Practical Course: SMT Solving

SMT Solving for Equality Logic and Uninterpreted Functions

Erika Ábrahám, Rebecca Haehn, Gereon Kremer, Stefan Schupp

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Satisfiability Checking

Propositional logic formula in CNF

Let $p_1, \ldots, p_n$ be Boolean variables (or propositions).

$$\varphi = \bigwedge_{i=1}^{m} \underbrace{\bigvee_{j=1}^{m_i} l_{ij}}_{\text{clause}}, \text{ with literals } l_{ij} \text{ with } l_{ij} = p_k \text{ or } l_{ij} = \neg p_k.$$

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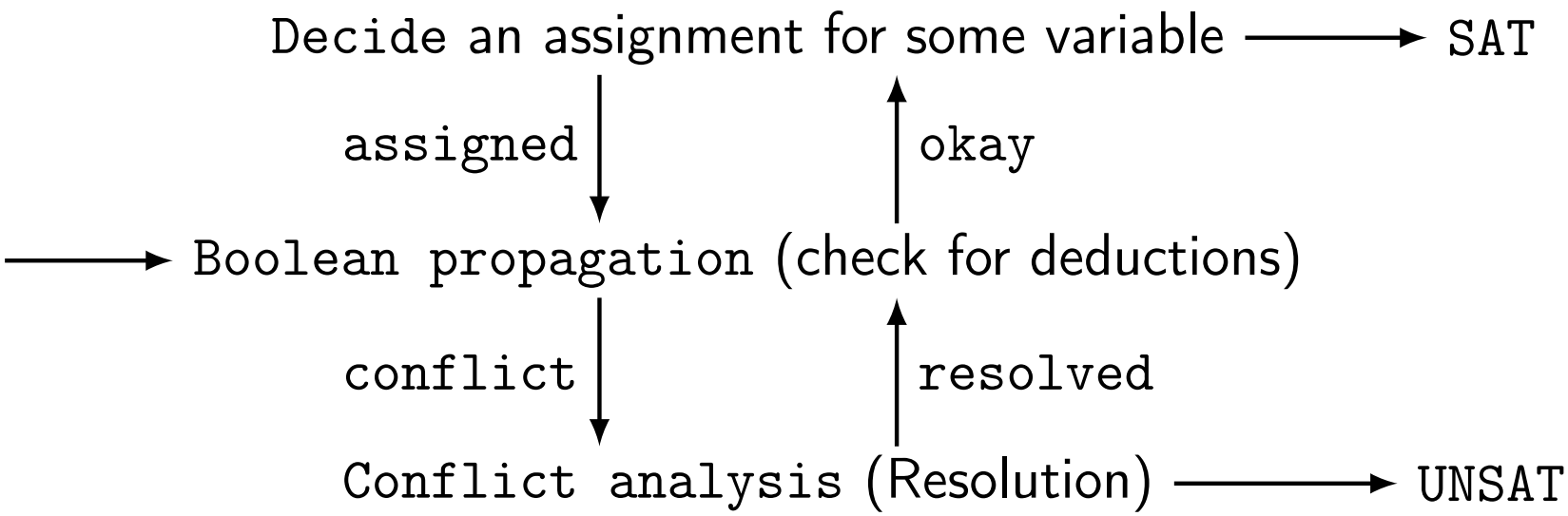
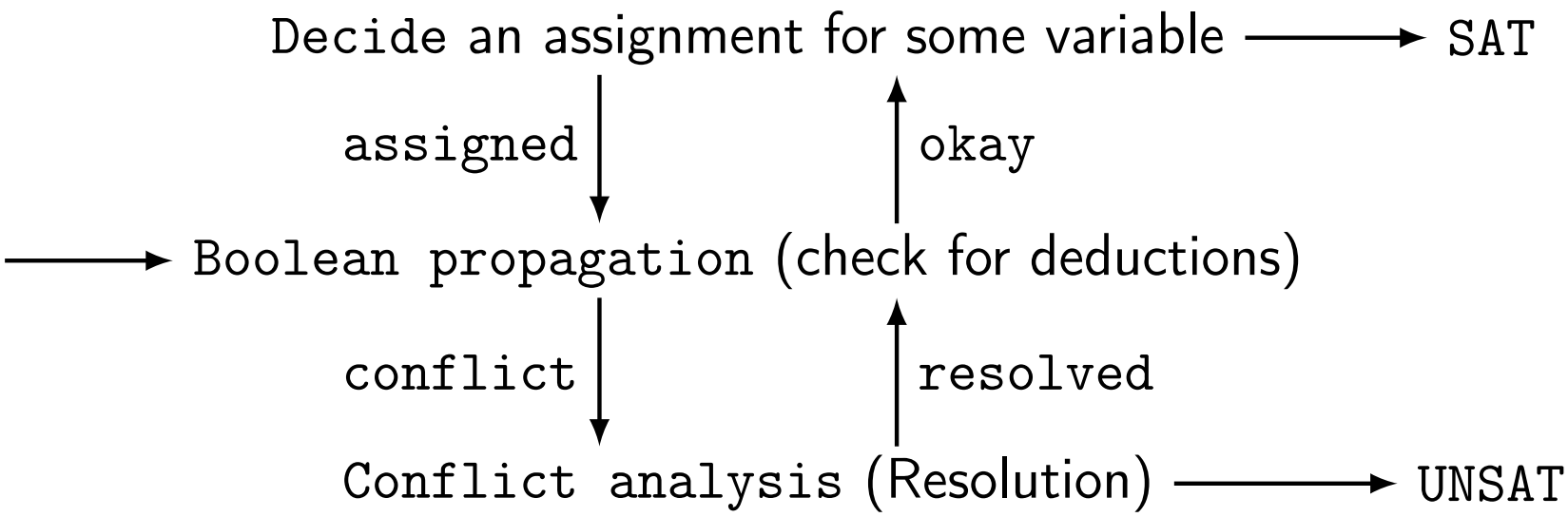
  $\to$ Davis-Putnam-Logemann-Loveland (DPLL) algorithm
  $\to$ Conflict-Driven-Clause-Learning (CDCL) algorithm

CDCL Algorithm



CDCL Algorithm



$$\begin{aligned}\varphi &= c_1 \wedge c_2 \wedge c_3 \wedge c_4 \\ &= p_1 \wedge (p_2 \vee p_3) \wedge (\neg p_3 \vee p_4) \wedge (\neg p_1 \vee p_2 \vee \neg p_4)\end{aligned}$$

$$\alpha = \emptyset$$

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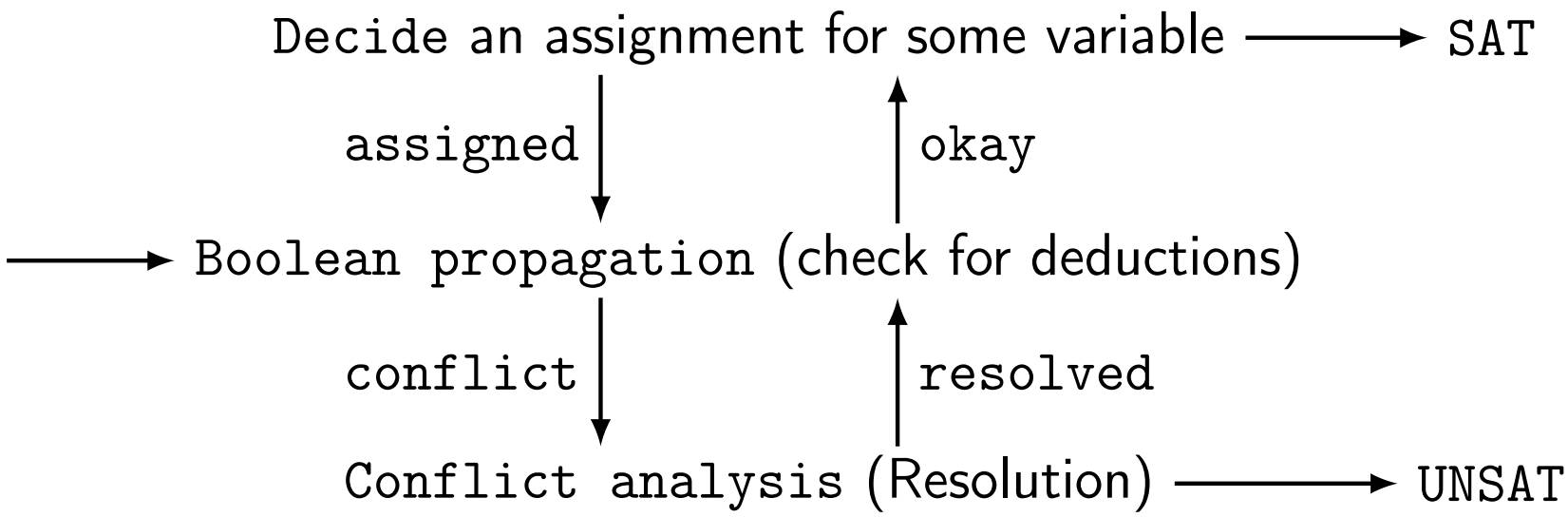

$$\varphi = c_1 \wedge c_2 \wedge c_3 \wedge c_4$$
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$$\alpha = \{p_1 \mapsto \mathit{true}\}$$

- ▷ Propagate

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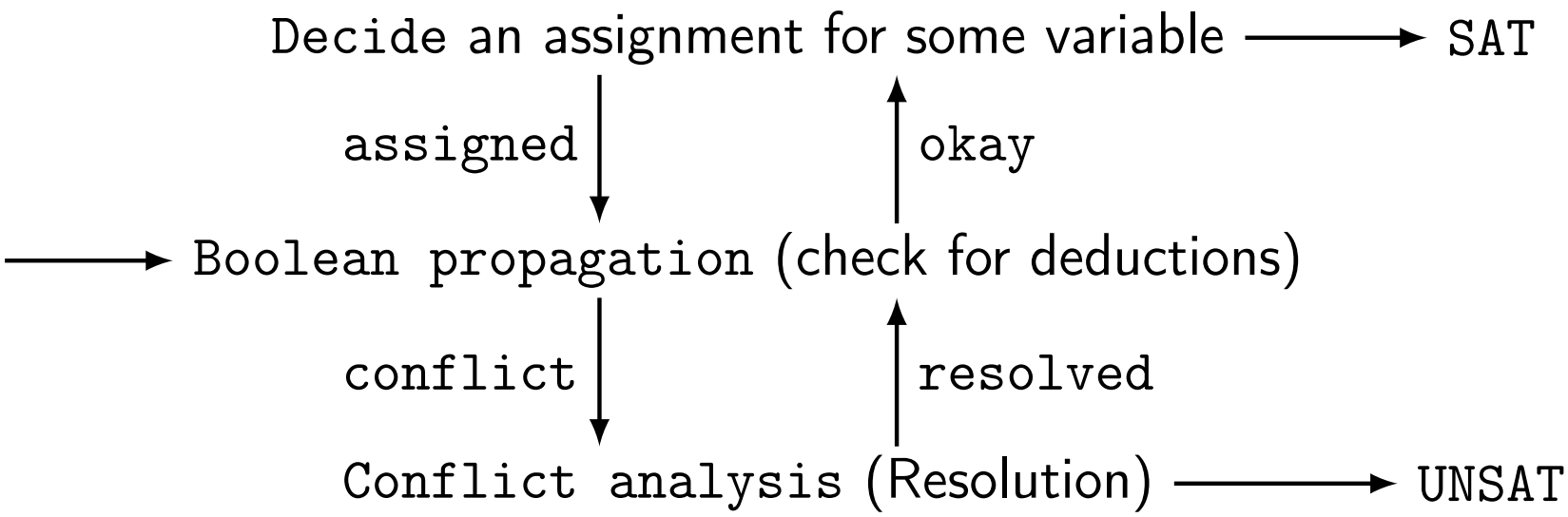

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$$= (p_2 \vee p_3) \wedge (\neg p_3 \vee p_4) \wedge (p_2 \vee \neg p_4)$$

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- ▷ Decide

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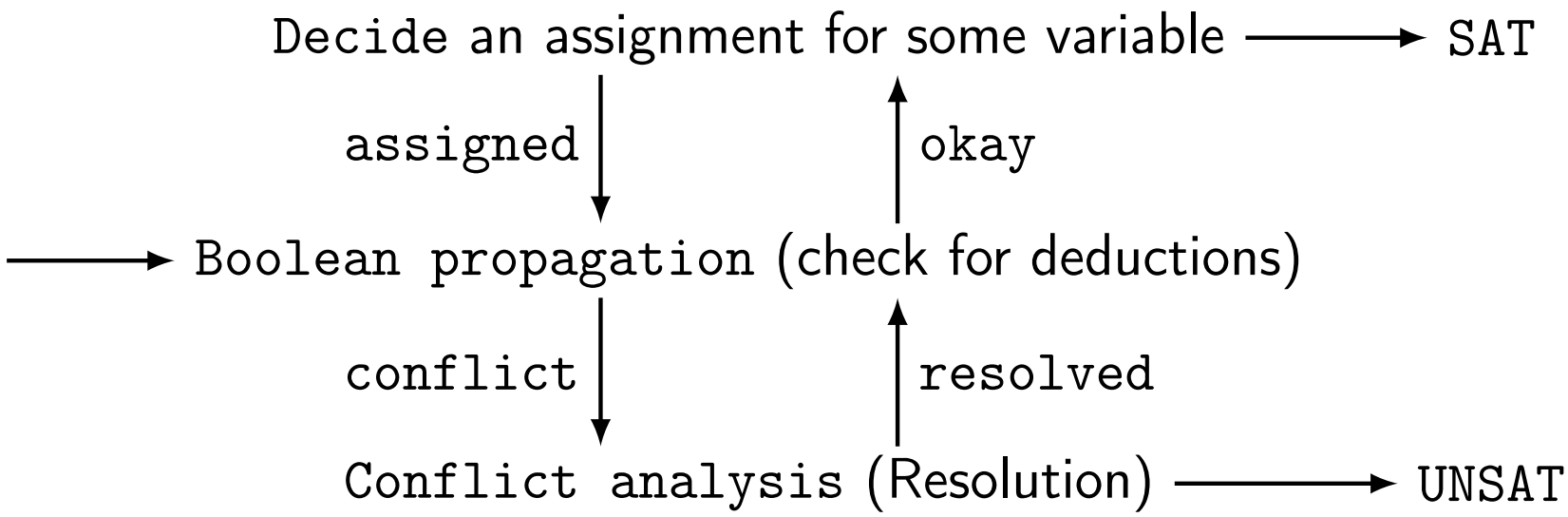

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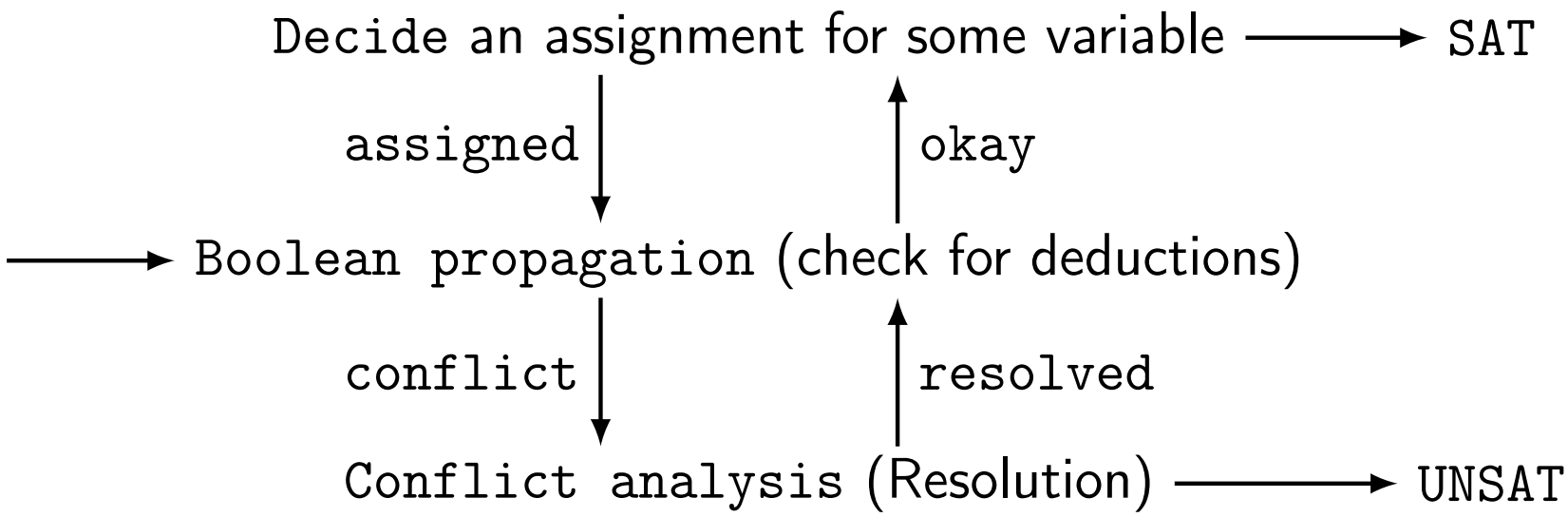

$$\varphi = c_3 \wedge c_4$$
$$= (\quad p_4) \wedge (\quad \neg p_4)$$

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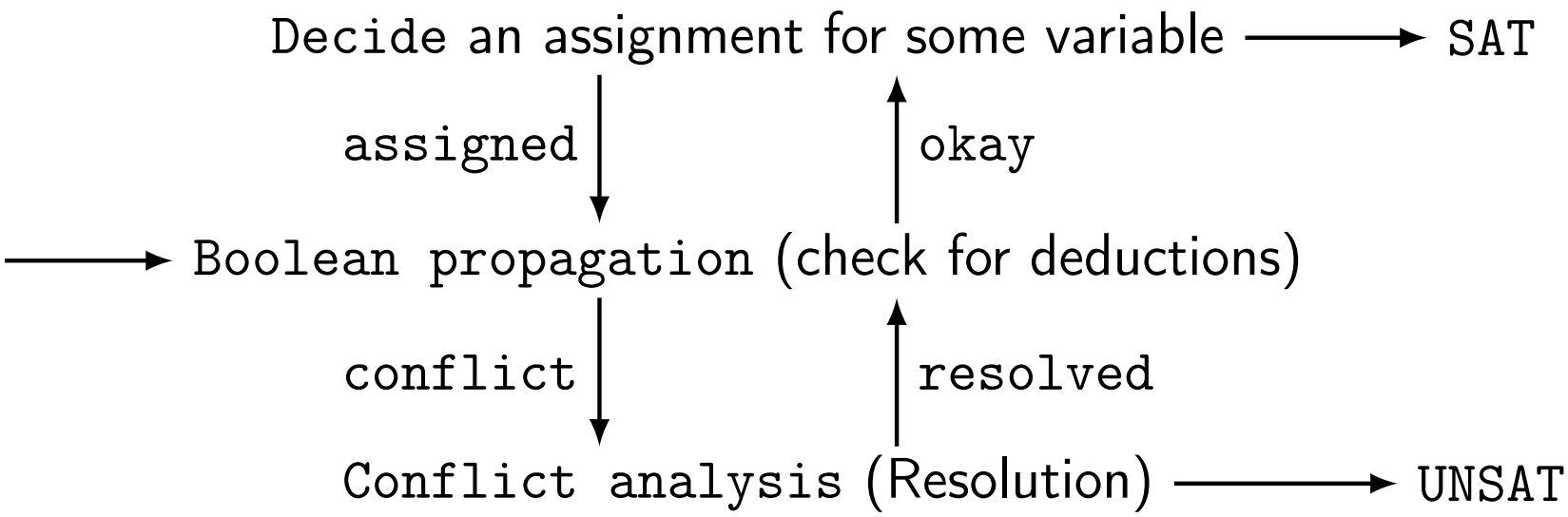

$\varphi \;=\; c_4$

$\;=\; \mathit{false}$

$\alpha \;=\; \{p_1 \mapsto \mathit{true},\; p_2 \mapsto \mathit{false},\; p_3 \mapsto \mathit{true},\; p_4 \mapsto \mathit{true}\}$

▷ `Conflict!` Conflict clause: $c_5 = \texttt{Resolvent}(c_3, c_4) = \neg p_1 \vee p_2 \vee \neg p_3$

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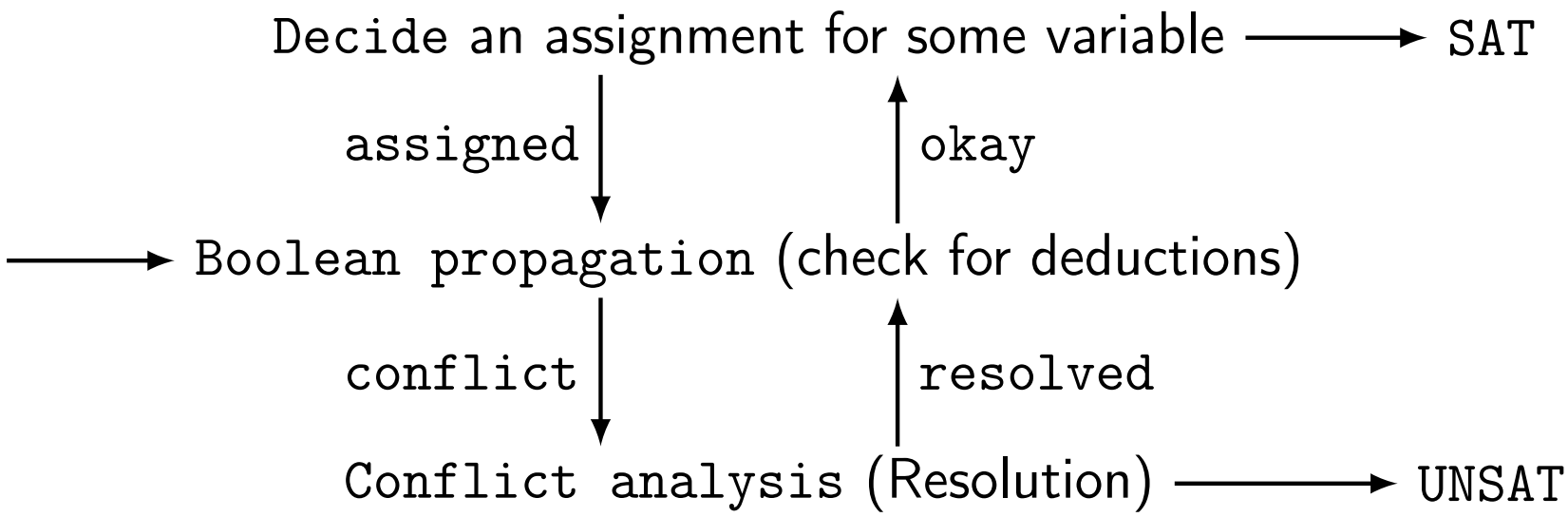

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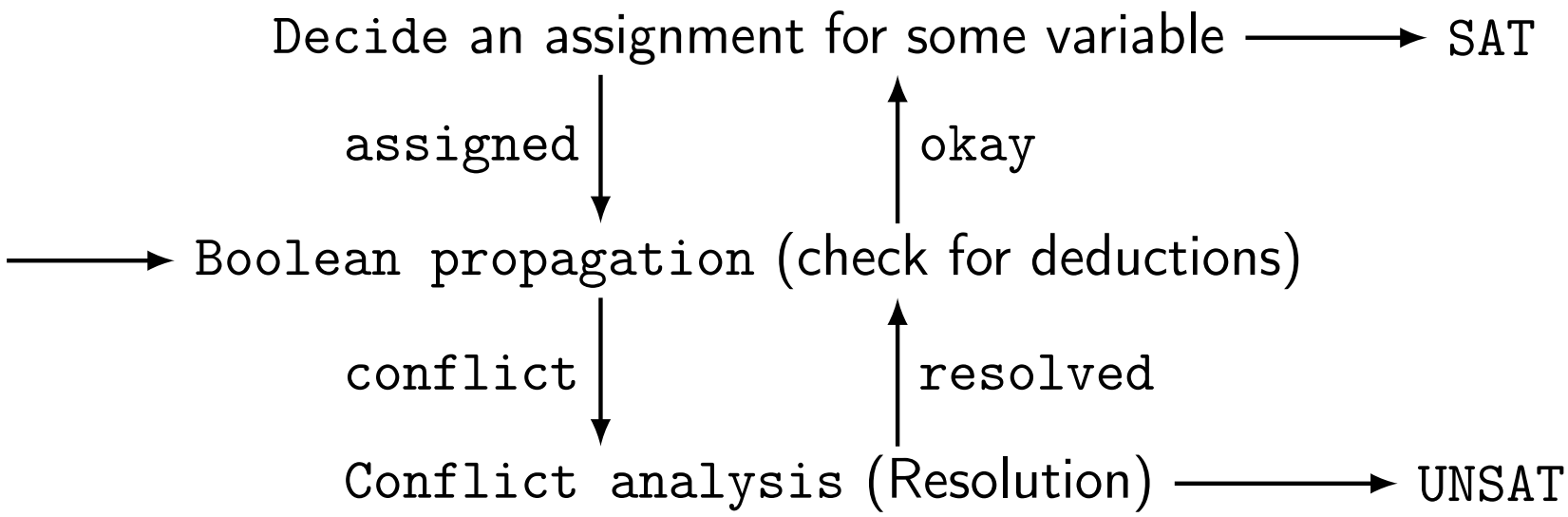
- ▷ Resolution

CDCL Algorithm



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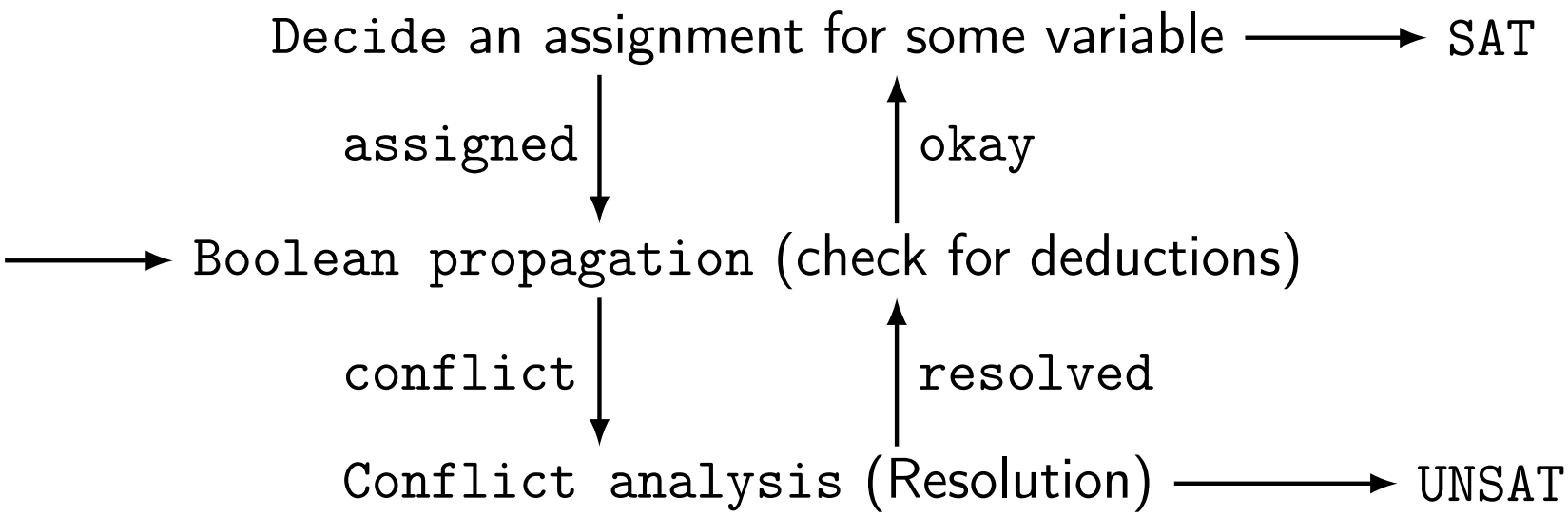

$\varphi = \mathit{true}$

$\alpha = \{p_1 \mapsto \mathit{true},\ p_2 \mapsto \mathit{true},\ p_3 \mapsto \mathit{true},\ p_4 \mapsto \mathit{true}\}$

▷ SAT

Equality Logic

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- constraints can also be equalities between constants and variables with values from some domain, e.g. $\varphi = a = b \wedge b = c \wedge a \neq c, a, b, c \in D$
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  $\rightarrow$ **Equality logic**
- additionally to constants and variables, function symbols can be used, e.g. $\varphi = (f(a) = b \wedge b < f(c)) \Rightarrow f(a) = f(c), a, b, c \in D, f$ function symbol
  $\rightarrow$ **Equality logic with uninterpreted functions**

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e.g. $a \neq b \wedge f(a) = f(b)$ is a satisfiable formula but
$a = b \wedge f(a) \neq f(b)$ is unsatisfiable

Satisfiability Modulo Theories

Satisfiability modulo theories formula in CNF

$$\varphi = \bigwedge_{i=1}^{n} \underbrace{\bigvee_{j=1}^{m_i} c_{ij}}_{clause}, \text{ with constraints } c_{ij} \text{ over some theory.}$$

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- SAT modulo theories problem (SMT): is a given φ satisfiable?

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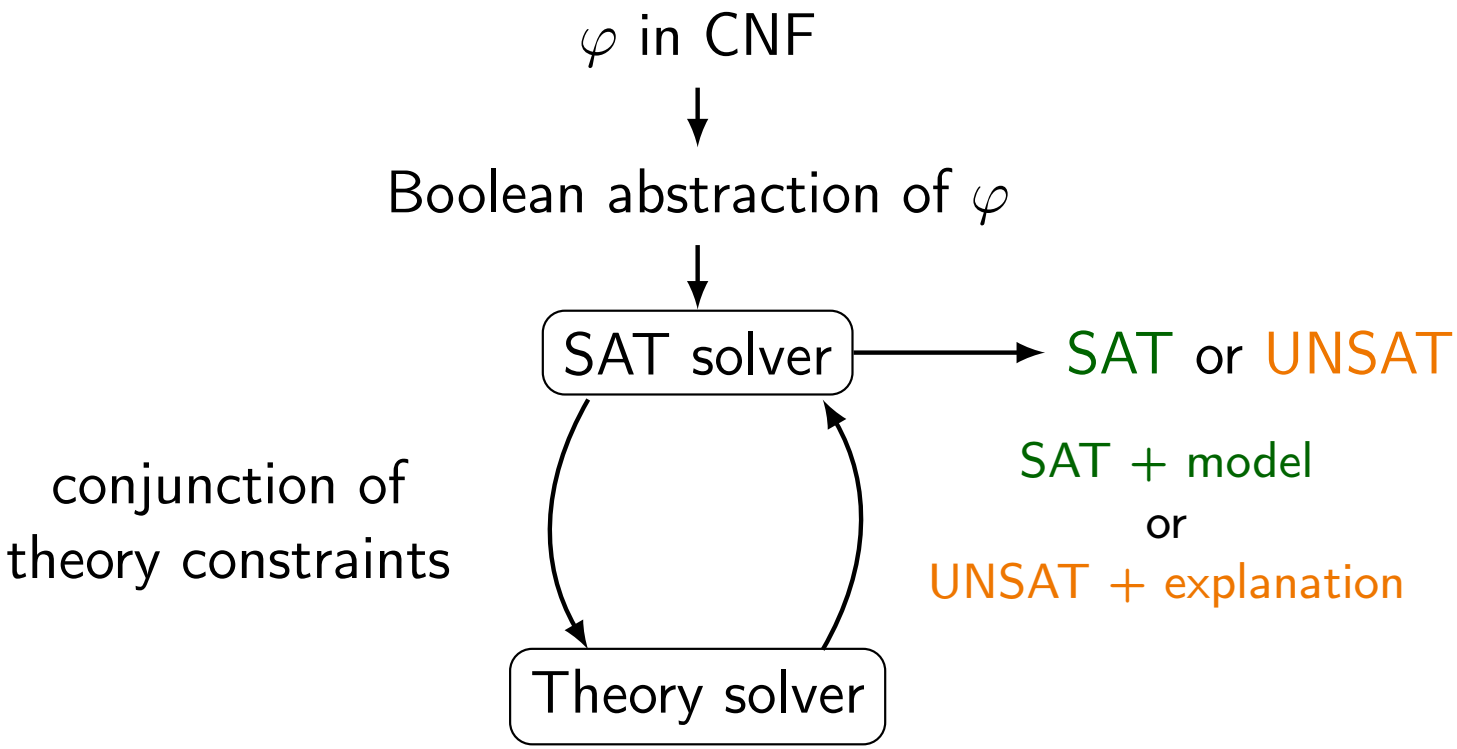
Lazy: combine decision procedures to solve SMT problems.



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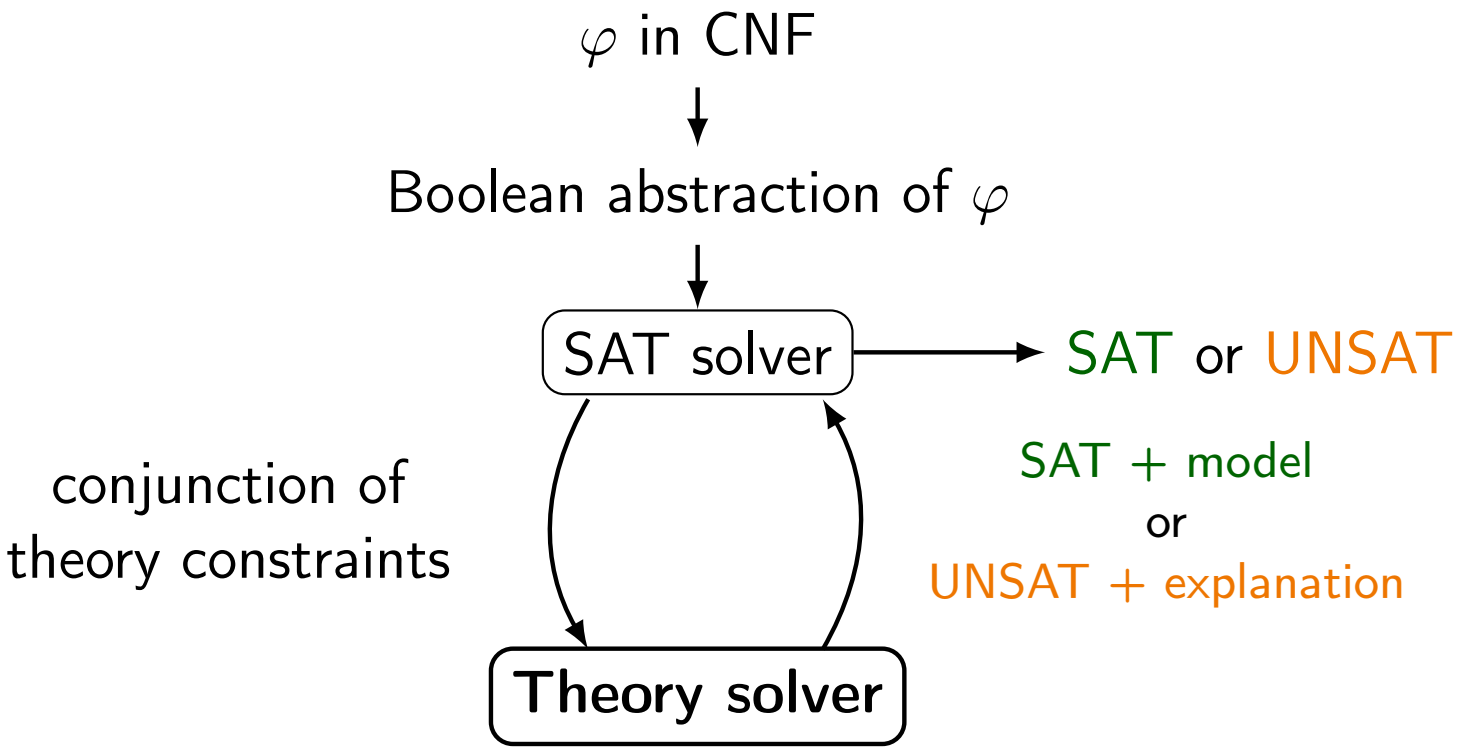

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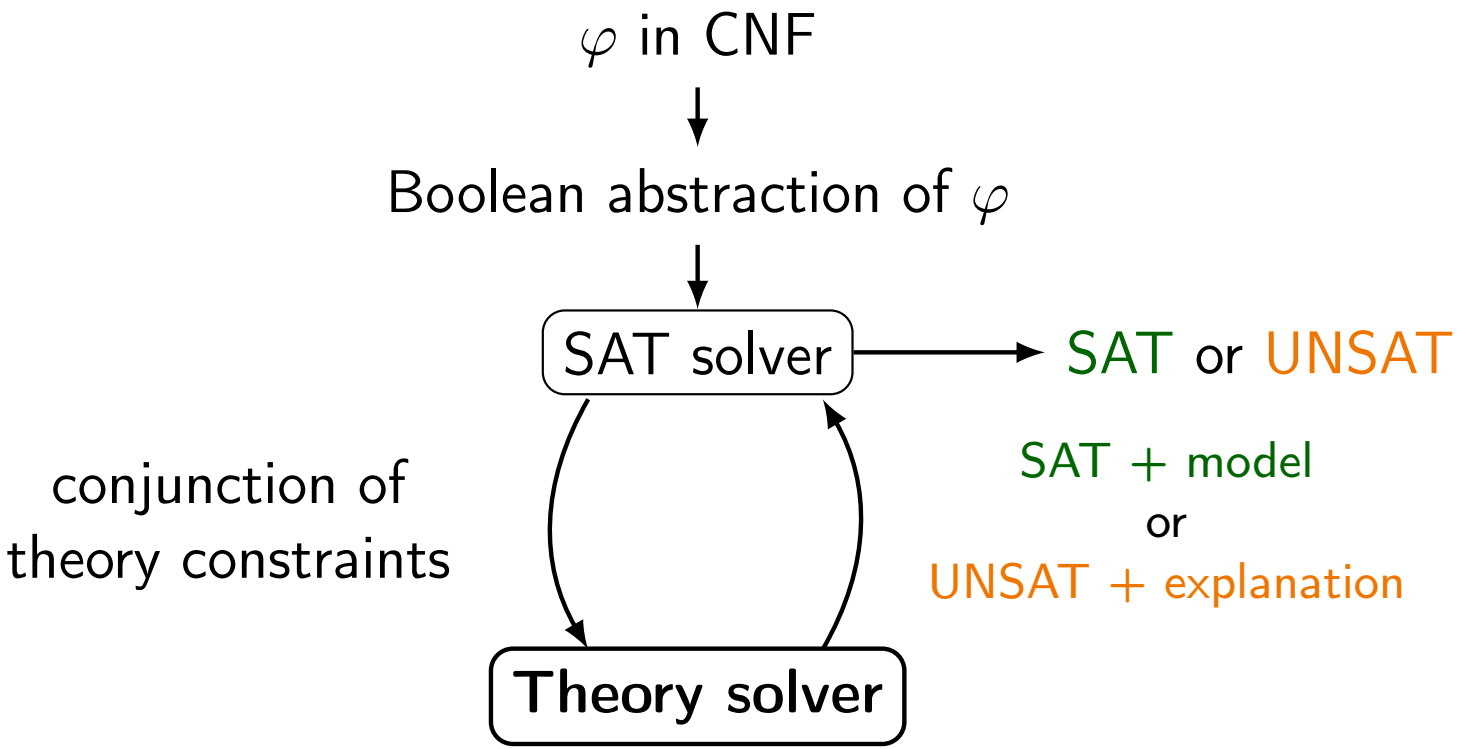

- full lazy: theory calls only for full Boolean solutions
- **less** lazy: theory call after every conflict-free `Propagate` execution

SMT-RAT

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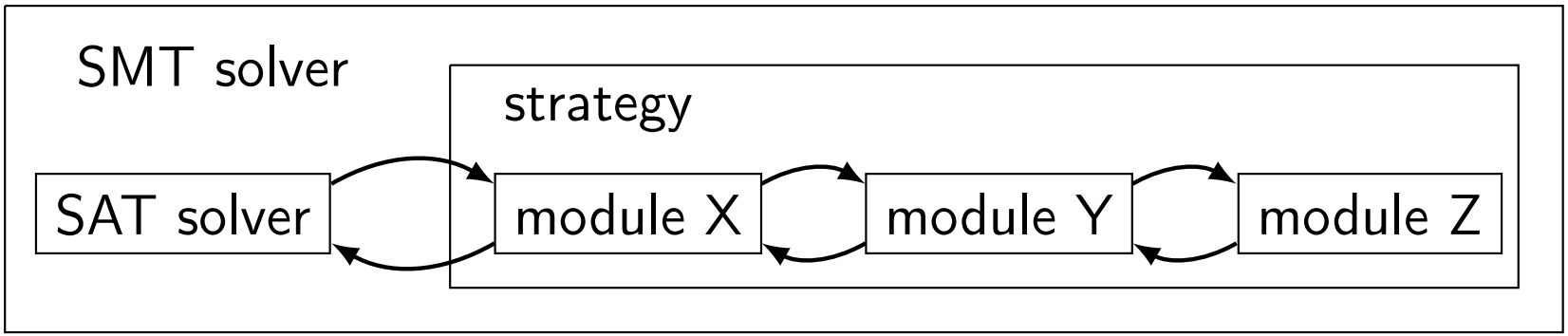
- consists of **modules**
- modules can be composed to an SMT solver according to a **strategy**



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- can give sets of formulas to other modules according to the strategy: `addSubformulaToPassedFormula`, `removeSubformula` and `runBackends`

Theory Solver

- for *conjunctions / sets* of equations and disequations over terms

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Theory solver input

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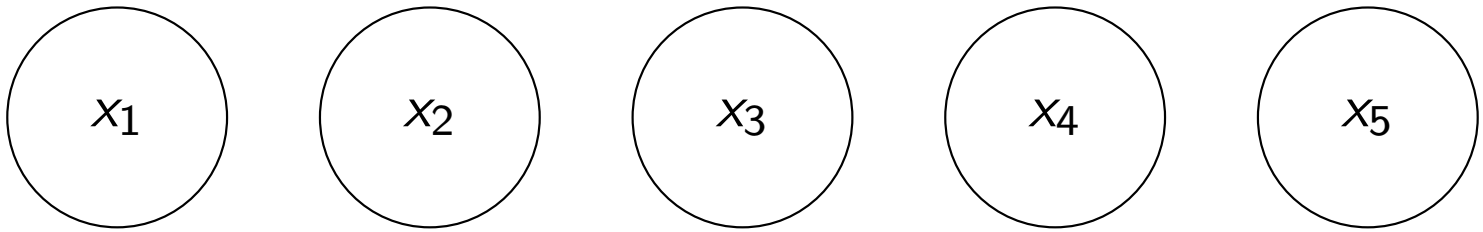
We assume an infinite domain. Assure reflexivity



all subterms

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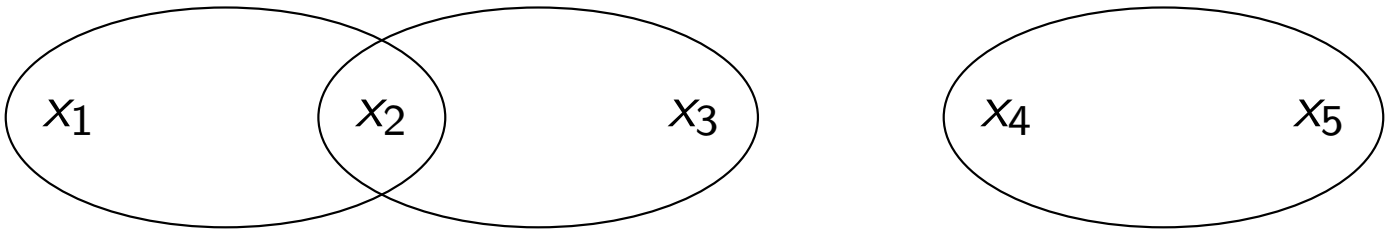

consider equalities

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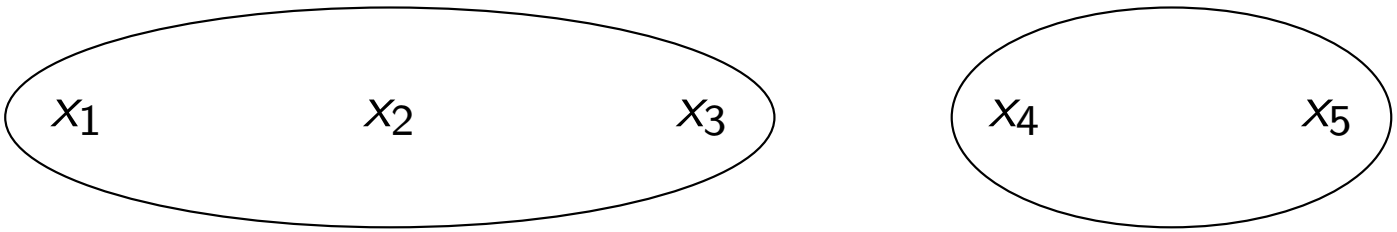

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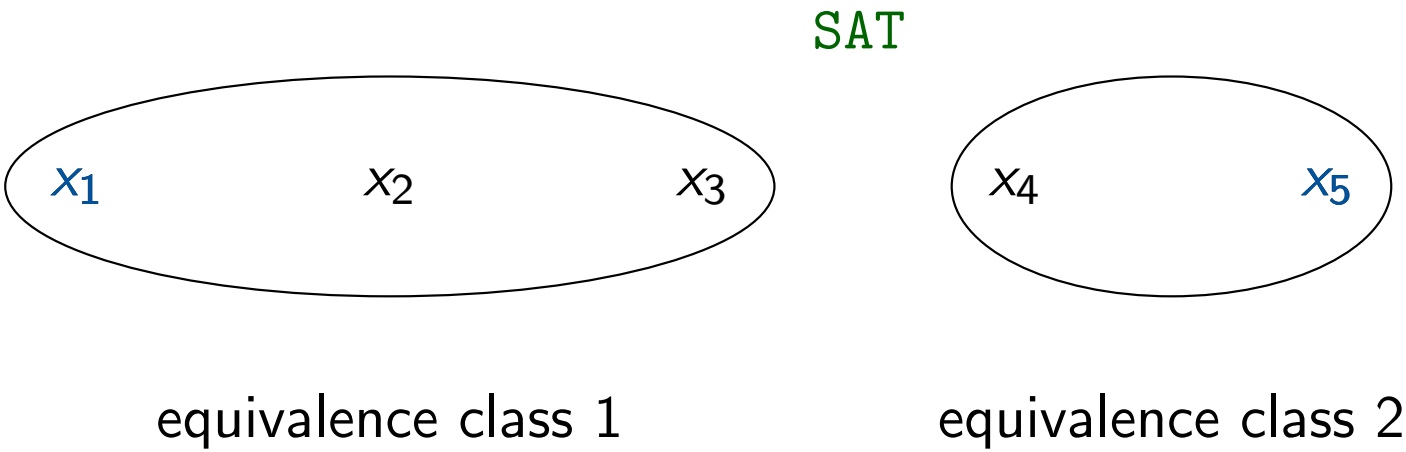

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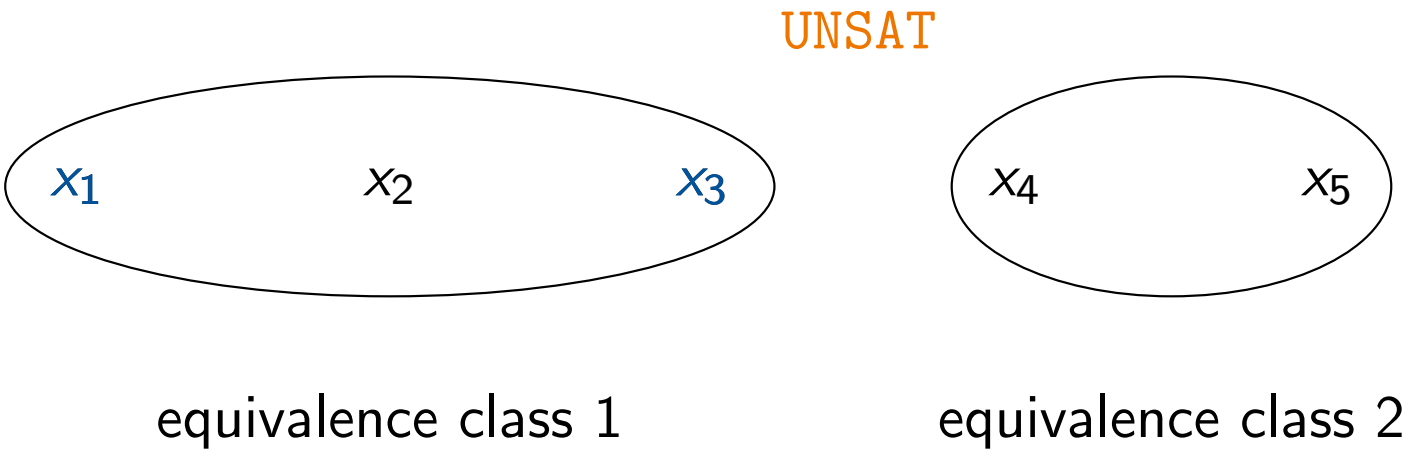

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EL + uninterpreted functions:

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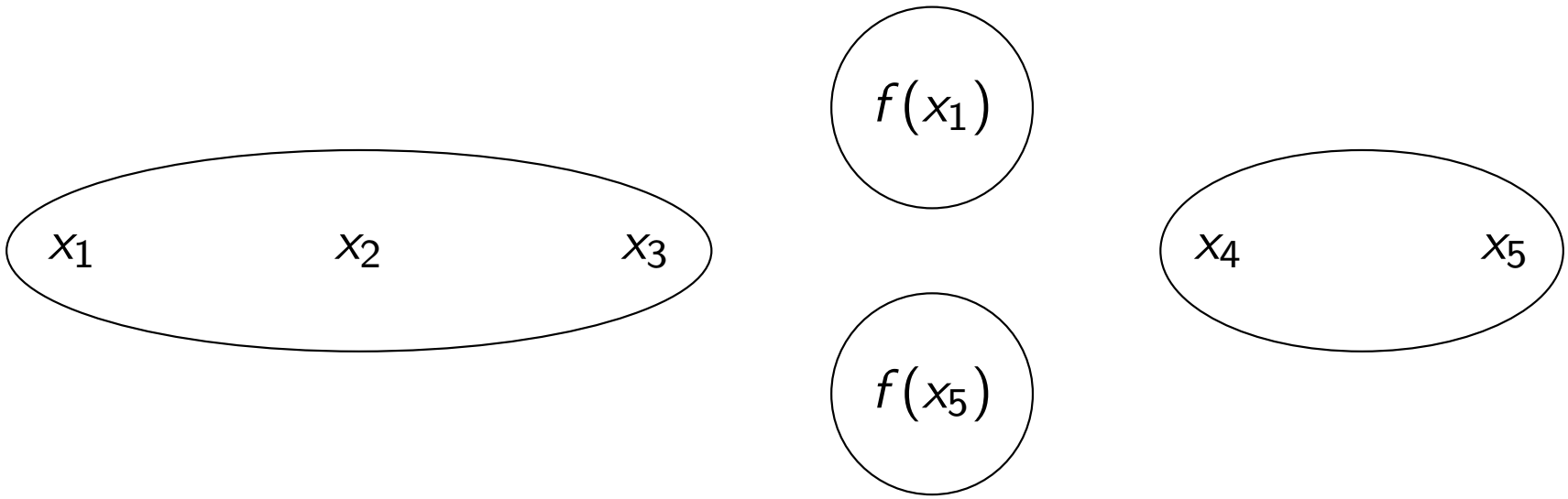

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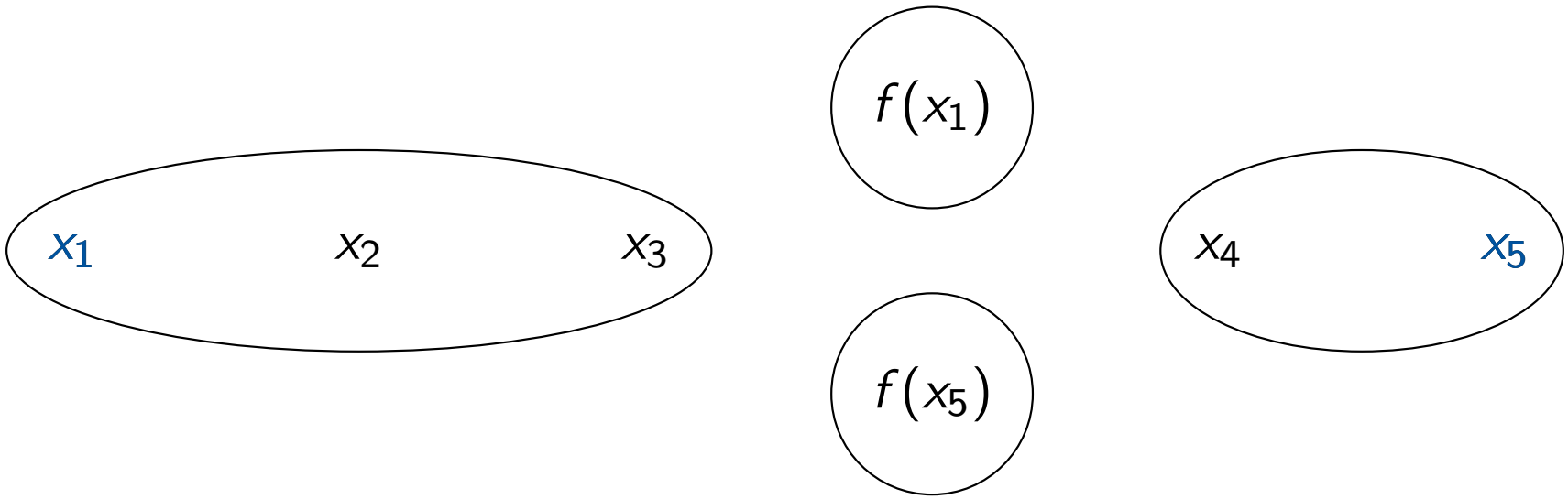

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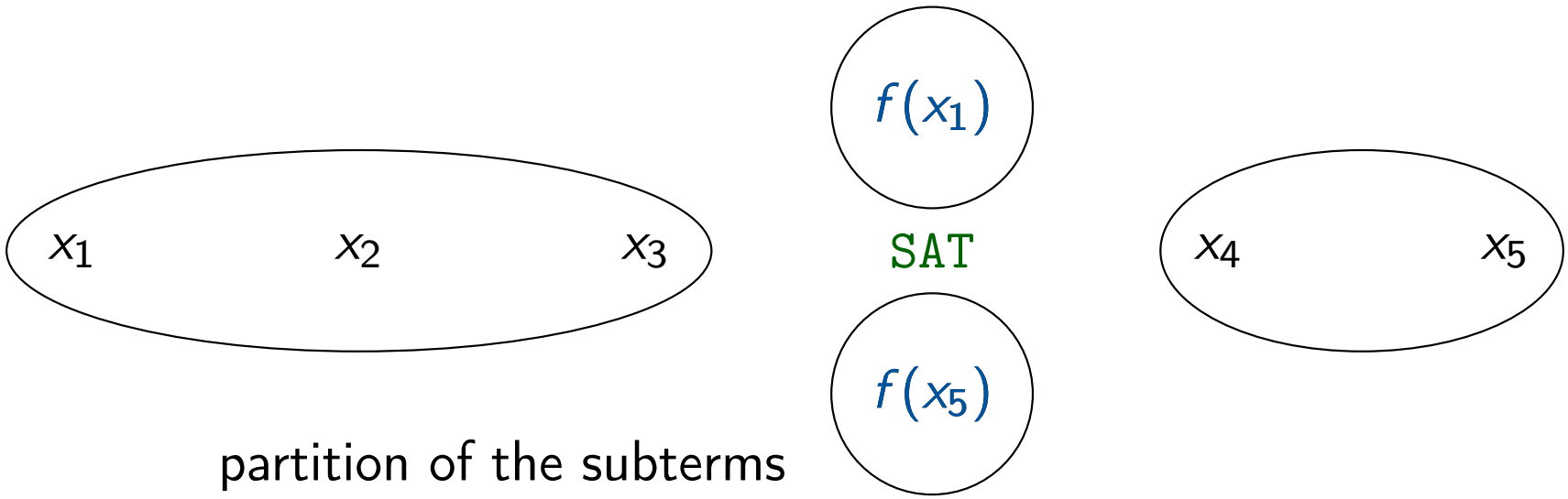

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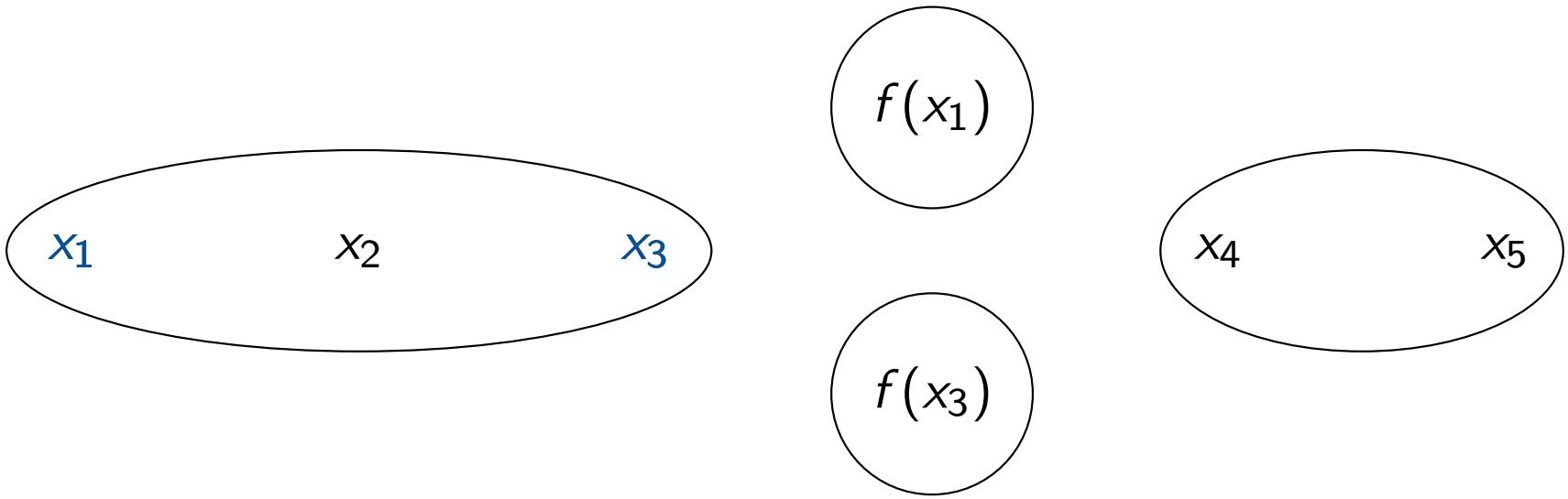

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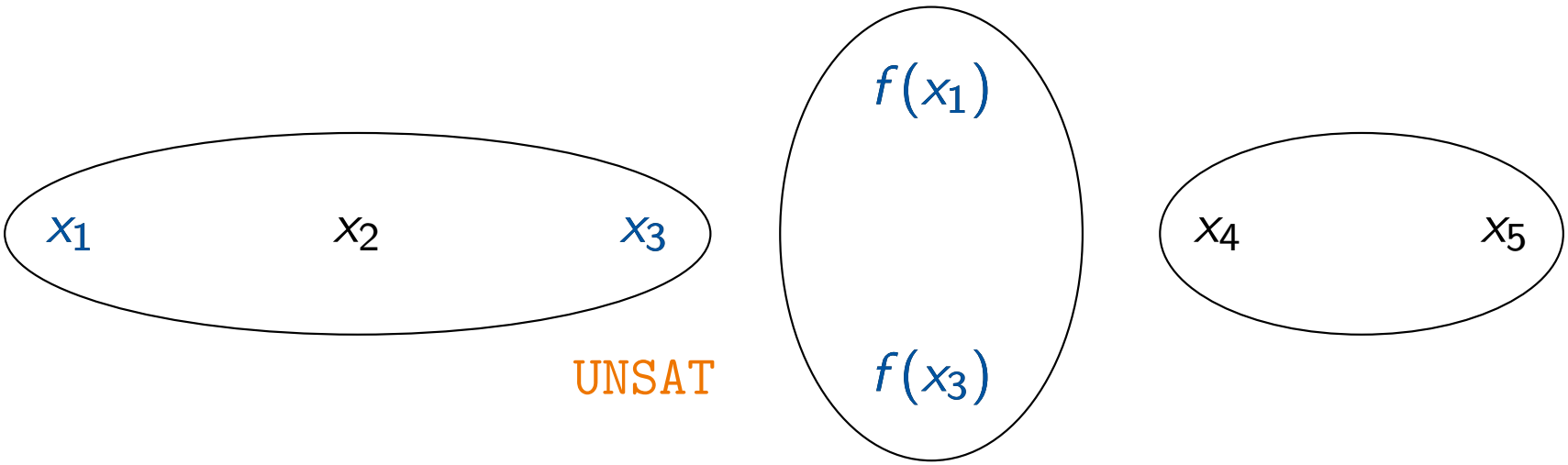


Initialization

Algorithm: `initialize`

Input: Set T of all subterms that can be used in (dis)equations
Output: -

1. no constraints received yet:
   $E := \emptyset$;
2. initial partition over T:
   $\mathcal{C} := \{\{t\} \mid t \in T\}$;
3. current state of satisfiability:
   $state := SAT$;

Equations

Algorithm: `addEquation`

Input: Equation $t_1 = t_2$ with subterms from T
Output: Satisfiability of the conjunction of all received (and not yet removed) equations and disequations

1. remember equation: $E := E \cup \{t_1 = t_2\}$;
2. `if` $(state = UNSAT)$ `then return UNSAT;`
3. update partition and re-check satisfiability of disequations:

```
if ([t1] ≠ [t2]) then
  C := (C \ {[t1], [t2]}) ∪ {[t1] ∪ [t2]};
  while ∃ F(t), F(t') ∈ T : [t]=[t'] ∧ [F(t)]≠[F(t')]
    C := (C \ {[F(t)], [F(t')]}) ∪ {[F(t)] ∪ [F(t')]};
  for each (t ≠ t') ∈ E
    if ([t] = [t']) then {state := UNSAT; return UNSAT;}
return SAT;
```

Disequations

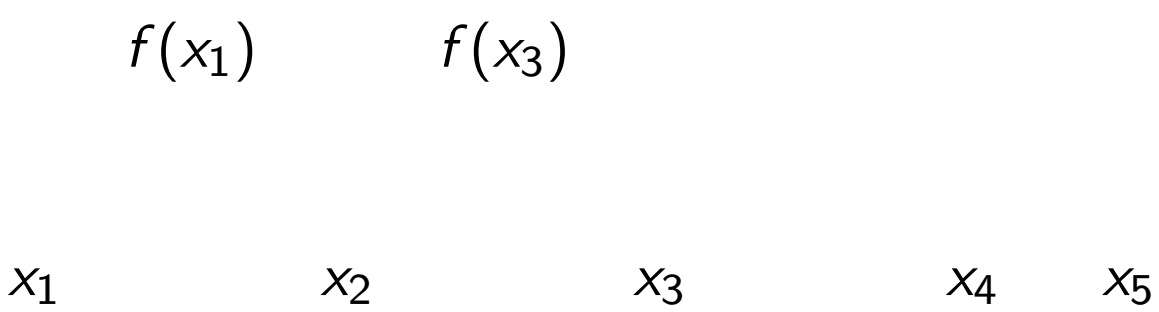
Algorithm: `addDisequation`

Input: Disequation $t_1 \neq t_2$ with subterms from T
Output: Satisfiability of the conjunction of all received (and not yet removed) equations and disequations

1. remember disequation:
   $E := E \cup \{t_1 \neq t_2\}$;
2. `if` $(e = \mathit{UNSAT})$ `then return UNSAT;`
3. check satisfiability of disequation:
   `if` $([t_1] = [t_2])$ `then`
   $\mathit{state} := \mathit{UNSAT}$;
   `return UNSAT;`
   `return SAT;`

E-Graph

- $x_1 = x_2 \wedge x_2 = x_3 \wedge x_4 = x_5 \wedge f(x_1) \neq f(x_3)$



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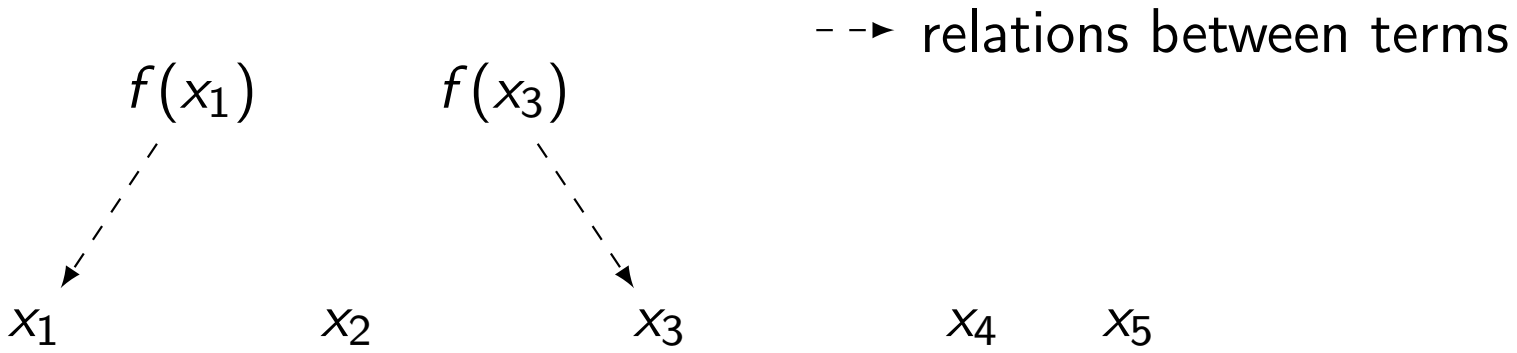

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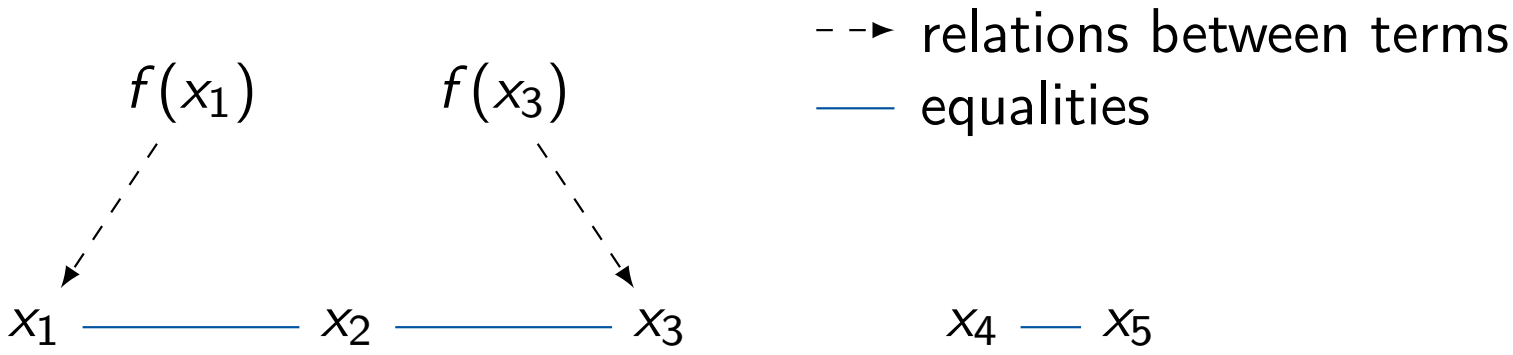

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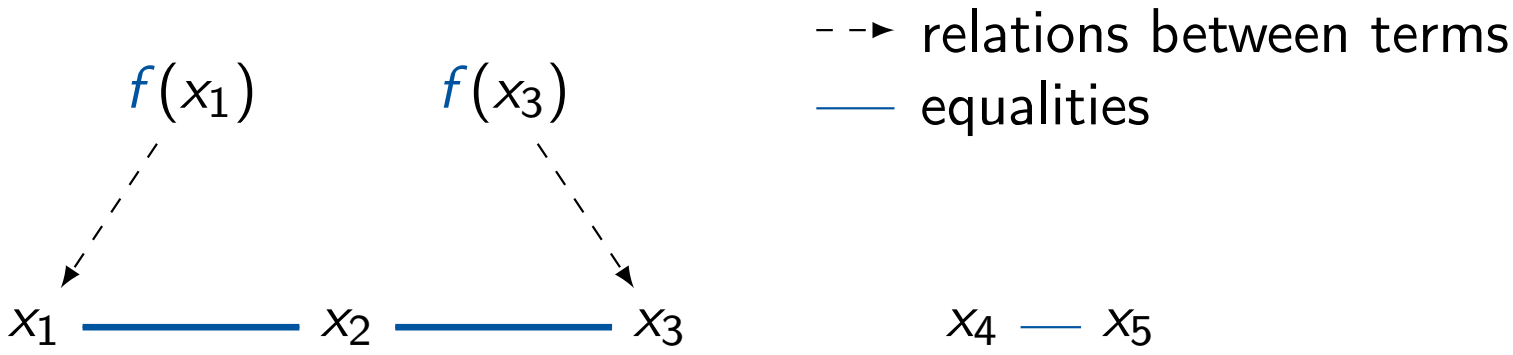

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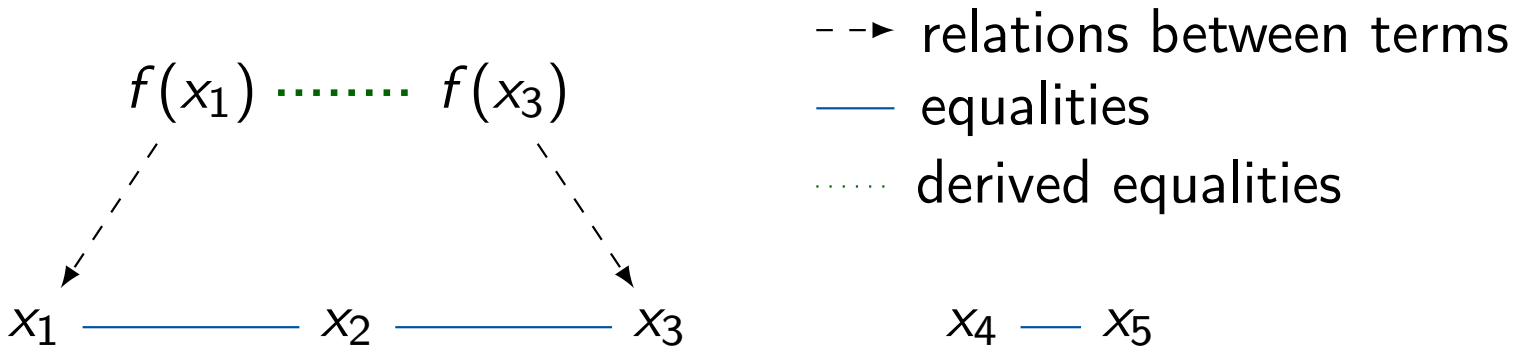

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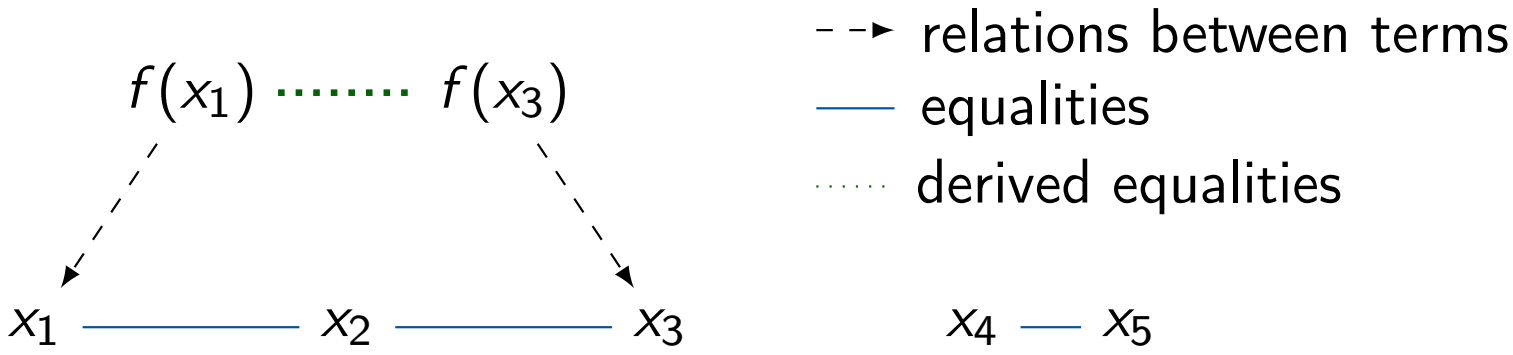
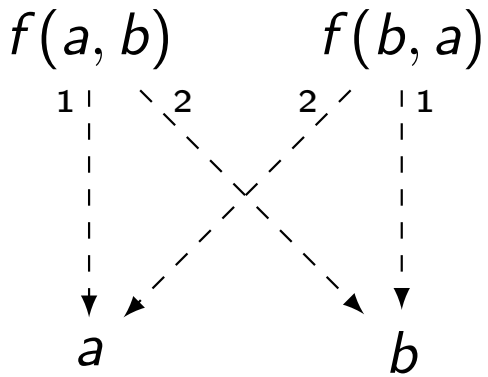


- Consider that $f(a, b) \neq f(b, a)$ is satisfiable!



Boost Graph Library

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- examples for both can be found on the respective pages in the above linked documentation

Models

Models (satisfying assignments)

- can be inferred from the E-graph

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Models enumerate equivalence classes and assign each variable and uninterpreted function the number of their corresponding equivalence class.

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- `Bool` is a finite domain!
  E.g. $a \neq b \wedge b \neq c \wedge c \neq a$ is unsatisfiable for $a,\ b,\ c \in$ `Bool`, although there is no explicit equality.

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- later more about *minimal* infeasible subsets

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Algorithm: `backtrack`

Input: Equation and disequation set S
Output: -

1. remove the equations and disequations: $E := E \setminus S$;
2. compute satisfiability for T and the conjunction of all equations and disequations from E
3. remember satisfiability result in `state`
4. return satisfiability result

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