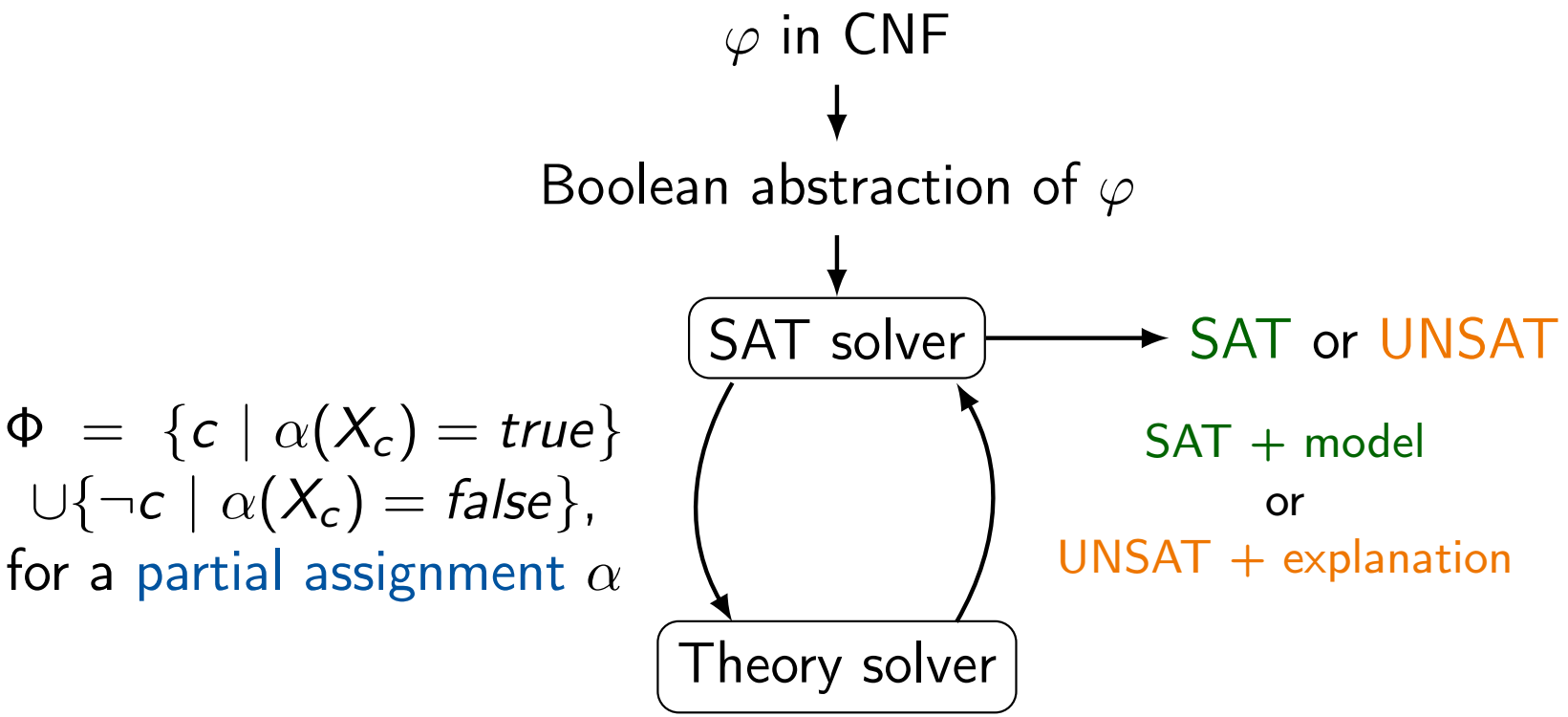
Practical Course: SMT Solving

Incrementality, Backtracking and Minimal Infeasible Subsets

Erika Ábrahám, Rebecca Haehn, Gereon Kremer, Stefan Schupp

Winter term 2018/19

Reminder: Less Lazy SMT Solving + DPLL



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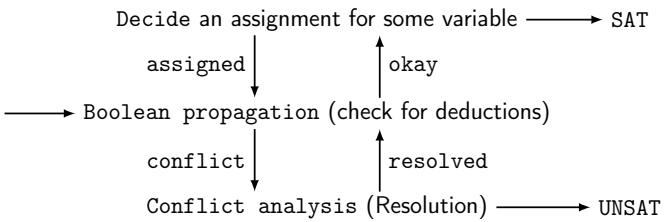

φ in CNF

Boolean abstraction of φ

SAT solver → SAT or UNSAT

SAT + model
or
UNSAT + explanation

Theory solver

$$\Phi = \{c \mid \alpha(X_c) = \mathit{true}\} \cup \{\neg c \mid \alpha(X_c) = \mathit{false}\},$$

for a partial assignment α

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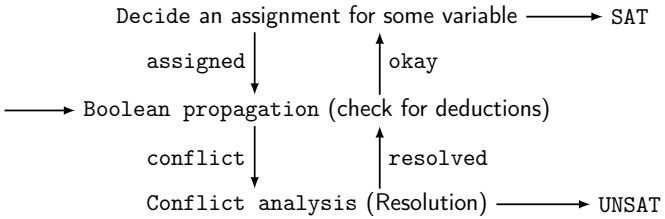

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→ many theory calls for similar sets of constraints

SMT-compliancy

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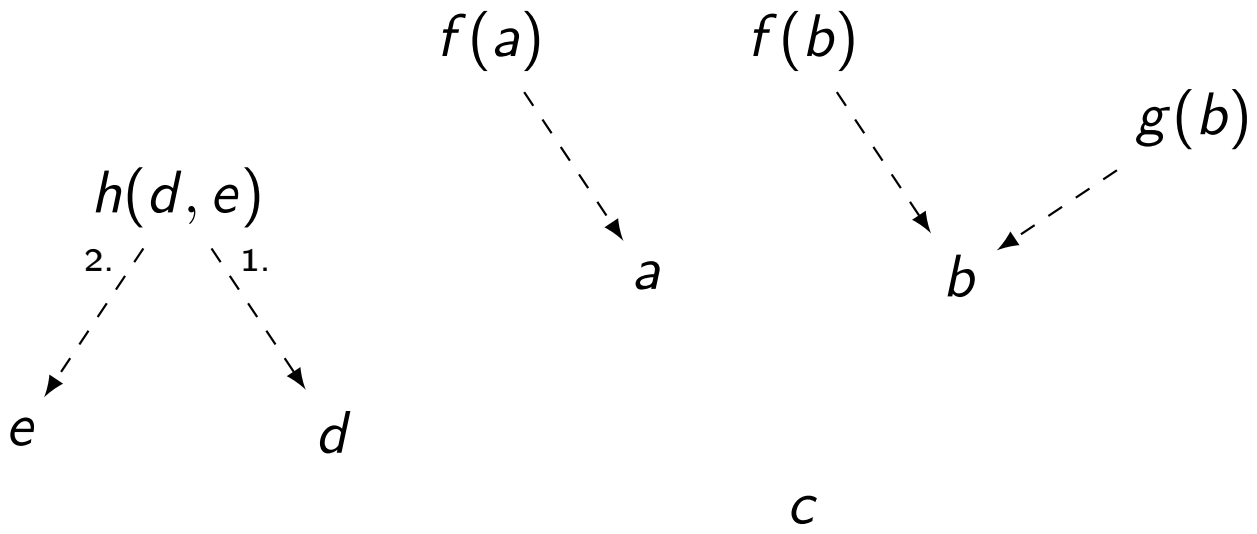
Backtracking

$\mathcal{T} = \{a,\ b,\ c,\ d,\ e,\ f(a),\ f(b),\ g(b),\ h(d,e)\}$
$\mathcal{C} = \{\}$



⇢ relations between terms

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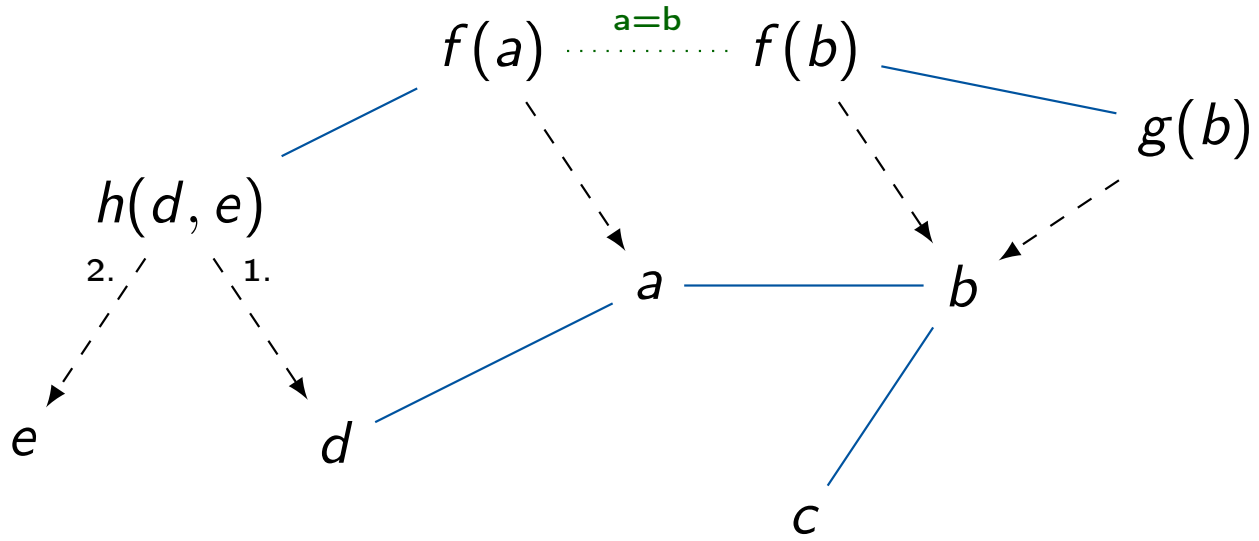

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derived equalities

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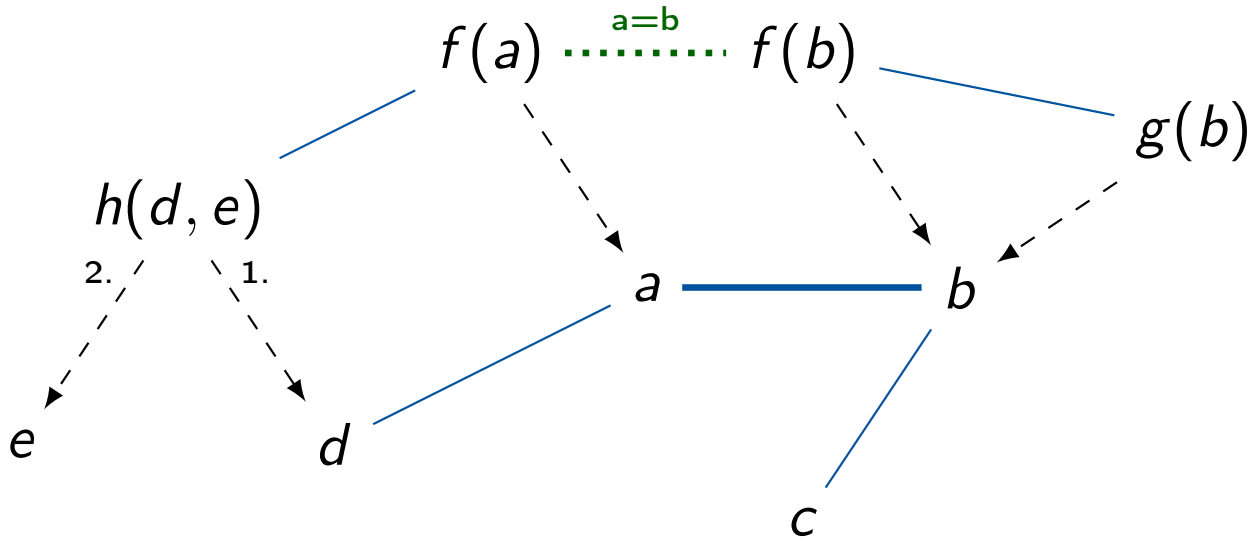
Remove $a = b$



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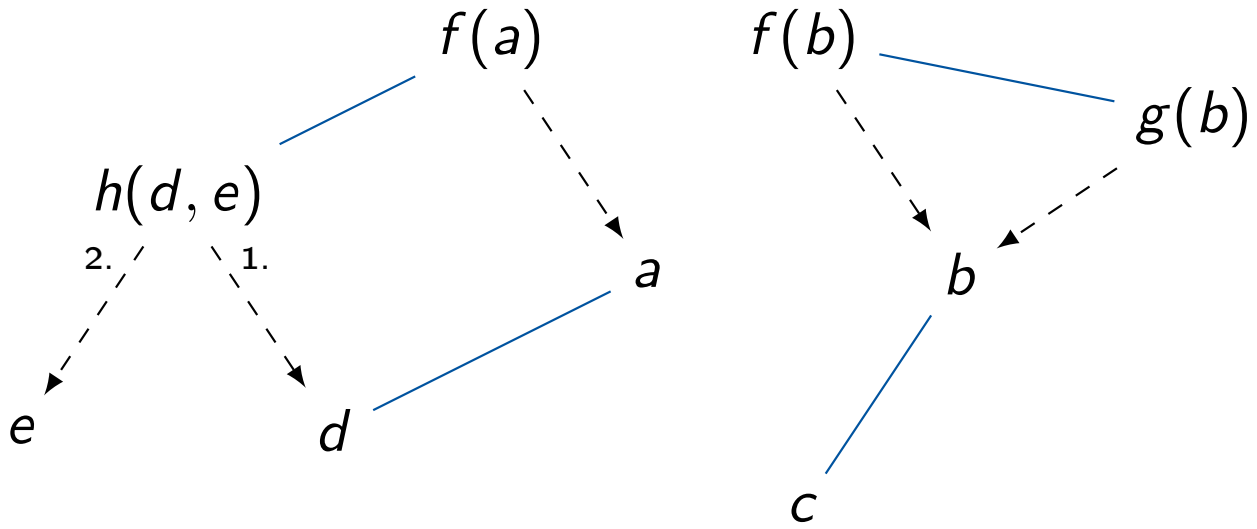

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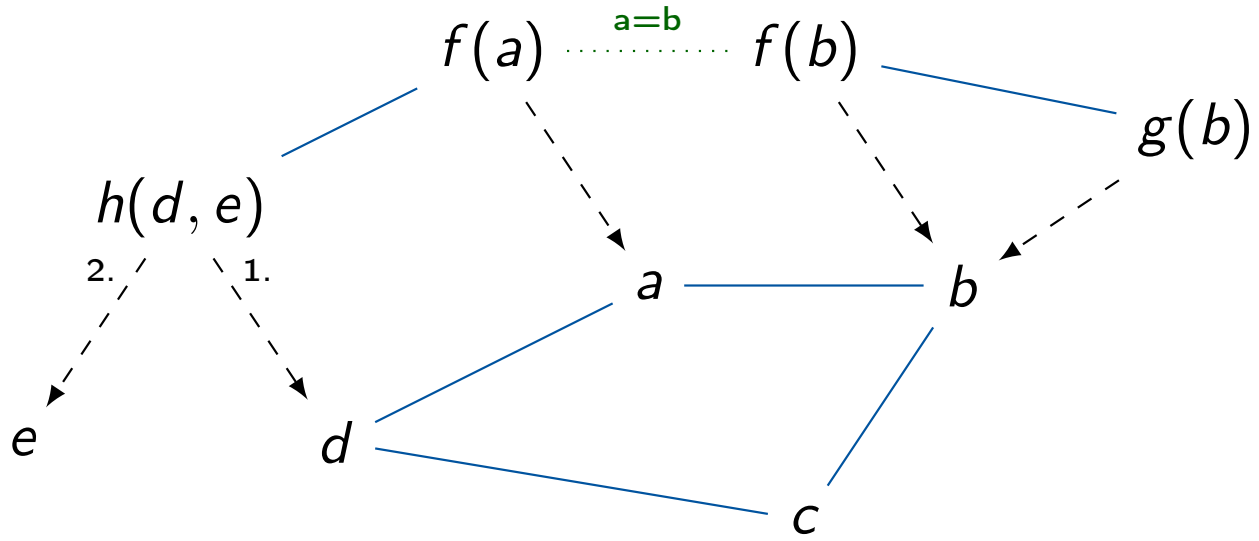

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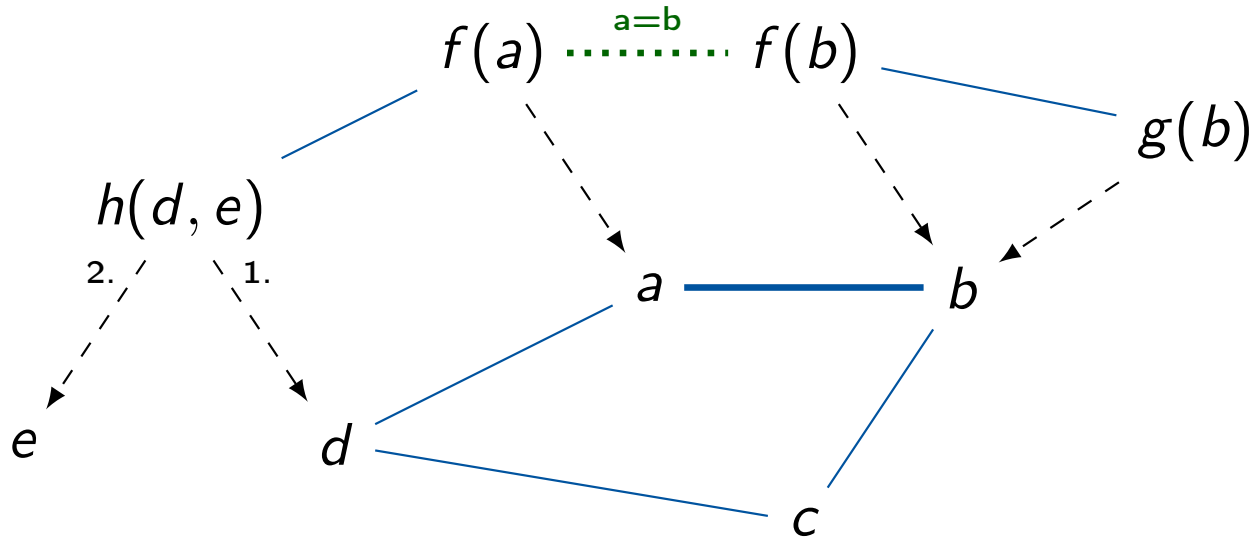

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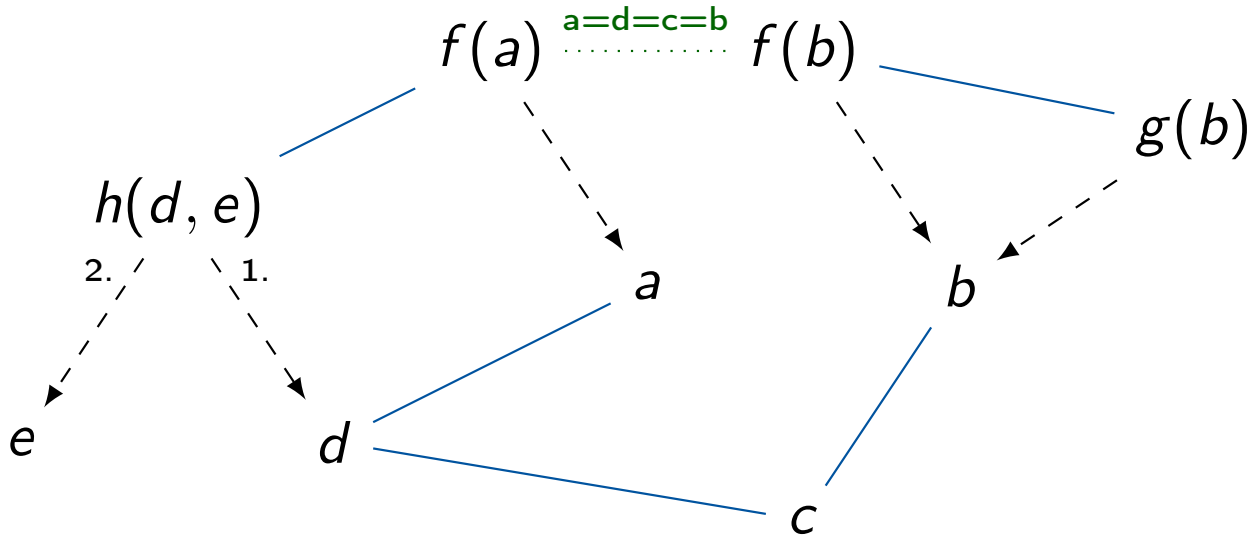

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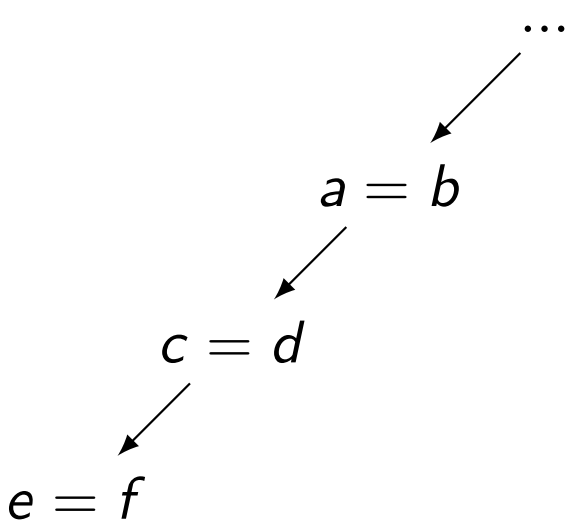
3. finding a (preferably minimal) **infeasible subset** as explanation for inconsistent constraint sets

Infeasible Subsets

Why not just use the trivial infeasible subset?

$$\ldots \wedge (\ldots \vee a = b) \wedge (e \neq f \vee f(a) \neq f(b)) \\ \wedge (c = d \vee f(a) \neq f(b)) \wedge (e = f \vee c \neq d \vee f(a) \neq f(b))$$



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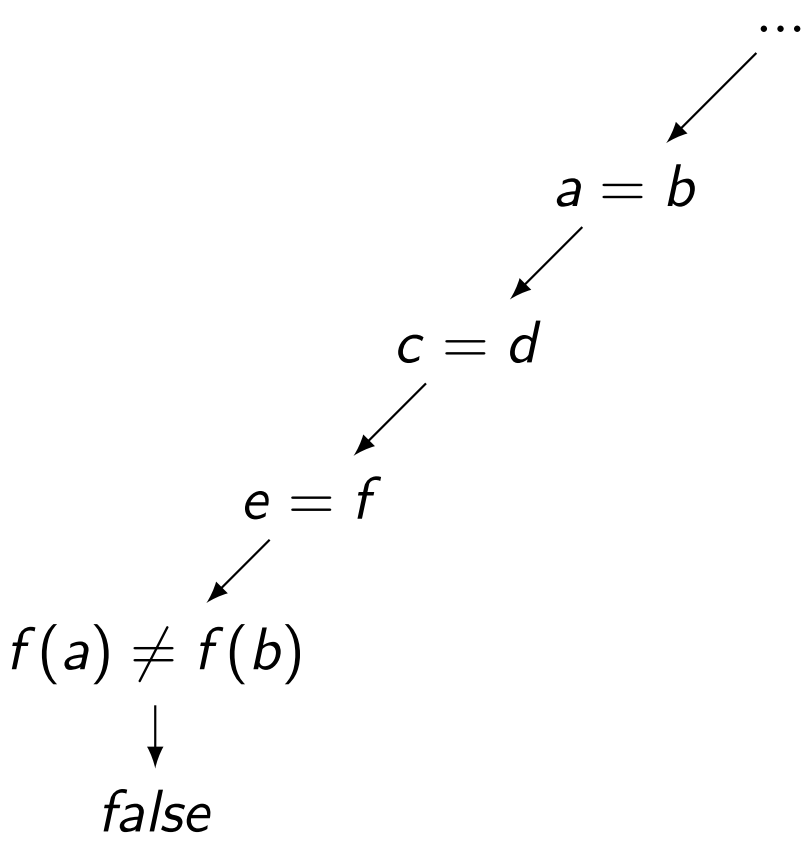

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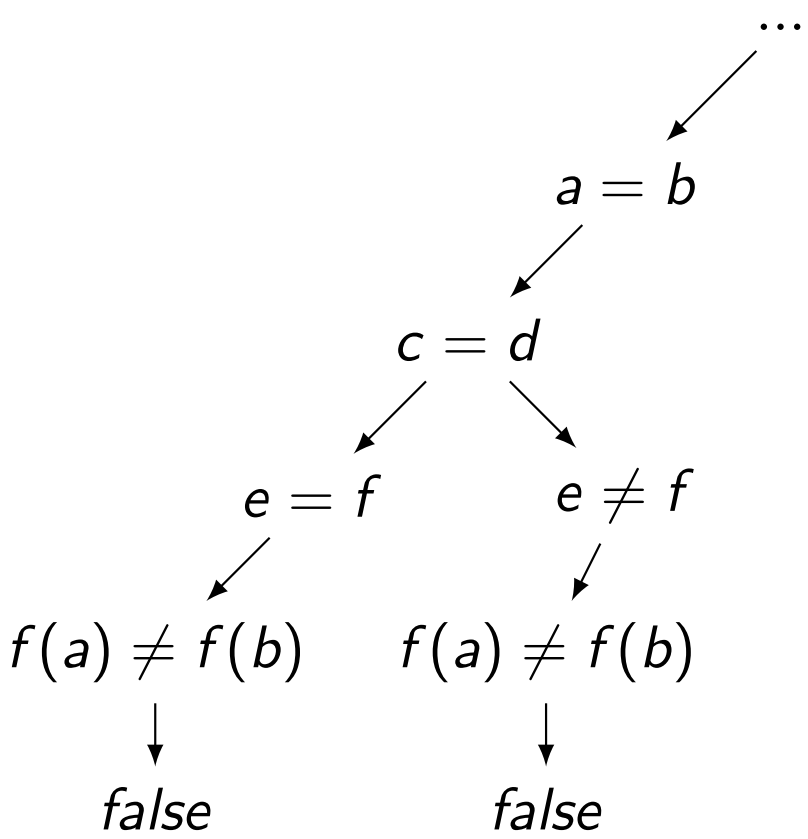

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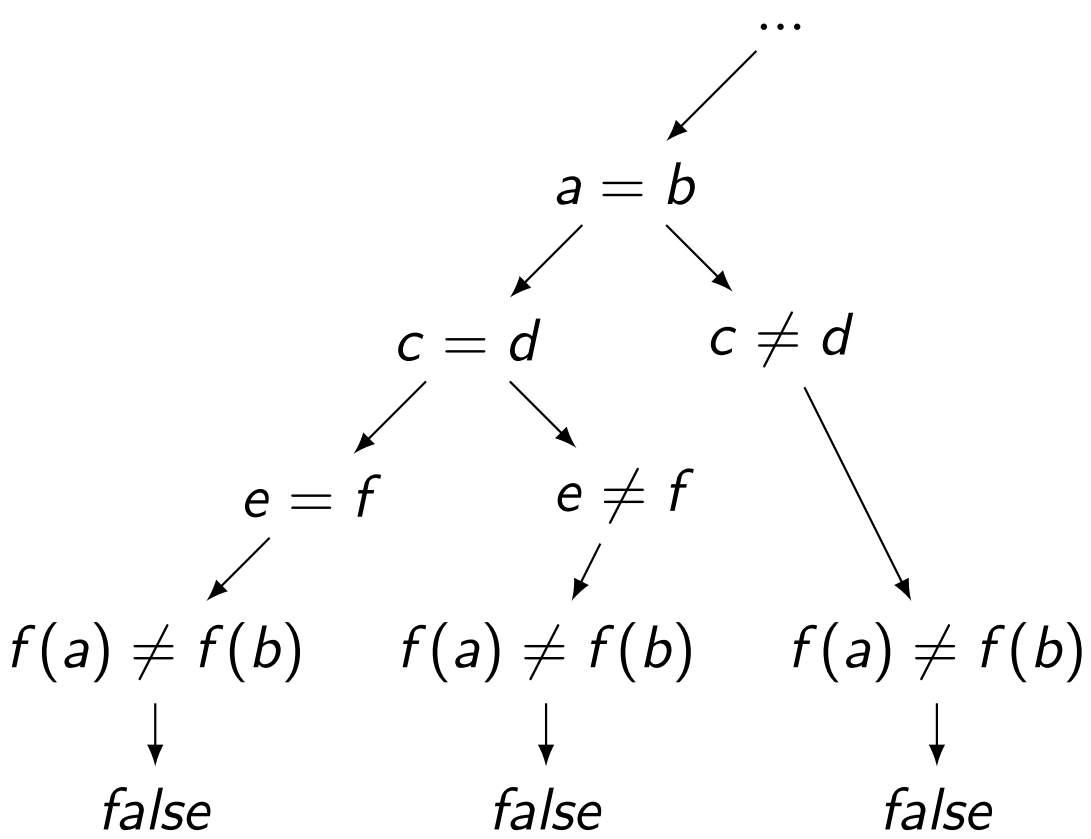

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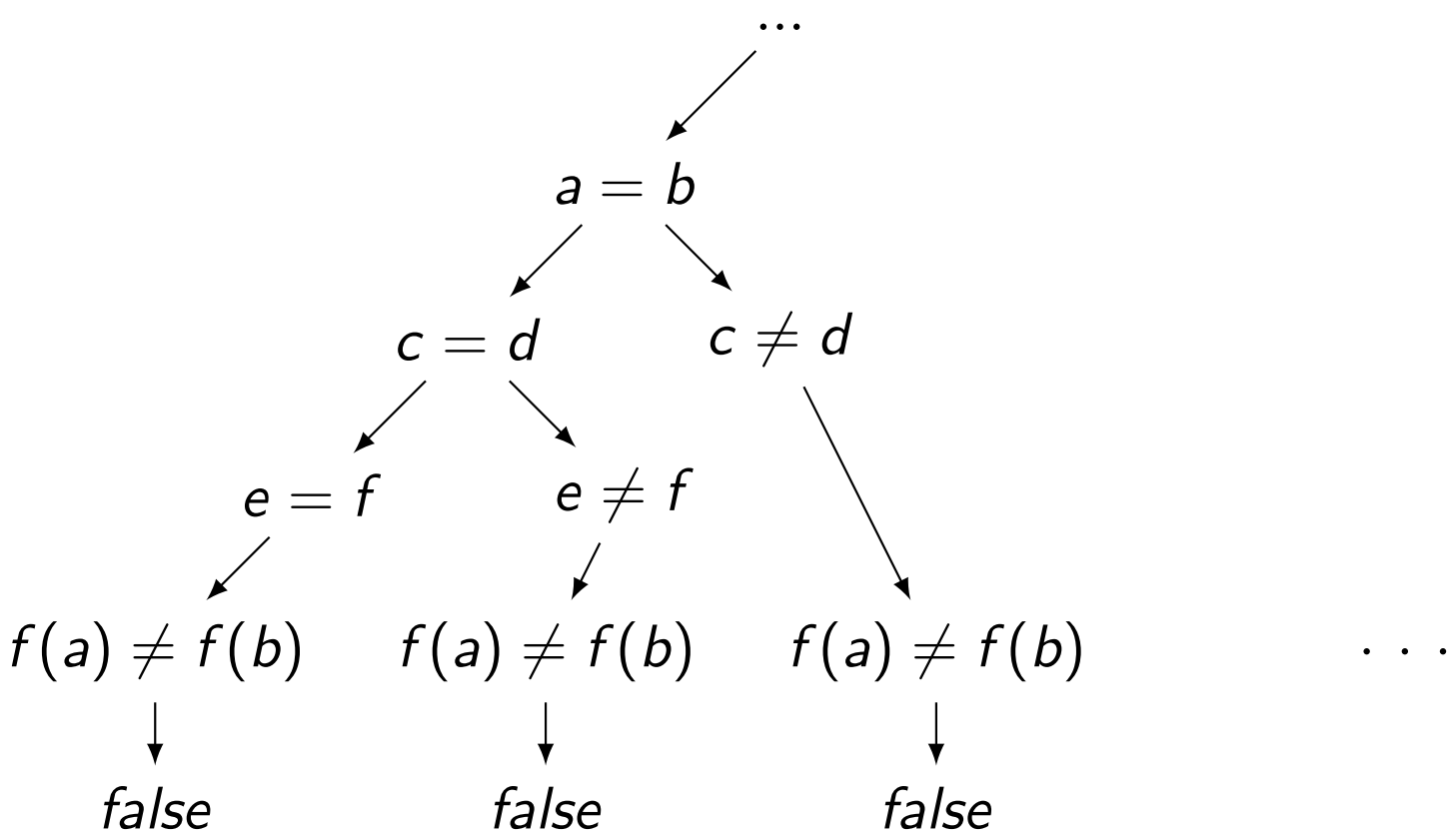

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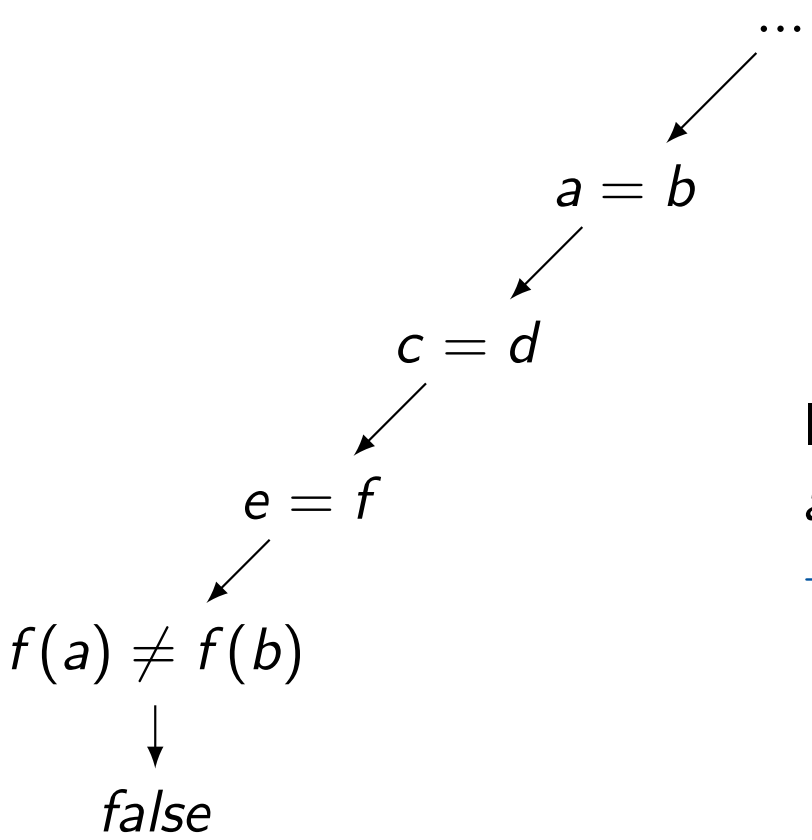

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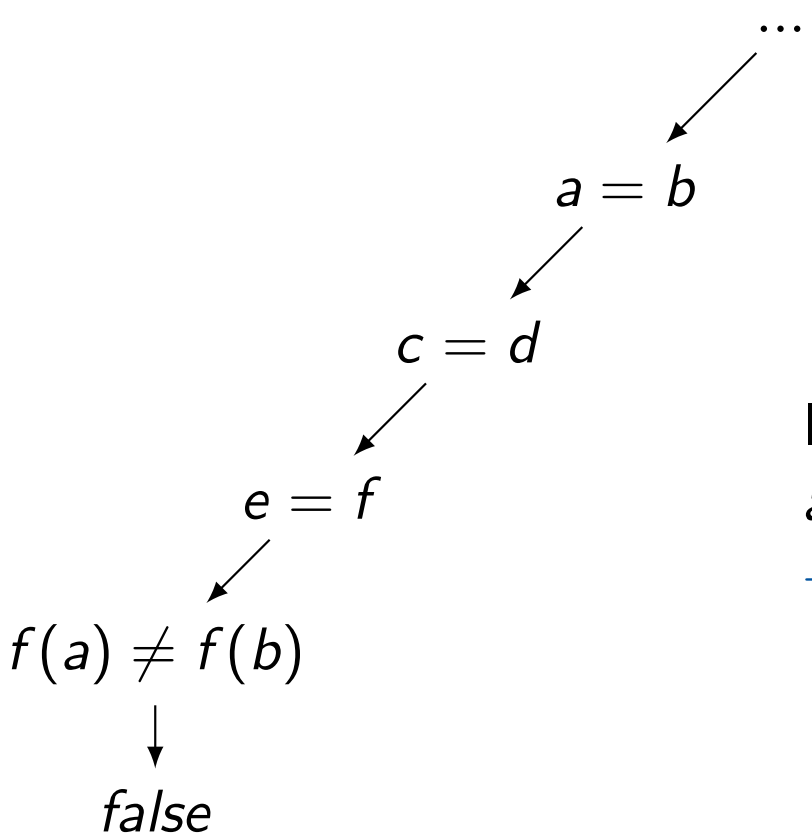

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   - build set of $t \neq t'$ and (a minimal number of) equations that imply $t = t'$ by transitivity and congruence

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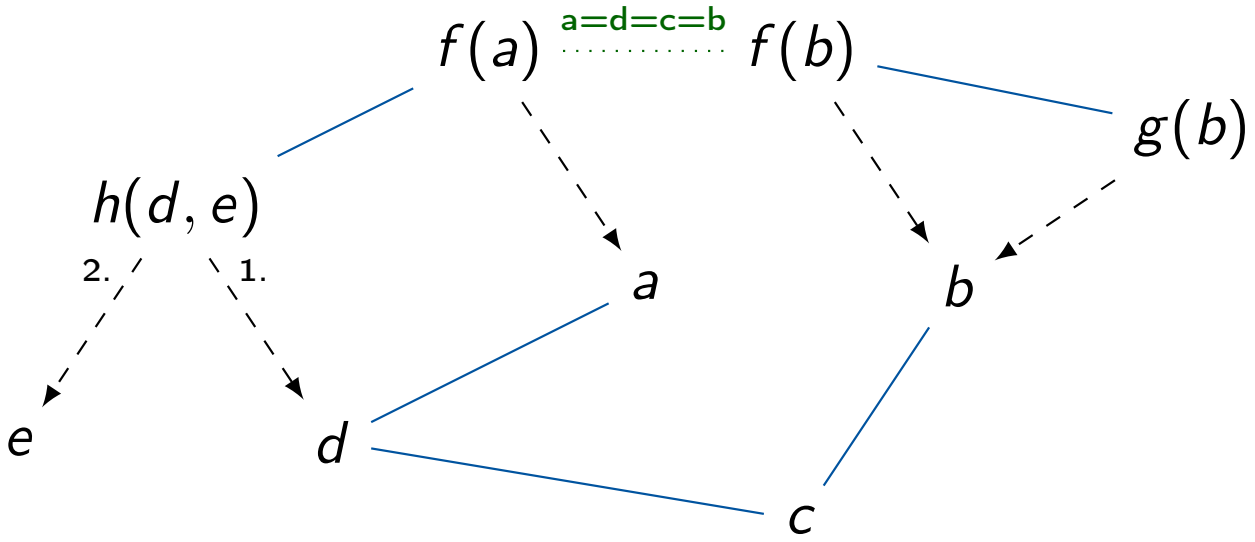

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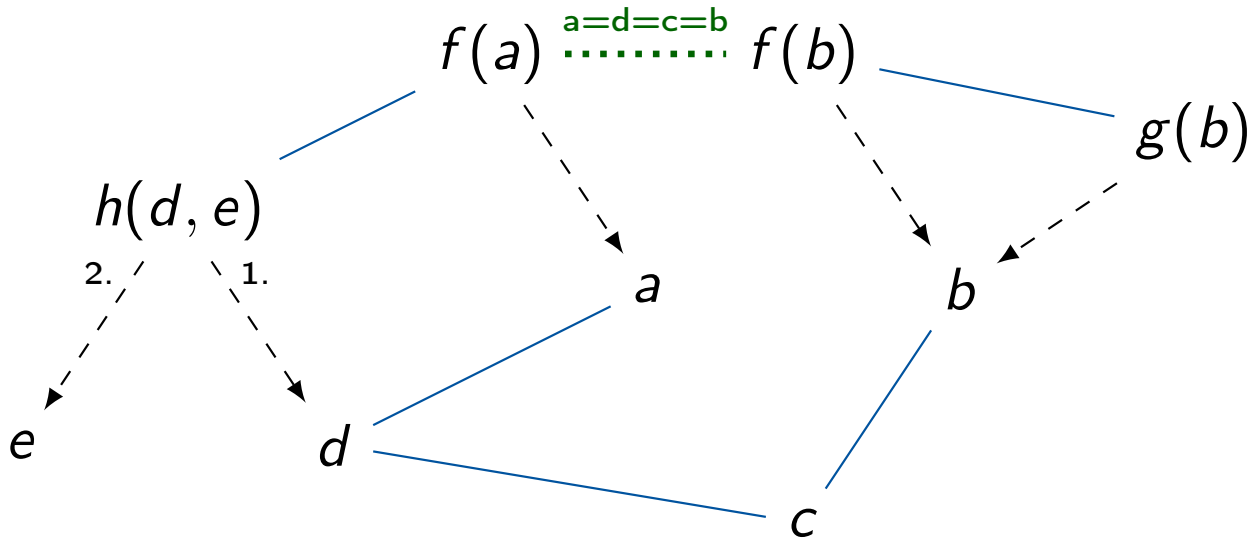
Minimal infeasible subset: shortest 'path' between $f(a)$ and $f(b)$



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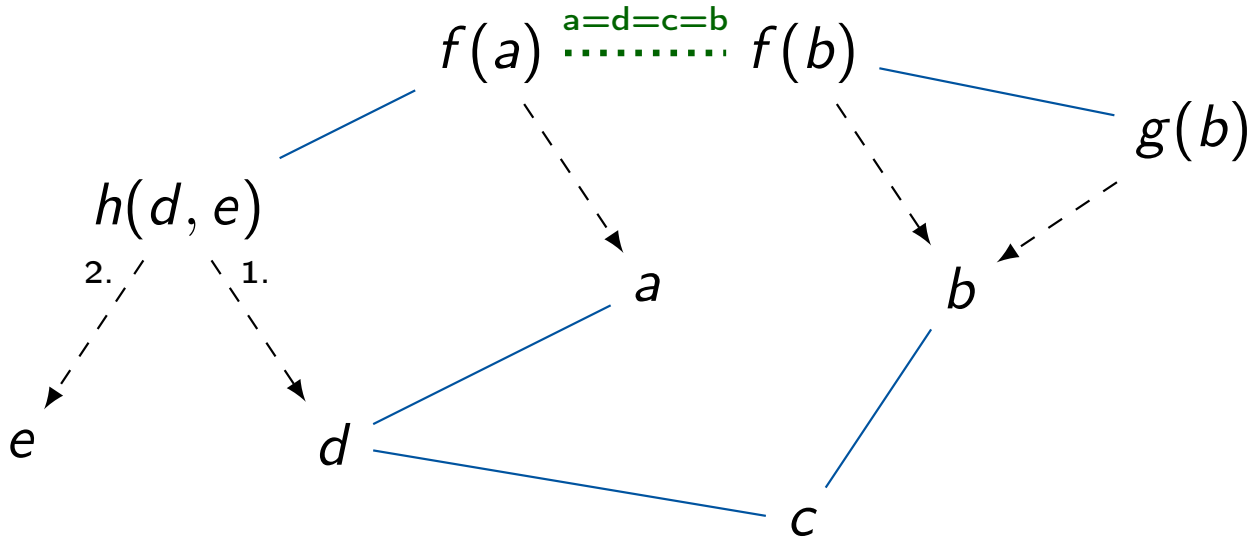

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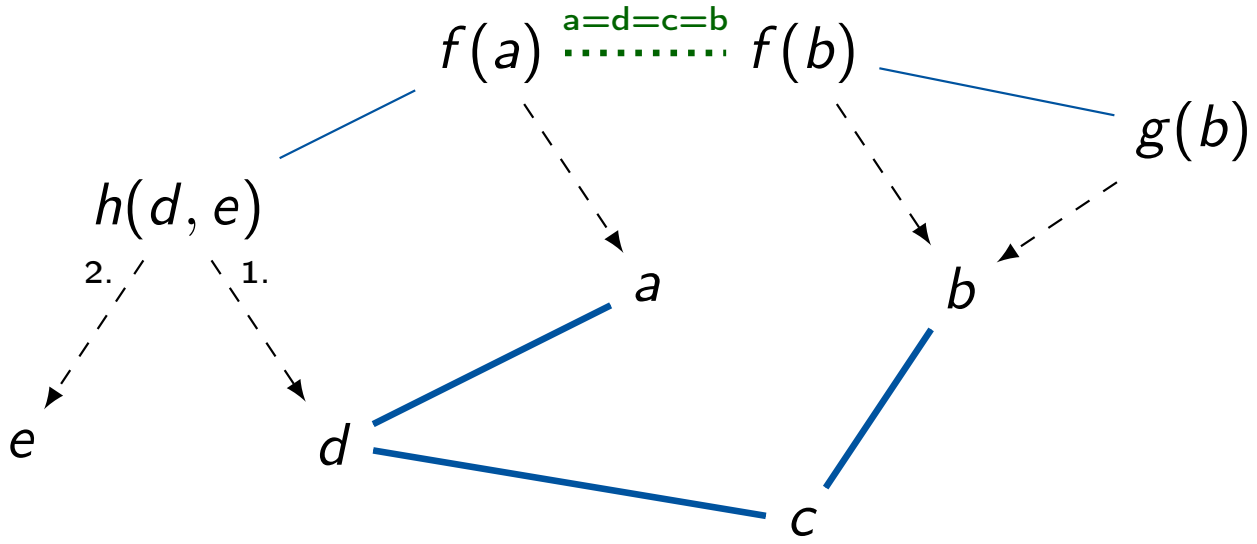

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- consists only of equality edges $\rightarrow$ done!

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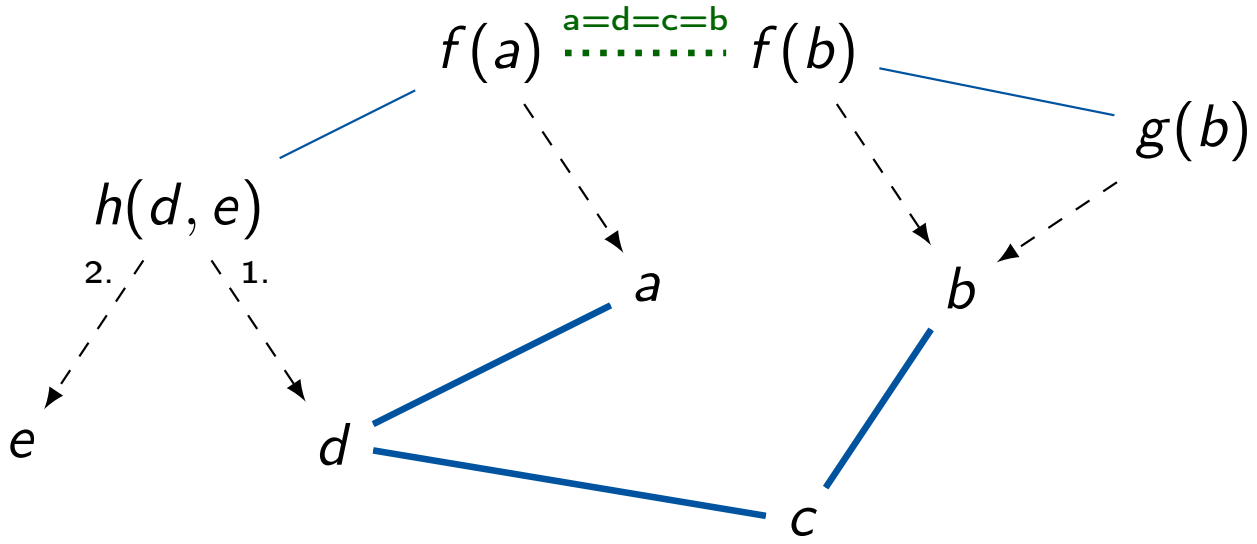

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- consists only of equality edges $\rightarrow$ done!
- Note that the resulting infeasible subset is not always minimal.

Theory Conflict

Algorithm: `getExplanation`

Input: State after some execution of `addEquation` or `addDisequation`
Output: If the current state is `UNSAT` an unsatisfiable subset MIS of the previously received equations and disequations else the empty set

1. $MIS := \emptyset$; `if (state = SAT) then return` MIS;
2. find a conflicting disequation: $t_1 \neq t_2$
3. find the shortest path P in the E-graph between the nodes t_1 and t_2
4. `while` $P \neq \emptyset$ `let` $e = (t, t') \in P$
   `if` $t = t' \in E$ `then` $MIS := MIS \cup \{t = t'\}$;
   `else let` $t = f(x_1, ..., x_n)$, $t' = f(y_1, ..., y_n)$ `for some` x, y `and` f
   `for` $k = 1$, $k < n$, $k{+}{+}$
   `find the shortest path` P_k `between` x_k `and` y_k
   $P := P \cup P_k$;

`return` MIS;