t \quad \nu' \models Inv(l)}{(l,\nu) \xrightarrow{t} (l,\nu')} \quad \texttt{Rule}_{\texttt{Time}}$$

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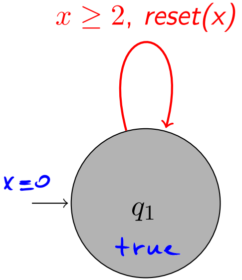
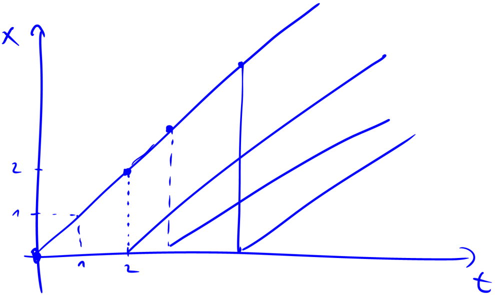
- Execution step: $\rightarrow = \xrightarrow{a} \cup \xrightarrow{t}$
- Path: $\sigma_0 \rightarrow \sigma_1 \rightarrow \sigma_2 \ldots$ with $\sigma_0 = (l_0, \nu_0)$ and $\nu_0 \in Inv(l_0)$
- Initial path: path $\sigma_0 \rightarrow \sigma_1 \rightarrow \sigma_2 \ldots$ with $\sigma_0 = (l_0, \nu_0)$, $l_0 \in Init$ and $\nu_0(x) = 0$ for all $x \in \mathcal{C}$
- Reachability of a state: exists an initial path leading to the state

Example: Timed Automaton





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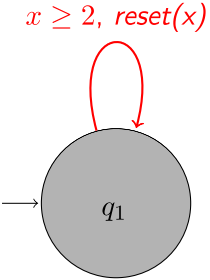

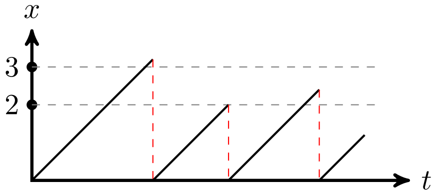

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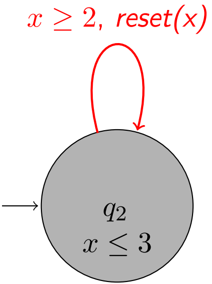

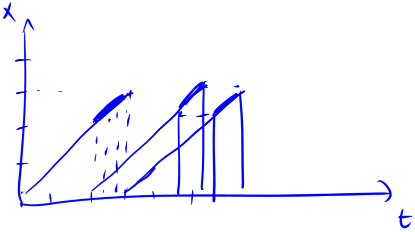

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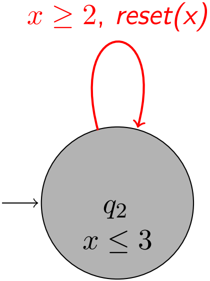

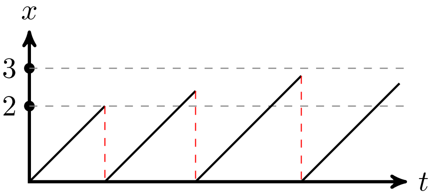

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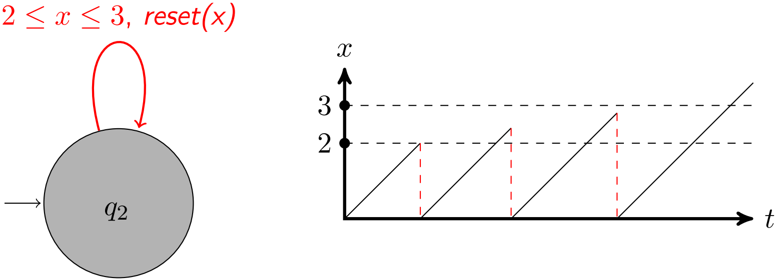
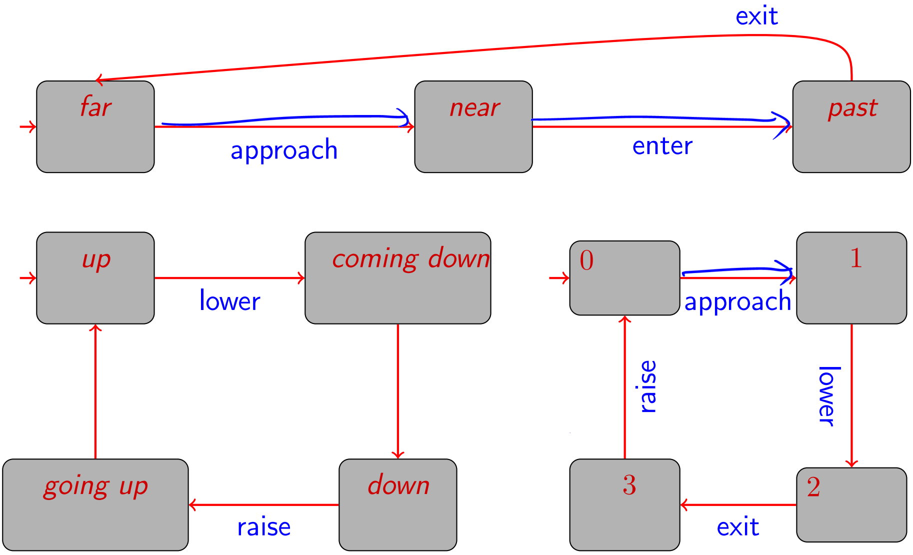


Example: Railroad Crossing



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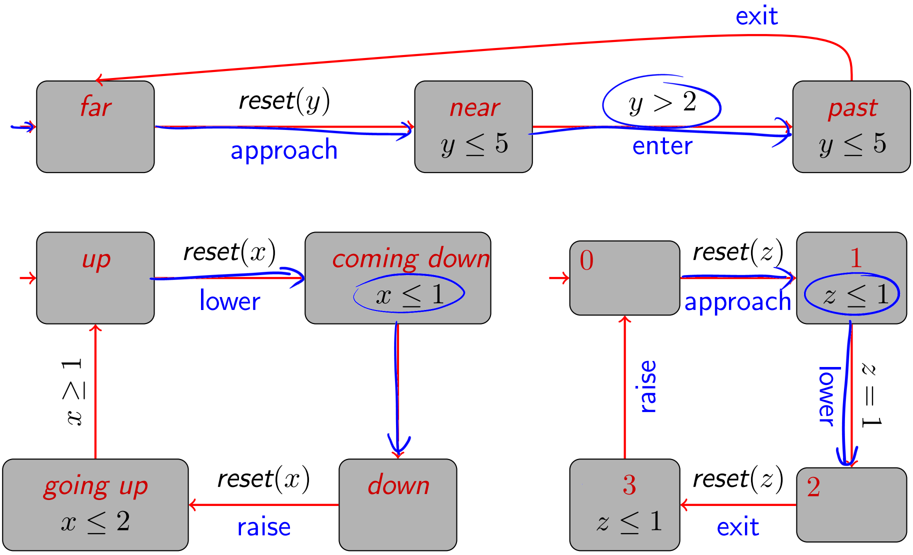
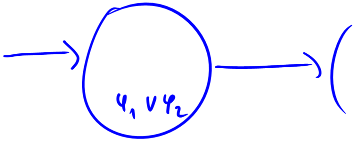
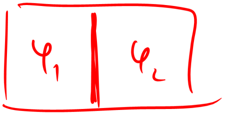
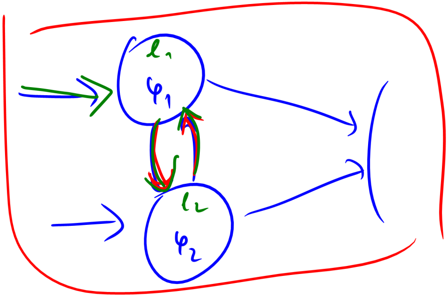
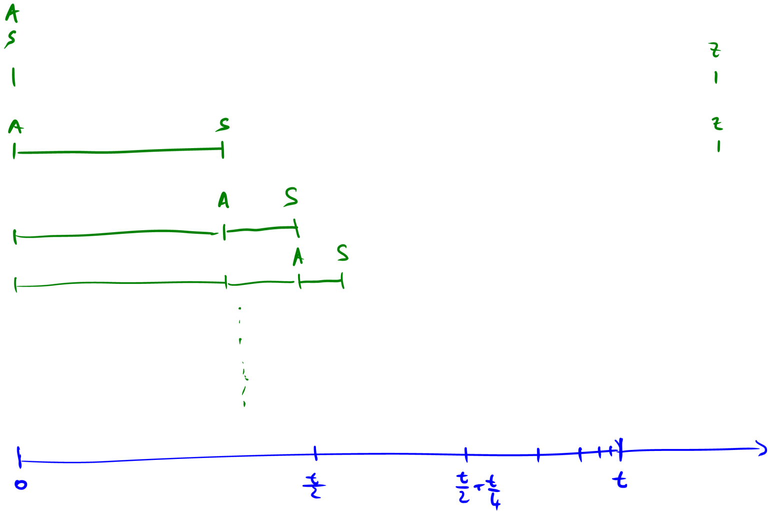


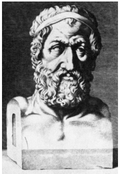

$\varphi_1 \vee \varphi_2$


φ_1
φ_2


ℓ_1
φ_1
ℓ_2
φ_2

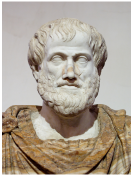

A
S
A
S
A
S
A
S
Z
Z
0
t/2
t/2 + t/4
t

Time divergence, timelock, and Zenoness



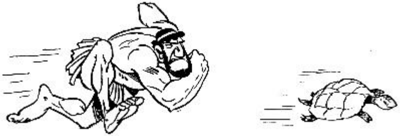
Zeno of Elea

(ca.490 BC-ca.430 BC)



Aristotle

(384 BC-322 BC)



Paradox:
Achilles and the tortoise

(Achilles was the great Greek hero of Homer's The Iliad.)

"In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point where the pursued started, so that the slower must always hold a lead." –Aristotle, Physics VI:9, 239b15

- Not all paths of a timed automata represent realistic behaviour.
- Three essential phenomena: time convergence, timelock, Zenoness.

Time convergence

Definition

For a timed automaton $\mathcal{T} = (Loc, \mathcal{C}, Lab, Edge, Inv, Init)$. we define $ExecTime : (Lab \cup \mathbb{R}^{\geq 0}) \to \mathbb{R}^{\geq 0}$ with

- $ExecTime(a) = 0$ for $a \in Lab$ and
- $ExecTime(d) = d$ for $d \in \mathbb{R}^{\geq 0}$.

Furthermore, for $\rho = s_0 \stackrel{\alpha_0}{\rightarrow} s_1 \stackrel{\alpha_1}{\rightarrow} s_2 \stackrel{\alpha_2}{\rightarrow} \ldots$ we define

$$ExecTime(\rho) = \sum_{i=0}^{\infty} ExecTime(\alpha_i).$$

A path is time-divergent iff $ExecTime(\rho) = \infty$, and time-convergent otherwise.

- Time-convergent paths are not realistic, and are not considered in the semantics.
- Note: their existence cannot be avoided (in general).

Timelock

Definition

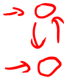
For a state $\sigma \in \Sigma$ let $Paths_{div}(\sigma)$ be the set of time-divergent paths starting in σ.
A state $\sigma \in \Sigma$ contains a timelock iff $Paths_{div}(\sigma) = \emptyset$.
A timed automaton is timelock-free iff none of its reachable states contains a timelock.

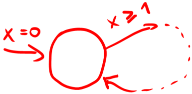
Timelocks are modelling flows and should be avoided.

Definition

An infinite path fragment π is Zeno iff it is time-convergent and infinitely many discrete actions are executed within π.
A timed automaton is non-Zeno iff no Zeno path starts in an initial state.

- Zeno paths represent non-realisable behaviour, since their execution would require infinitely fast processors.
- Though Zeno paths are modelling flows, they are not always easy to avoid.
- To check whether a timed automaton is non-Zeno is algorithmically difficult.
- Instead, sufficient conditions are considered that are simple to check, e.g., by static analysis.





Checking non-Zenoness

Theorem (Sufficient condition for non-Zenoness)

Let $\mathcal{T}$ be a timed automaton with clocks $\mathcal{C}$ such that for every control cycle

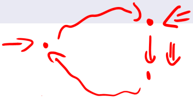
$$l_0 \overset{a_1:g_1,R_1}{\hookrightarrow} l_1 \overset{a_2:g_2,R_2}{\hookrightarrow} l_2 \ldots \overset{a_n:g_n,R_n}{\hookrightarrow} l_n = l_0$$

in $\mathcal{T}$ there exists a clock $x \in \mathcal{C}$ such that

- $x \in R_i$ *for some* $0 < i \leq n$, *and*
- *for all evaluations* $\nu \in V$ *there exist some* $0 < j \leq n$ *and* $d \in \mathbb{N}^{>0}$ *with*

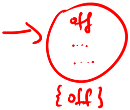
$$\nu(x) < d \quad \textit{implies} \quad (\nu \not\models Inv(l_j) \text{ or } \nu \not\models g_j).$$

Then $\mathcal{T}$ is non-Zeno.



Contents

- How to describe the behaviour of timed automata?
- Logic: TCTL, a real-time variant of CTL



- Syntax:

  State formulae

  $$\psi \quad ::= \quad \textit{true} \quad | \quad a \quad | \quad g \quad | \quad \psi \wedge \psi \quad | \quad \neg\psi \quad | \quad \mathbf{E}\varphi \quad | \quad \mathbf{A}\varphi$$

  Path formulae:

  $$\varphi \quad ::= \quad \psi \, \mathcal{U}^{J} \psi$$

  with $J \subseteq \mathbb{R}^{\geq 0}$ is an interval with integer bounds (open right bound may be ∞).
- Note: no next-time operator

TCTL syntax

Syntactic sugar:

$$
\begin{array}{lll}
\mathcal{F}^J \psi & := & \textit{true}\ \mathcal{U}^J\ \psi \\
\mathbf{E}\mathcal{G}^J \psi & := & \neg \mathbf{A}\mathcal{F}^J \neg \psi \\
\mathbf{A}\mathcal{G}^J \psi & := & \neg \mathbf{E}\mathcal{F}^J \neg \psi \\
\psi_1\ \mathcal{U}\ \psi_1 & := & \psi_1\ \mathcal{U}^{[0,\infty)}\ \psi_2 \\
\mathcal{F} \psi & := & \mathcal{F}^{[0,\infty)} \psi \\
\mathcal{G} \psi & := & \mathcal{G}^{[0,\infty)} \psi
\end{array}
$$

TCTL semantics

Definition (TCTL continuous semantics)

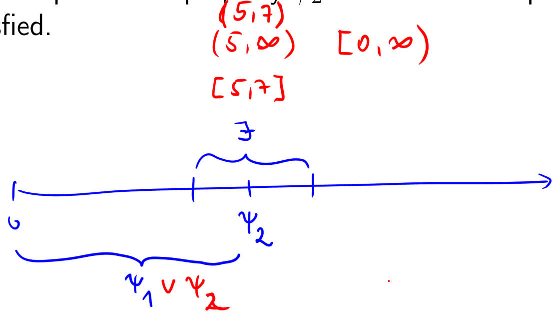
Let $\mathcal{T} = (Loc, \mathcal{C}, Lab, Edge, Inv, Init)$ be a timed automaton, AP a set of atomic propositions, and $L : Loc \to 2^{AP}$ a state labelling function. The function $\models$ assigns a truth value to each TCTL state and path formulae as follows:

$\models\ \subseteq (T \times \Sigma) \times TCTL$

$$
\begin{array}{llll}
\mathcal{T},\ \sigma & \models \mathit{true} & & \\
(\ell, \nu) = \sigma & \models a & \text{iff} & a \in L(\sigma) = L(\ell) \\
\sigma & \models g & \text{iff} & \sigma \models g \iff \nu(g) = \mathit{true} \\
\sigma & \models \neg\psi & \text{iff} & \sigma \not\models \psi \\
\sigma & \models \psi_1 \wedge \psi_2 & \text{iff} & \sigma \models \psi_1 \text{ and } \sigma \models \psi_2 \\
\sigma & \models \mathbf{E}\varphi & \text{iff} & \pi \models \varphi \text{ for some } \pi \in Paths_{div}(\sigma) \\
\sigma & \models \mathbf{A}\varphi & \text{iff} & \pi \models \varphi \text{ for all } \pi \in Paths_{div}(\sigma).
\end{array}
$$

where $\sigma \in \Sigma$, $a \in AP$, $g \in ACC(\mathcal{C})$, ψ, ψ_1 and ψ_2 are TCTL state formulae, and φ is a TCTL path formula.

Meaning of $\mathcal{U}$: a time-divergent path satisfies $\psi_1 \; \mathcal{U}^J \; \psi_2$ whenever at some time point in J property ψ_2 holds and at all previous time instants ψ_1 is satisfied.



TCTL semantics

Definition (TCTL continuous semantics)

For a time-divergent path $\pi = (\ell_0, \nu_0) \xrightarrow{\alpha_0} (\ell_1, \nu_1) \xrightarrow{\alpha_1} \ldots$ we define $\pi \models \psi_1 \, \mathcal{U}^J \, \psi_2$ iff

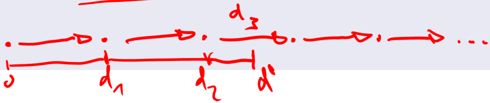
- $\exists i \geq 0. \; (\ell_i, \nu_i + d) \models \psi_2$ for some $d \in [0, d_i]$ with

$$(\sum_{k=0}^{i-1} d_k) + d \in J, \textit{ and}$$

- $\forall j \leq i. \; (\ell_j, \nu_j + d') \models \psi_1$ for any $d' \in [0, d_j]$ with

$$(\sum_{k=0}^{j-1} d_k) + d' \leq (\sum_{k=0}^{i-1} d_k) + d$$

where $d_i = \mathit{ExecTime}(\alpha_i)$.



Satisfaction set

Definition

For a timed automaton $\mathcal{T}$ with clocks $\mathcal{C}$ and locations Loc, and a TCTL state formula ψ the satisfaction set $Sat(\psi)$ is defined by

$$Sat(\psi) = \{s \in \Sigma \mid s \models \psi\}.$$

$\mathcal{T}$ satisfies ψ iff ψ holds in all initial states:

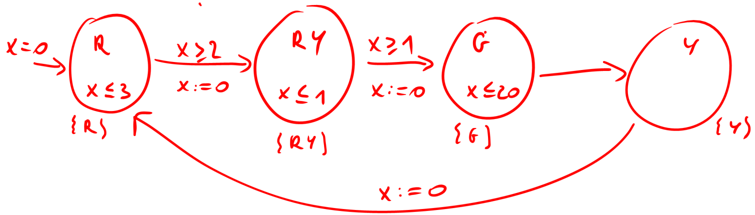
$$\mathcal{T} \models \psi \quad \textit{iff} \quad \forall l_0 \in Init.\ (l_0, \nu_0) \models \psi$$

where $\nu_0(x) = 0$ for all $x \in \mathcal{C}$.

TCTL vs. CTL

- TCTL formulae with intervals $[0, \infty)$ may be considered as CTL formulae
- However, there is a difference due to time-convergent paths
- TCTL ranges over time-divergent paths, whereas CTL over all paths!

$T:$



$T \overset{?}{\models} A\,(R \vee RY)\,\mathcal{U}^{\leq 4}\, G$ ✓

$T \overset{?}{\models} A\,F\,Y$ ✓

$T \models A\,G\,A\,F\,Y$ ✓

$T \overset{?}{\models} A\,F\,x \geq 1$

$T \models A\,G\,A\,F^{=1}\,\text{true}$

$\quad\; A\,G\,E\,F^{=1}\,\text{true}$

$$\frac{v' \models Inv(\ell) \quad v' = v + t \quad t > 0}{(\ell, v) \xrightarrow{t} (\ell, v')}$$

$$\underbrace{x = 0 \xrightarrow{1/2} x = \tfrac{1}{2} \xrightarrow{1/4} x = \tfrac{3}{4} \rightarrow \ldots}_{R}$$