

Satisfiability Checking

Simplex as a Theory Module in SMT

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Informatik 2
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WS 19/20

- 1 Full lazy SMT-solving with Simplex
- 2 Less lazy SMT-solving with Simplex

1 Full lazy SMT-solving with Simplex

2 Less lazy SMT-solving with Simplex

The Xmas problem

There are three types of Xmas presents Santa Claus can make.

- Santa Claus wants to reduce the overhead by making only two types.
- He needs at least 100 presents.
- He needs at least 5 of either type 1 or type 2.
- He needs at least 10 of the third type.
- Each present of type 1, 2, and 3 need 1, 2, resp. 5 minutes to make.
- Santa Claus is late, and he has only 3 hours left.
- Each present of type 1, 2, and 3 costs 3, 2, resp. 1 EUR.
- He has 300 EUR for presents in total.

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- He has 300 EUR for presents in total.

$$\begin{aligned} & (p_1 = 0 \vee p_2 = 0 \vee p_3 = 0) \wedge p_1 + p_2 + p_3 \geq 100 \wedge \\ & (p_1 \geq 5 \vee p_2 \geq 5) \wedge p_3 \geq 10 \wedge p_1 + 2p_2 + 5p_3 \leq 180 \wedge \\ & \qquad \qquad \qquad 3p_1 + 2p_2 + p_3 \leq 300 \end{aligned}$$

The Xmas problem

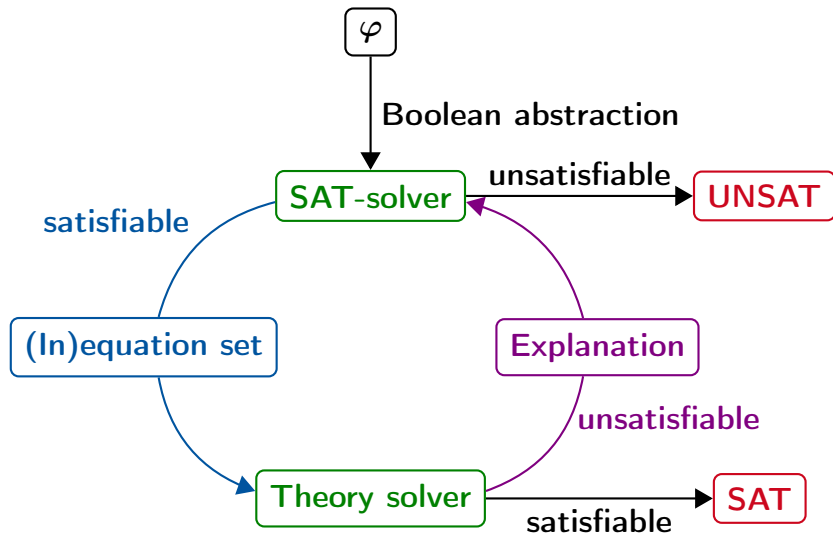
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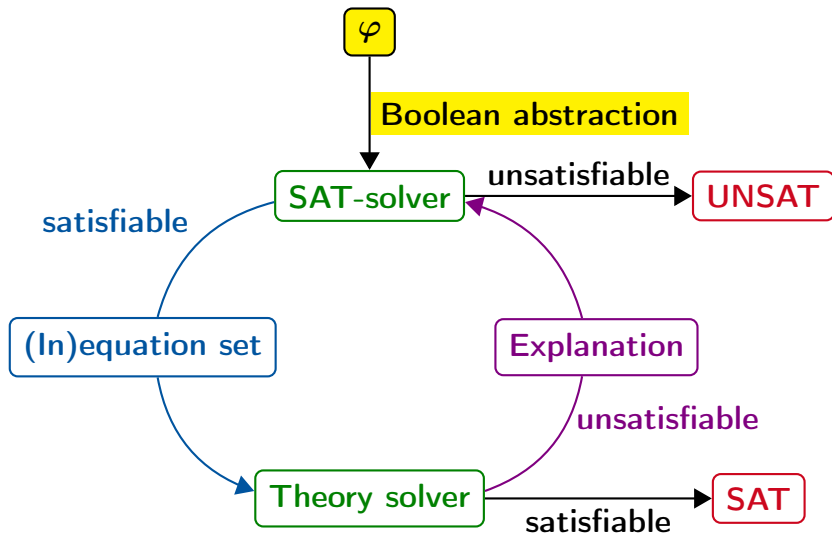
$$(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0) \wedge p_1 + p_2 + p_3 \geq 100 \wedge \\ (p_1 \geq 5 \vee p_2 \geq 5) \wedge p_3 \geq 10 \wedge p_1 + 2p_2 + 5p_3 \leq 180 \wedge \\ 3p_1 + 2p_2 + p_3 \leq 300$$

For the moment we **relax the integrality constraints**, i.e., we search for a **real-valued** solution.

Full lazy SMT-solving



Full lazy SMT-solving



Arithmetic formula:

$$\underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9}$$

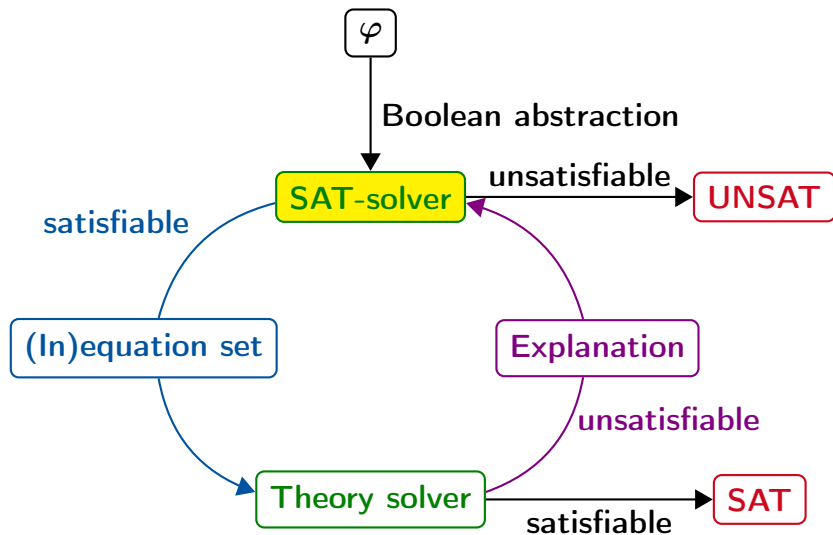
Arithmetic formula:

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
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$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9}$$

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Full lazy SMT-solving



Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

DL1 : $a_1 : 0$

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

DL1 : $a_1 : 0$

DL2 : $a_2 : 0, a_3 : 1$

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

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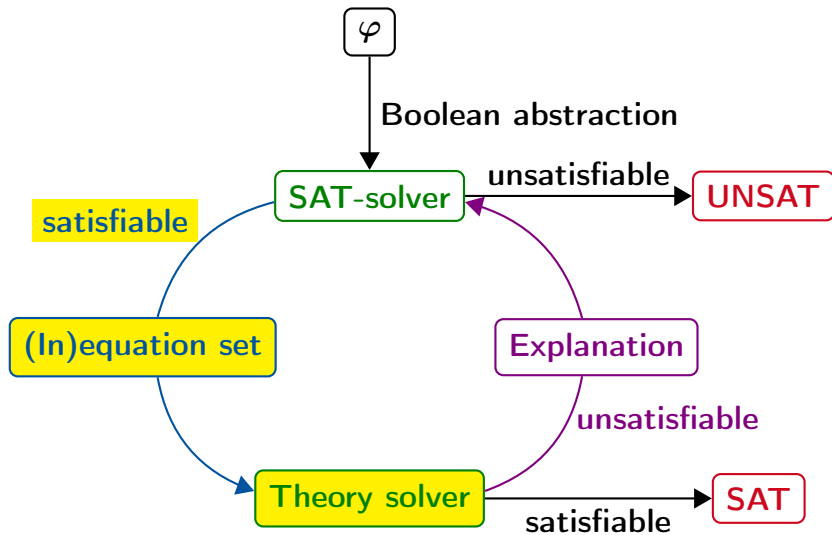
DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

DL1 : $a_1 : 0$

DL2 : $a_2 : 0, a_3 : 1$

DL3 : $a_5 : 0, a_6 : 1$

Full lazy SMT-solving



Full lazy theory solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1, \quad DL3 : a_5 : 0, a_6 : 1$

Full lazy theory solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1, \quad DL3 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_3, a_6$

Full lazy theory solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1, \quad DL3 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_3, a_6$

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_3 \geq 10}_{a_6} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_7} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_8}$$

Full lazy theory solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1, \quad DL3 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_3, a_6$

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9}$$

Encoding:

$p_1 + p_2 + p_3 \geq 100, p_3 \geq 10,$

$p_1 + 2p_2 + 5p_3 \leq 180, 3p_1 + 2p_2 + p_3 \leq 300, p_3 = 0, p_2 \geq 5$

Full lazy theory solving

$$\begin{array}{llllll} p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\ p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\ p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\ 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\ p_3 = 0 & \rightarrow & s_5 = & & & p_3 & s_5 = 0 \\ p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$							
$s_1(0)$	1	1	1							
$s_2(0)$	0	0	1							
$s_3(0)$	1	2	5							
$s_4(0)$	3	2	1							
$s_5(0)$	0	0	1							
$s_6(0)$	0	1	0							

Full lazy theory solving

$$\begin{array}{llll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + p_2 + p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + 2p_2 + 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + 2p_2 + p_3 & s_4 \leq 300 \\
 p_3 = 0 & \rightarrow & s_5 = & & p_3 & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$				
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1				
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1				
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4				
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2				
$s_5(0)$	0	0	1	$s_5(0)$	0	0	1				
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0				

Full lazy theory solving

$$\begin{array}{llll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + p_2 + p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + 2p_2 + 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + 2p_2 + p_3 & s_4 \leq 300 \\
 p_3 = 0 & \rightarrow & s_5 = & p_3 & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2
$s_5(0)$	0	0	1	$s_5(0)$	0	0	1	$s_5(10)$	0	0	1
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0

Full lazy theory solving

$$\begin{array}{llll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + p_2 + p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + 2p_2 + 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + 2p_2 + p_3 & s_4 \leq 300 \\
 p_3 = 0 & \rightarrow & s_5 = & p_3 & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5
 \end{array}$$

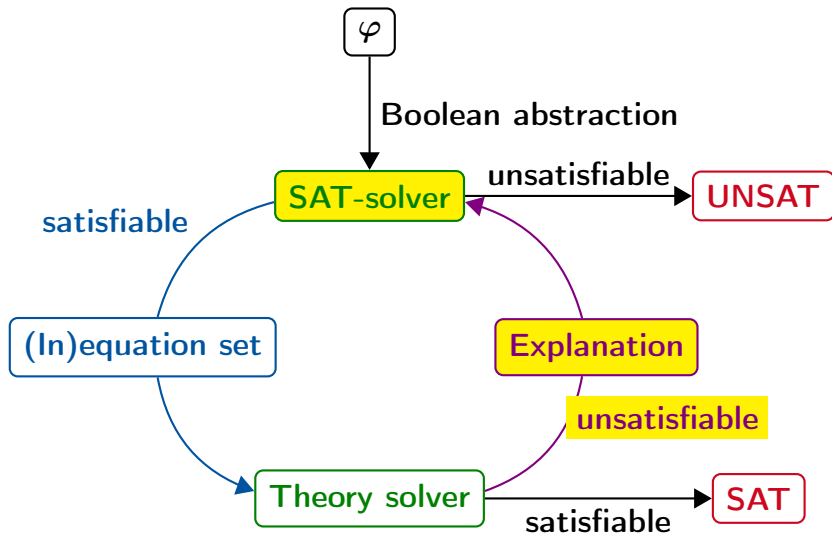
Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2
$s_5(0)$	0	0	1	$s_5(0)$	0	0	1	$s_5(10)$	0	0	1
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0

Conflict: the constraints for the basic variable of the conflicting row and all non-basic variables with non-zero coefficients in the conflicting row together are unsatisfiable.

Thus $\underbrace{p_3 = 0}_{a_3} \wedge \underbrace{p_3 \geq 10}_{a_7}$ is not satisfiable.

Full lazy SMT-solving



Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_3 \vee \neg a_7)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_3 \vee \neg a_7)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL0$ and use the new clause for propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_3 \vee \neg a_7)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL0$ and use the new clause for propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

Full lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_3 \vee \neg a_7)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL0$ and use the new clause for propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

Full lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

$DL3 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_3 \vee \neg a_7)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

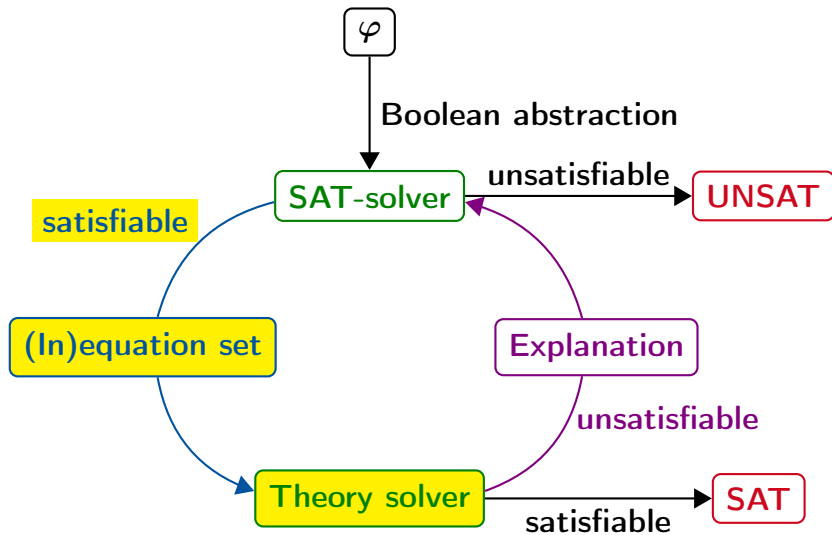
No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL0$ and use the new clause for propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Full lazy SMT-solving



Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_6$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_6$

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \wedge (\neg a_3 \vee \neg a_7)$$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_6$

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \wedge (\neg a_3 \vee \neg a_7)$$

Encoding:

$$p_1 + p_2 + p_3 \geq 100, p_3 \geq 10,$$

$$p_1 + 2p_2 + 5p_3 \leq 180, 3p_1 + 2p_2 + p_3 \leq 300, p_2 = 0, p_2 \geq 5$$

Full lazy theory solving

$$\begin{array}{llllll} p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\ p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\ p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\ 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\ p_2 = 0 & \rightarrow & s_5 = & & p_2 & & s_5 = 0 \\ p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$												
$s_1(0)$	1	1	1												
$s_2(0)$	0	0	1												
$s_3(0)$	1	2	5												
$s_4(0)$	3	2	1												
$s_5(0)$	0	1	0												
$s_6(0)$	0	1	0												

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & p_2 & & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$							
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1							
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1							
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4							
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2							
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0							
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0							

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & p_2 & & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$				
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1				
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1				
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4				
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2				
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(0)$	0	1	0				
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0				

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & p_2 & & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$		$s_1(100)$	$s_6(5)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1	$p_1(85)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4	$s_3(145)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2	$s_4(275)$	3	-1	-2
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(5)$	0	1	0
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$p_2(5)$	0	1	0

Full lazy theory solving

$$\begin{array}{llll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + p_2 + p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + 2p_2 + 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + 2p_2 + p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & p_2 & s_5 = 0 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5
 \end{array}$$

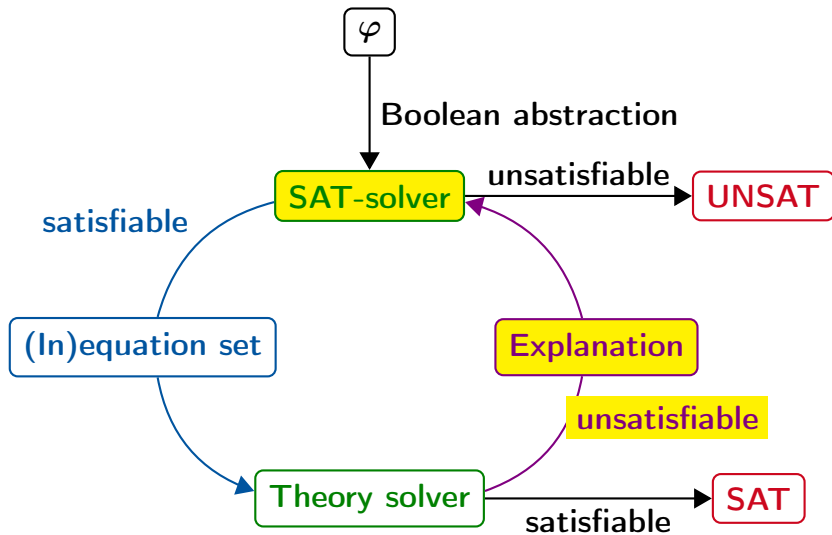
Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$		$s_1(100)$	$s_6(5)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1	$p_1(85)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4	$s_3(145)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2	$s_4(275)$	3	-1	-2
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(5)$	0	1	0
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$p_2(5)$	0	1	0

Conflict: the constraints for the basic variable of the conflicting row and all non-basic variables with non-zero coefficients in the conflicting row together are unsatisfiable.

Thus $\underbrace{p_2 = 0}_{a_2} \wedge \underbrace{p_2 \geq 5}_{a_6}$ is not satisfiable.

Full lazy SMT-solving



Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6)$$

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting.

Full lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL1$ and apply propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

Full lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Learn new clause: $(\neg a_2 \vee \neg a_6)$.

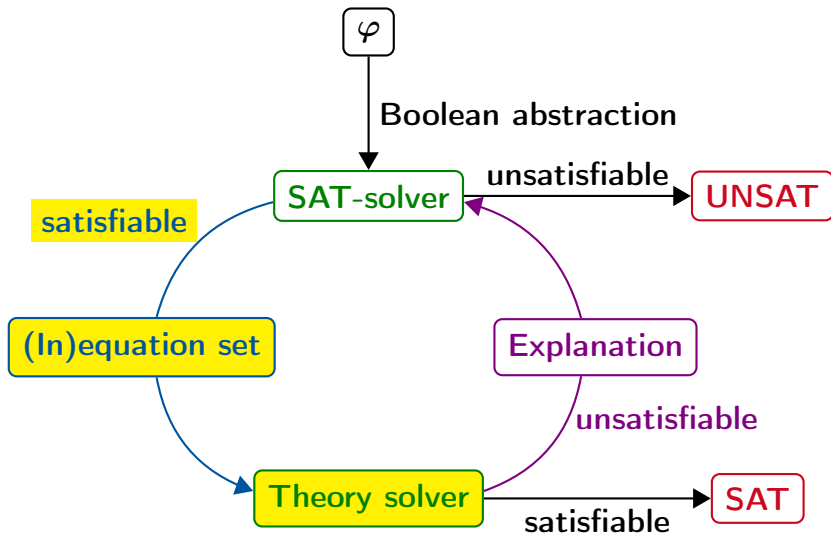
$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting.
Backtrack to decision level $DL1$ and apply propagation.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Full lazy SMT-solving



Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0, \quad DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_5$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0, \quad DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_5$

$$\begin{aligned} & \underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge \\ & \underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge \\ & \underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9} \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6) \end{aligned}$$

Full lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0, \quad DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

True theory constraints: $a_4, a_7, a_8, a_9, a_2, a_5$

$$\begin{aligned} & \underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge \\ & \underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge \\ & \underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9} \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6) \end{aligned}$$

Encoding:

$$p_1 + p_2 + p_3 \geq 100, p_3 \geq 10,$$

$$p_1 + 2p_2 + 5p_3 \leq 180, 3p_1 + 2p_2 + p_3 \leq 300, p_2 = 0, p_1 \geq 5$$

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & & p_2 & s_5 = 0 \\
 p_1 \geq 5 & \rightarrow & s_6 = & p_1 & & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$							
$s_1(0)$	1	1	1							
$s_2(0)$	0	0	1							
$s_3(0)$	1	2	5							
$s_4(0)$	3	2	1							
$s_5(0)$	0	1	0							
$s_6(0)$	1	0	0							

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & & p_2 & s_5 = 0 \\
 p_1 \geq 5 & \rightarrow & s_6 = & p_1 & & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$				
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1				
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1				
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4				
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2				
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0				
$s_6(0)$	1	0	0	$s_6(100)$	1	-1	-1				

Full lazy theory solving

$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & & p_2 & s_5 = 0 \\
 p_1 \geq 5 & \rightarrow & s_6 = & p_1 & & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(0)$	0	1	0
$s_6(0)$	1	0	0	$s_6(100)$	1	-1	-1	$s_6(90)$	1	-1	-1

Full lazy theory solving

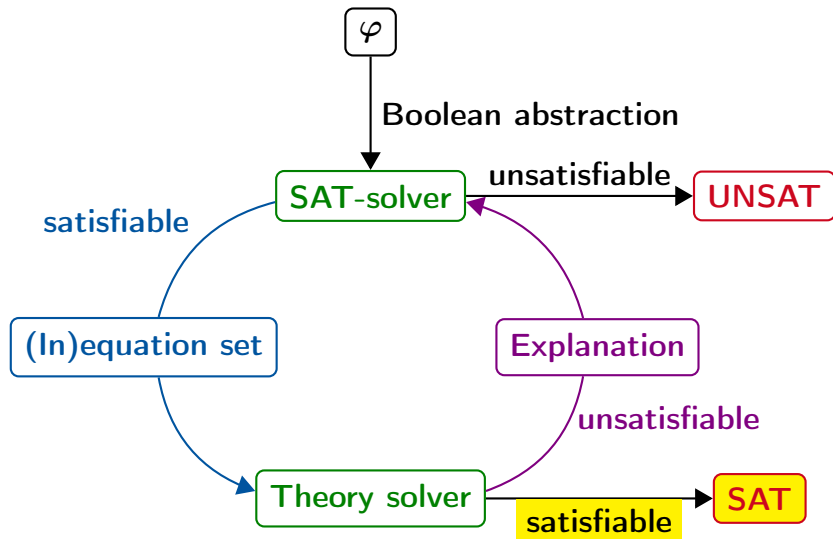
$$\begin{array}{llllll}
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_1 = & p_1 + & p_2 + & p_3 & s_1 \geq 100 \\
 p_3 \geq 10 & \rightarrow & s_2 = & & & p_3 & s_2 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_3 = & p_1 + & 2p_2 + & 5p_3 & s_3 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_4 = & 3p_1 + & 2p_2 + & p_3 & s_4 \leq 300 \\
 p_2 = 0 & \rightarrow & s_5 = & & & p_2 & s_5 = 0 \\
 p_1 \geq 5 & \rightarrow & s_6 = & p_1 & & & s_6 \geq 5
 \end{array}$$

Variable order: $s_1 < \dots < s_6 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$p_3(0)$		$s_1(100)$	$p_2(0)$	$s_2(10)$
$s_1(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_2(0)$	0	0	1	$s_2(0)$	0	0	1	$p_3(10)$	0	0	1
$s_3(0)$	1	2	5	$s_3(100)$	1	1	4	$s_3(140)$	1	1	4
$s_4(0)$	3	2	1	$s_4(300)$	3	-1	-2	$s_4(280)$	3	-1	-2
$s_5(0)$	0	1	0	$s_5(0)$	0	1	0	$s_5(0)$	0	1	0
$s_6(0)$	1	0	0	$s_6(100)$	1	-1	-1	$s_6(90)$	1	-1	-1

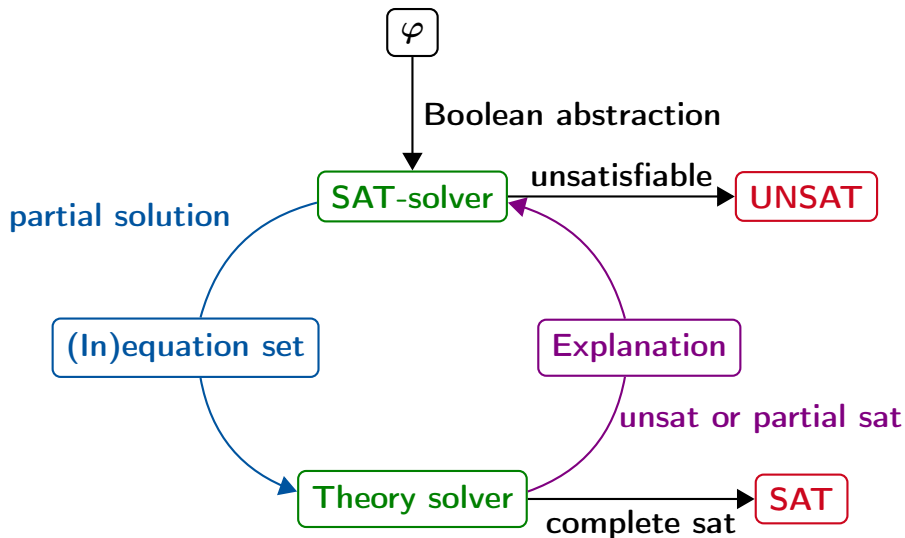
Solution: $p_1 = 90$, $p_2 = 0$, $p_3 = 10$.

Full lazy SMT-solving



- In **full lazy** SMT-solving, the SAT solver asks the theory solver whether found **complete** satisfying assignments for the abstraction are consistent in the theory.
- In **less lazy** SMT-solving, the SAT solver asks for consistency checks in the theory more frequently, also for **partial** assignments.
- Usually, this happens after each completed decision level.

Less lazy SMT-solving



Requirements on the theory solver

- (Minimal) infeasible subsets (to explain infeasibility)
- Incrementality (to add constraints stepwise)
- Backtracking (to mimic backtracking in the SAT solver)

Requirements on the theory solver

- (Minimal) infeasible subsets (to explain infeasibility)
- Incrementality (to add constraints stepwise)
- Backtracking (to mimic backtracking in the SAT solver)

Minimal infeasible subsets in Simplex:

- As seen in full lazy SMT solving
- The constraints corresponding to the basic variable of the contradictory row and all non-basic variables with non-zero coefficients in this row are together unsatisfiable.

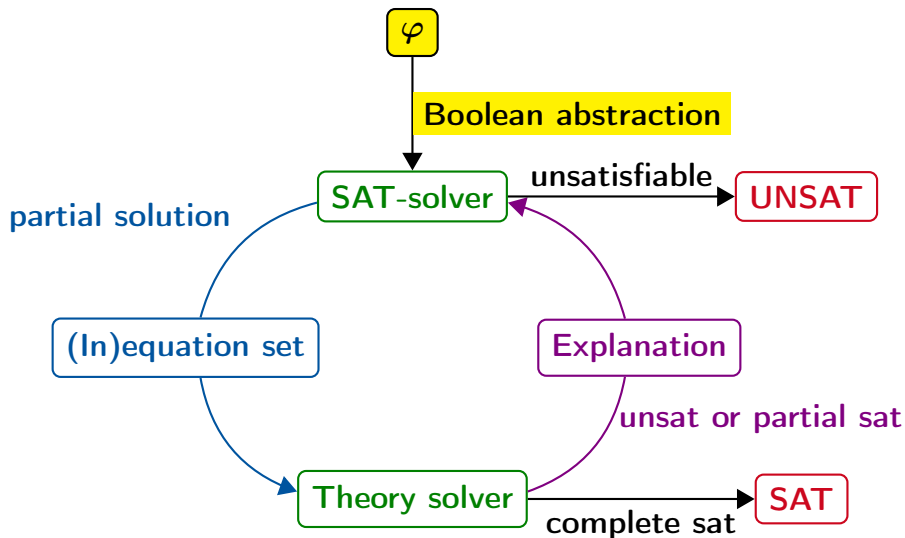
Incrementality in Simplex:

- Add all constraints but **without bounds** on non-active constraints.
- If a constraint becomes true, **activate** its bound.

Backtracking in Simplex:

- Remove bounds of unassigned constraints

Less lazy SMT-solving



Arithmetic formula:

$$\underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge$$
$$\underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9}$$

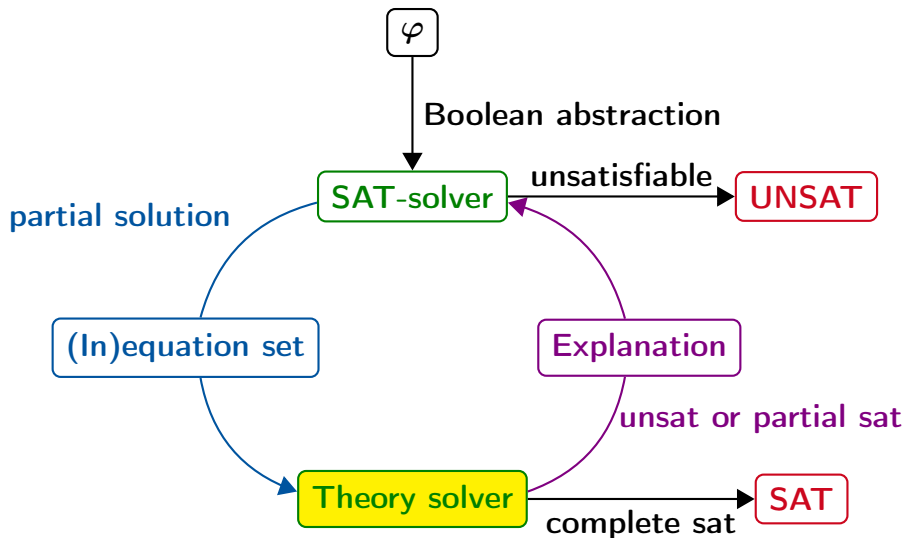
Arithmetic formula:

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9}$$

Boolean abstraction:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Less lazy SMT-solving



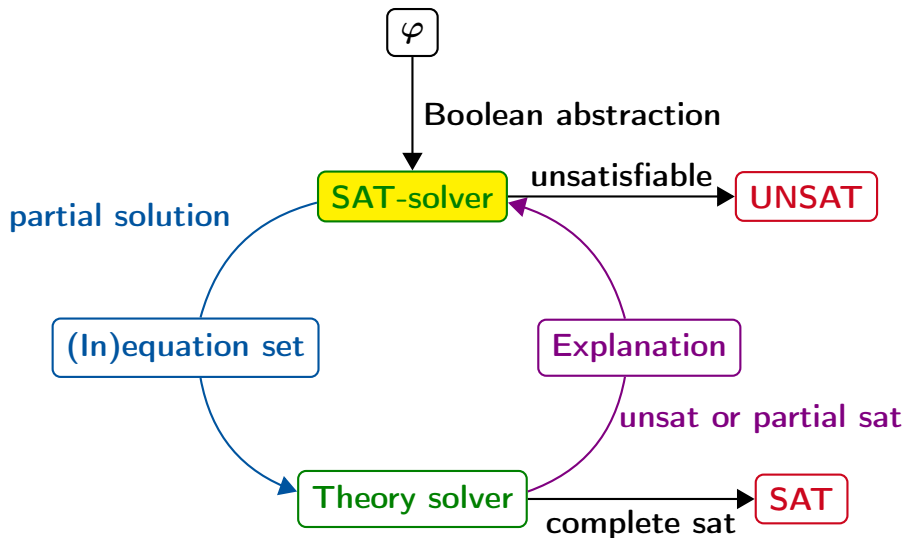
Less lazy theory solving

Initialize the Simplex tableau with all equalities but **without any bounds**.

$$\begin{array}{llll} p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\ p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\ p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\ p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\ p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\ p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\ p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\ p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\ 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300 \end{array}$$

	$p_1(0)$	$p_2(0)$	$p_3(0)$
$s_1(0)$	1	0	0
$s_2(0)$	0	1	0
$s_3(0)$	0	0	1
$s_4(0)$	1	1	1
$s_5(0)$	1	0	0
$s_6(0)$	0	1	0
$s_7(0)$	0	0	1
$s_8(0)$	1	2	5
$s_9(0)$	3	2	1

Less lazy SMT-solving



$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

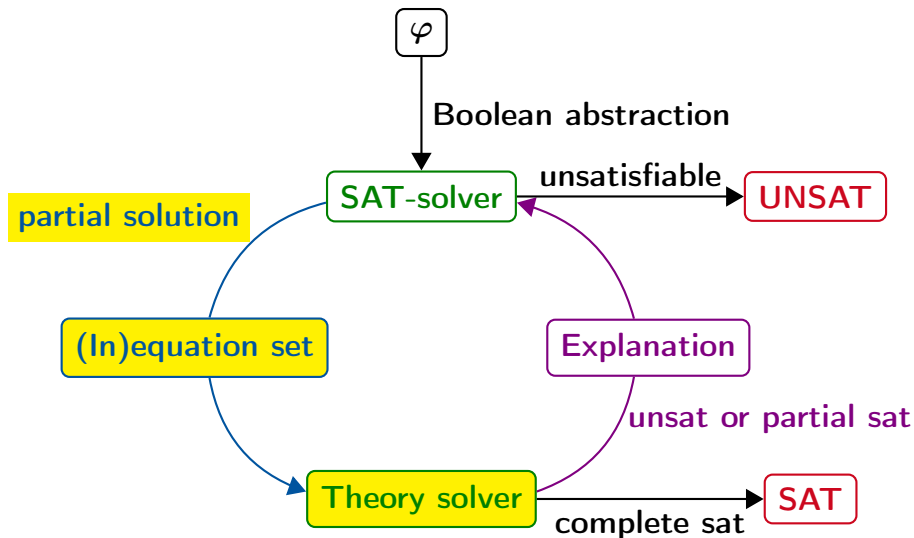
$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

Less lazy SMT-solving



Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

New true theory constraints: a_4, a_7, a_8, a_9

Less lazy theory solving

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

New true theory constraints: a_4, a_7, a_8, a_9

$$\begin{aligned} & (\underbrace{p_1 = 0}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3}) \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge \\ & (\underbrace{p_1 \geq 5}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6}) \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge \\ & \underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \end{aligned}$$

Less lazy theory solving

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

New true theory constraints: a_4, a_7, a_8, a_9

$$\underbrace{(p_1 = 0 \vee p_2 = 0 \vee p_3 = 0)}_{a_1} \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5 \vee p_2 \geq 5)}_{a_5} \wedge \underbrace{p_2 \geq 5}_{a_6} \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge$$
$$\underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9}$$

Encoding:

$$s_4 \geq 100, s_7 \geq 10, s_8 \leq 180, s_9 \leq 300$$

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

Variable order: $s_1 < \dots < s_9 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$							
$s_1(0)$	1	0	0							
$s_2(0)$	0	1	0							
$s_3(0)$	0	0	1							
$s_4(0)$	1	1	1							
$s_5(0)$	1	0	0							
$s_6(0)$	0	1	0							
$s_7(0)$	0	0	1							
$s_8(0)$	1	2	5							
$s_9(0)$	3	2	1							

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & & & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + & p_2 + & p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & & & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & & & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + & 2p_2 + & 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + & 2p_2 + & p_3 & s_9 \leq 300
 \end{array}$$

Variable order: $s_1 < \dots < s_9 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_4(100)$	$p_2(0)$	$p_3(0)$				
$s_1(0)$	1	0	0	$s_1(100)$	1	-1	-1				
$s_2(0)$	0	1	0	$s_2(0)$	0	1	0				
$s_3(0)$	0	0	1	$s_3(0)$	0	0	1				
$s_4(0)$	1	1	1	$p_1(100)$	1	-1	-1				
$s_5(0)$	1	0	0	$s_5(100)$	1	-1	-1				
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0				
$s_7(0)$	0	0	1	$s_7(0)$	0	0	1				
$s_8(0)$	1	2	5	$s_8(100)$	1	1	4				
$s_9(0)$	3	2	1	$s_9(300)$	3	-1	-2				

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & & & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + & p_2 + & p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & & & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & & & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + & 2p_2 + & 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + & 2p_2 + & p_3 & s_9 \leq 300
 \end{array}$$

Variable order: $s_1 < \dots < s_9 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_4(100)$	$p_2(0)$	$p_3(0)$		$s_4(100)$	$p_2(0)$	$s_7(10)$
$s_1(0)$	1	0	0	$s_1(100)$	1	-1	-1	$s_1(90)$	1	-1	-1
$s_2(0)$	0	1	0	$s_2(0)$	0	1	0	$s_2(0)$	0	1	0
$s_3(0)$	0	0	1	$s_3(0)$	0	0	1	$s_3(10)$	0	0	1
$s_4(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_5(0)$	1	0	0	$s_5(100)$	1	-1	-1	$s_5(90)$	1	-1	-1
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0
$s_7(0)$	0	0	1	$s_7(0)$	0	0	1	$p_3(10)$	0	0	1
$s_8(0)$	1	2	5	$s_8(100)$	1	1	4	$s_8(140)$	1	1	4
$s_9(0)$	3	2	1	$s_9(300)$	3	-1	-2	$s_9(280)$	3	-1	-2

Less lazy theory solving

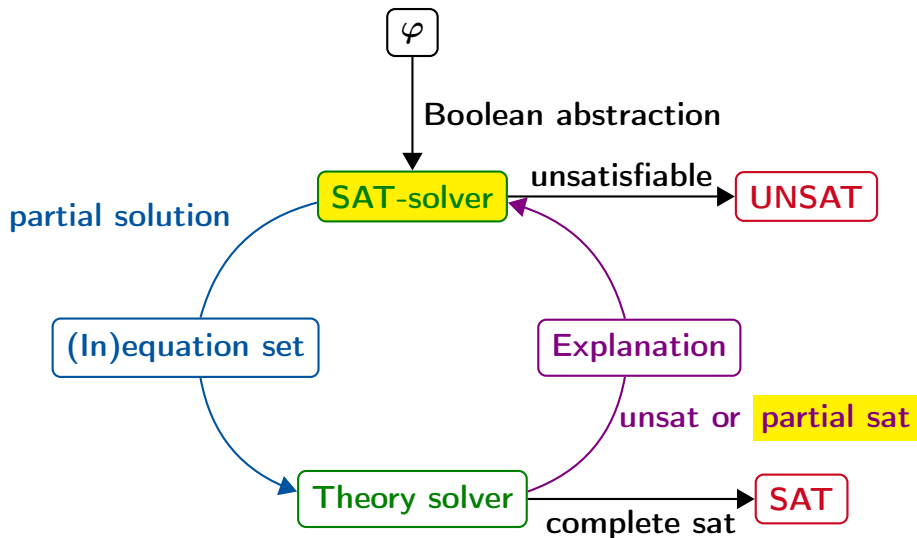
$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & & & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + & p_2 + & p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & & & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & & & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + & 2p_2 + & 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + & 2p_2 + & p_3 & s_9 \leq 300
 \end{array}$$

Variable order: $s_1 < \dots < s_9 < p_1 < p_2 < p_3$, the values of the variables are given in parentheses

	$p_1(0)$	$p_2(0)$	$p_3(0)$		$s_4(100)$	$p_2(0)$	$p_3(0)$		$s_4(100)$	$p_2(0)$	$s_7(10)$
$s_1(0)$	1	0	0	$s_1(100)$	1	-1	-1	$s_1(90)$	1	-1	-1
$s_2(0)$	0	1	0	$s_2(0)$	0	1	0	$s_2(0)$	0	1	0
$s_3(0)$	0	0	1	$s_3(0)$	0	0	1	$s_3(10)$	0	0	1
$s_4(0)$	1	1	1	$p_1(100)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_5(0)$	1	0	0	$s_5(100)$	1	-1	-1	$s_5(90)$	1	-1	-1
$s_6(0)$	0	1	0	$s_6(0)$	0	1	0	$s_6(0)$	0	1	0
$s_7(0)$	0	0	1	$s_7(0)$	0	0	1	$p_3(10)$	0	0	1
$s_8(0)$	1	2	5	$s_8(100)$	1	1	4	$s_8(140)$	1	1	4
$s_9(0)$	3	2	1	$s_9(300)$	3	-1	-2	$s_9(280)$	3	-1	-2

Return partial SAT.

Less lazy SMT-solving



$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

DL1 : $a_1 : 0$

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9$$

Assume a fixed variable order: a_1, \dots, a_9

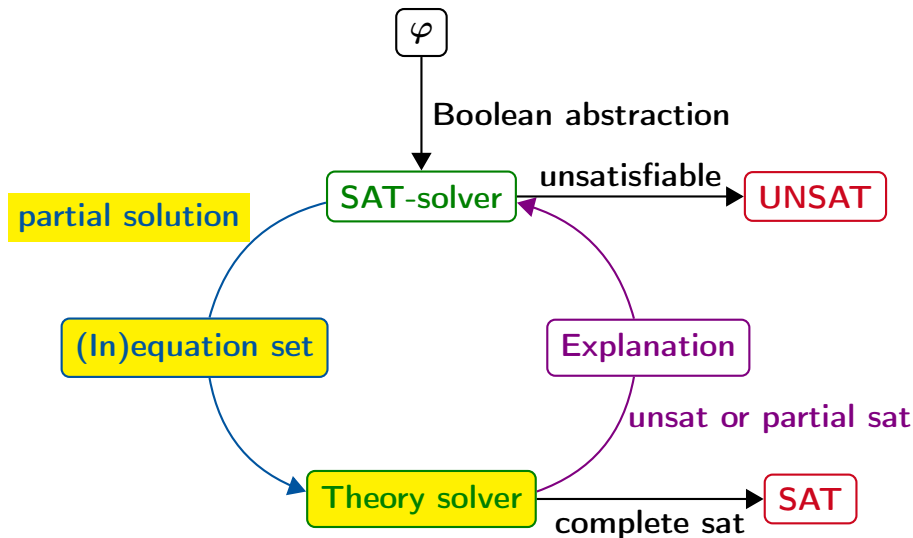
Assignment to decision variables: false

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

DL1 : $a_1 : 0$

DL2 : $a_2 : 0, a_3 : 1$

Less lazy SMT-solving



Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1,$ $DL1 : a_1 : 0,$
 $DL2 : a_2 : 0, a_3 : 1$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1$

Incrementality: add a_3

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1$

Incrementality: add a_3

$$\begin{aligned} & (\underbrace{p_1 = 0}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3}) \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge \\ & (\underbrace{p_1 \geq 5}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6}) \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge \\ & \underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \end{aligned}$$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, \quad DL1 : a_1 : 0,$

$DL2 : a_2 : 0, a_3 : 1$

Incrementality: add a_3

$$\begin{aligned} & (\underbrace{p_1 = 0}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3}) \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge \\ & (\underbrace{p_1 \geq 5}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6}) \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge \\ & \underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \end{aligned}$$

Encoding:

$$s_3 = 0$$

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & & & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + & p_2 + & p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & & & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & & p_2 & & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & & & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + & 2p_2 + & 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + & 2p_2 + & p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$
$s_1(90)$	1	-1	-1
$s_2(0)$	0	1	0
$s_3(10)$	0	0	1
$p_1(90)$	1	-1	-1
$s_5(90)$	1	-1	-1
$s_6(0)$	0	1	0
$p_3(10)$	0	0	1
$s_8(140)$	1	1	4
$s_9(280)$	3	-1	-2

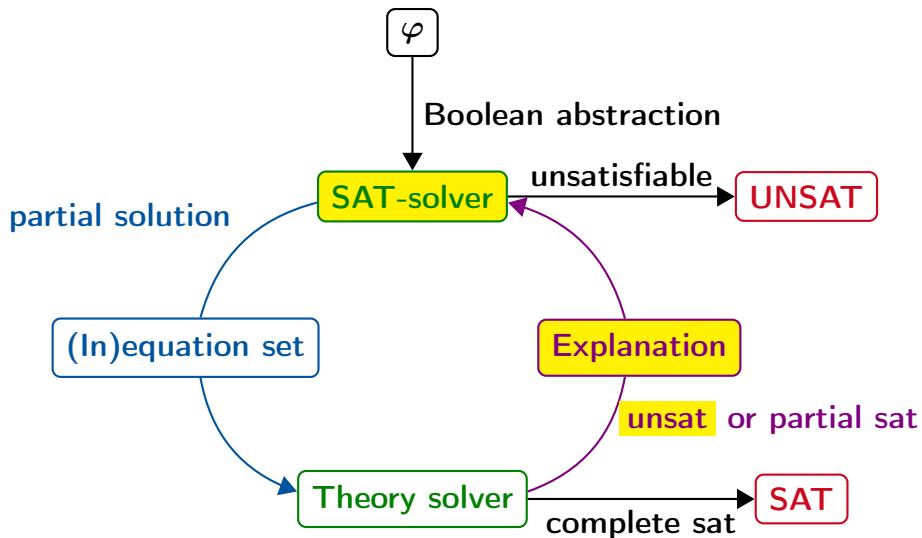
Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$
$s_1(90)$	1	-1	-1
$s_2(0)$	0	1	0
$s_3(10)$	0	0	1
$p_1(90)$	1	-1	-1
$s_5(90)$	1	-1	-1
$s_6(0)$	0	1	0
$p_3(10)$	0	0	1
$s_8(140)$	1	1	4
$s_9(280)$	3	-1	-2

Conflict: $\underbrace{p_3 = 0}_{a_3} \wedge \underbrace{p_3 \geq 10}_{a_7}$ is not satisfiable.

Less lazy SMT-solving



Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

Add clause $(\neg a_3 \vee \neg a_7)$:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

Add clause $(\neg a_3 \vee \neg a_7)$:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

Add clause $(\neg a_3 \vee \neg a_7)$:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtracking removes $DL1$ and $DL2$ first, then propagation is applied.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

Add clause $(\neg a_3 \vee \neg a_7)$:

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtracking removes $DL1$ and $DL2$ first, then propagation is applied.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1$

$DL1 : a_1 : 0$

$DL2 : a_2 : 0, a_3 : 1$

Add clause $(\neg a_3 \vee \neg a_7)$:

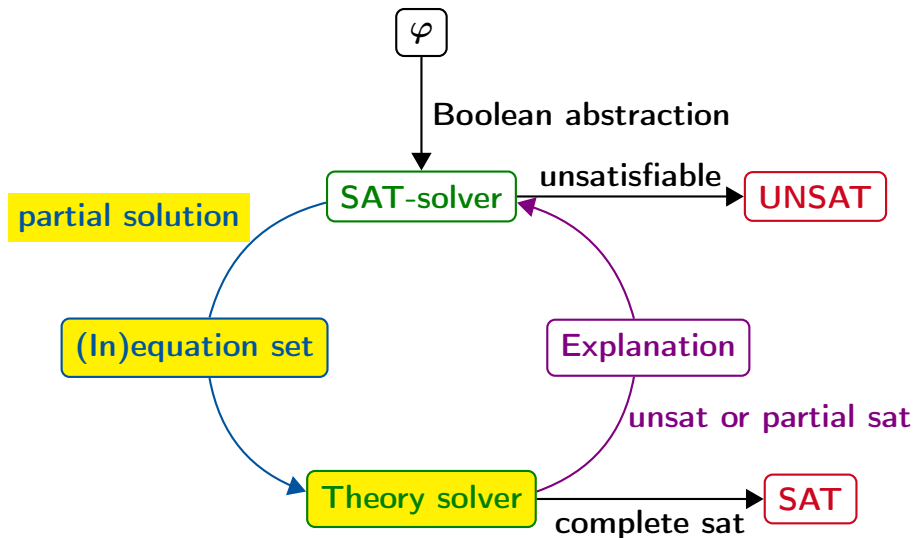
$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

No conflict resolution needed, since the new clause is already asserting.
Backtracking removes $DL1$ and $DL2$ first, then propagation is applied.

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

Less lazy SMT-solving



Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$ $DL1 : a_1 : 0, a_2 : 1$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$ $DL1 : a_1 : 0, a_2 : 1$

Backtracking: remove a_3 , Incrementality: add a_2

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$ $DL1 : a_1 : 0, a_2 : 1$

Backtracking: remove a_3 , Incrementality: add a_2

$$\underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge$$
$$\underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9}$$

Less lazy theory solving

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$ DL1 : $a_1 : 0, a_2 : 1$

Backtracking: remove a_3 , Incrementality: add a_2

$$\begin{aligned} & (\underbrace{p_1 = 0}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3}) \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge \\ & (\underbrace{p_1 \geq 5}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6}) \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge \\ & \underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \end{aligned}$$

Encoding:

remove $s_3 = 0$, add $s_2 = 0$

Less lazy theory solving

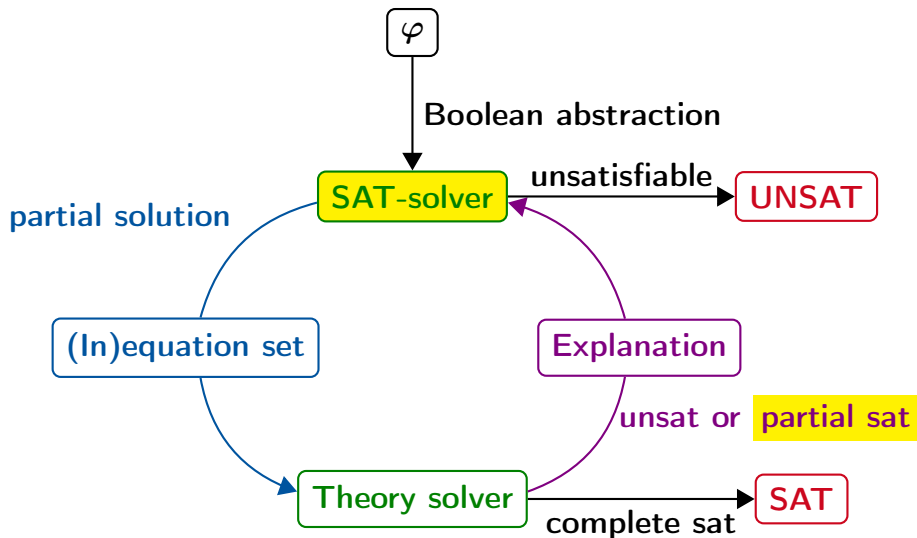
Backtracking: remove bound $s_3 = 0$, add bound $s_2 = 0$

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$
$s_1(90)$	1	-1	-1
$s_2(0)$	0	1	0
$s_3(10)$	0	0	1
$p_1(90)$	1	-1	-1
$s_5(90)$	1	-1	-1
$s_6(0)$	0	1	0
$p_3(10)$	0	0	1
$s_8(140)$	1	1	4
$s_9(280)$	3	-1	-2

Return partial SAT.

Less lazy SMT-solving



$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

DL1 : $a_1 : 0, a_2 : 1$

Less lazy SAT-solving

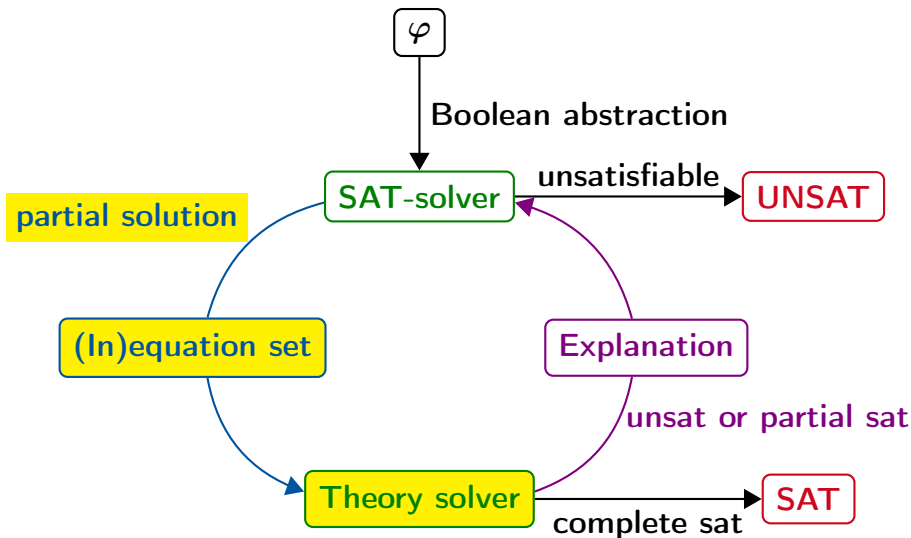
$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7)$$

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

DL1 : $a_1 : 0, a_2 : 1$

DL2 : $a_5 : 0, a_6 : 1$

Less lazy SMT-solving



Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

Incrementality: add a_6

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

Incrementality: add a_6

$$\begin{aligned} & (\underbrace{p_1 = 0}_{a_1} \vee \underbrace{p_2 = 0}_{a_2} \vee \underbrace{p_3 = 0}_{a_3}) \wedge \underbrace{p_1 + p_2 + p_3 \geq 100}_{a_4} \wedge \\ & (\underbrace{p_1 \geq 5}_{a_5} \vee \underbrace{p_2 \geq 5}_{a_6}) \wedge \underbrace{p_3 \geq 10}_{a_7} \wedge \underbrace{p_1 + 2p_2 + 5p_3 \leq 180}_{a_8} \wedge \\ & \underbrace{3p_1 + 2p_2 + p_3 \leq 300}_{a_9} \wedge (\neg a_3 \vee \neg a_7) \end{aligned}$$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1,$
 $DL2 : a_5 : 0, a_6 : 1$

Incrementality: add a_6

$$\underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge$$
$$\underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge$$
$$\underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9} \wedge (\neg a_3 \vee \neg a_7)$$

Encoding:

$$s_6 \geq 5$$

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$				
$s_1(90)$	1	-1	-1				
$s_2(0)$	0	1	0				
$s_3(10)$	0	0	1				
$p_1(90)$	1	-1	-1				
$s_5(90)$	1	-1	-1				
$s_6(0)$	0	1	0				
$p_3(10)$	0	0	1				
$s_8(140)$	1	1	4				
$s_9(280)$	3	-1	-2				

Less lazy theory solving

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$		$s_4(100)$	$s_6(5)$	$s_7(10)$
$s_1(90)$	1	-1	-1	$s_1(85)$	1	-1	-1
$s_2(0)$	0	1	0	$s_2(5)$	0	1	0
$s_3(10)$	0	0	1	$s_3(10)$	0	0	1
$p_1(90)$	1	-1	-1	$p_1(85)$	1	-1	-1
$s_5(90)$	1	-1	-1	$s_5(85)$	1	-1	-1
$s_6(0)$	0	1	0	$p_2(5)$	0	1	0
$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_8(140)$	1	1	4	$s_8(145)$	1	1	4
$s_9(280)$	3	-1	-2	$s_9(275)$	3	-1	-2

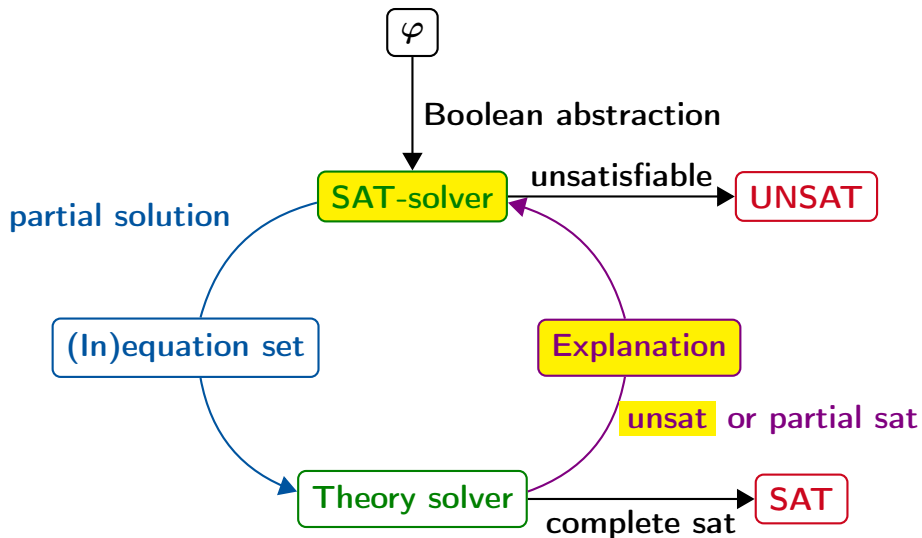
Less lazy theory solving

$$\begin{array}{lll}
 p_1 = 0 & \rightarrow & s_1 = p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$p_2(0)$	$s_7(10)$		$s_4(100)$	$s_6(5)$	$s_7(10)$
$s_1(90)$	1	-1	-1	$s_1(85)$	1	-1	-1
$s_2(0)$	0	1	0	$s_2(5)$	0	1	0
$s_3(10)$	0	0	1	$s_3(10)$	0	0	1
$p_1(90)$	1	-1	-1	$p_1(85)$	1	-1	-1
$s_5(90)$	1	-1	-1	$s_5(85)$	1	-1	-1
$s_6(0)$	0	1	0	$p_2(5)$	0	1	0
$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_8(140)$	1	1	4	$s_8(145)$	1	1	4
$s_9(280)$	3	-1	-2	$s_9(275)$	3	-1	-2

Conflict: $\underbrace{p_2 = 0}_{a_2} \wedge \underbrace{p_2 \geq 5}_{a_6}$ is not satisfiable.

Less lazy SMT-solving



Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Add clause $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \\ \wedge (\neg a_2 \vee \neg a_6)$$

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Add clause $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \\ \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting. Backtracking removes $DL2$ first, then propagation is used to imply new assignments (first using the new learnt clause).

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Add clause $(\neg a_2 \vee \neg a_6)$.

$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \\ \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting. Backtracking removes $DL2$ first, then propagation is used to imply new assignments (first using the new learnt clause).

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

Less lazy SAT-solving

Current assignment:

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1$

$DL2 : a_5 : 0, a_6 : 1$

Add clause $(\neg a_2 \vee \neg a_6)$.

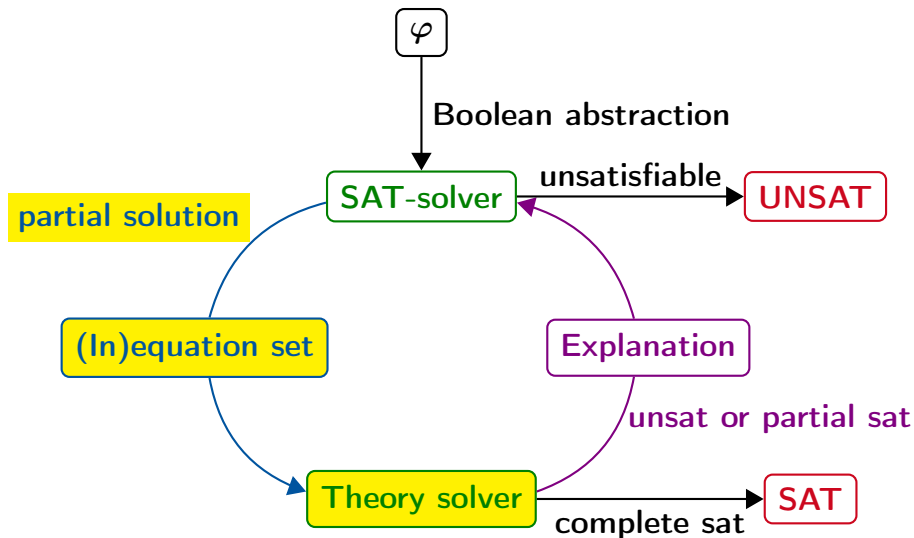
$$(a_1 \vee a_2 \vee a_3) \wedge a_4 \wedge (a_5 \vee a_6) \wedge a_7 \wedge a_8 \wedge a_9 \wedge (\neg a_3 \vee \neg a_7) \\ \wedge (\neg a_2 \vee \neg a_6)$$

No conflict resolution needed, since the new clause is already asserting. Backtracking removes $DL2$ first, then propagation is used to imply new assignments (first using the new learnt clause).

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$

$DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Less lazy SMT-solving



Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0,$ $DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0, \quad DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Backtracking: remove a_6 , Incrementality: add a_5

Less lazy theory solving

$DL0 : a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0, \quad DL1 : a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Backtracking: remove a_6 , Incrementality: add a_5

$$\begin{aligned} & \underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge \\ & \underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge \\ & \underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9} \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6) \end{aligned}$$

Less lazy theory solving

DL0 : $a_4 : 1, a_7 : 1, a_8 : 1, a_9 : 1, a_3 : 0$, DL1 : $a_1 : 0, a_2 : 1, a_6 : 0, a_5 : 1$

Backtracking: remove a_6 , Incrementality: add a_5

$$\begin{aligned} & \underbrace{(p_1 = 0)}_{a_1} \vee \underbrace{(p_2 = 0)}_{a_2} \vee \underbrace{(p_3 = 0)}_{a_3} \wedge \underbrace{(p_1 + p_2 + p_3 \geq 100)}_{a_4} \wedge \\ & \underbrace{(p_1 \geq 5)}_{a_5} \vee \underbrace{(p_2 \geq 5)}_{a_6} \wedge \underbrace{(p_3 \geq 10)}_{a_7} \wedge \underbrace{(p_1 + 2p_2 + 5p_3 \leq 180)}_{a_8} \wedge \\ & \underbrace{(3p_1 + 2p_2 + p_3 \leq 300)}_{a_9} \wedge (\neg a_3 \vee \neg a_7) \wedge (\neg a_2 \vee \neg a_6) \end{aligned}$$

Encoding: remove $s_6 \geq 5$, add $s_5 \geq 5$

Less lazy theory solving

Backtracking: remove $s_6 \geq 5$, Incrementality: add $s_5 \geq 5$

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$s_6(5)$	$s_7(10)$				
$s_1(85)$	1	-1	-1				
$s_2(5)$	0	1	0				
$s_3(10)$	0	0	1				
$p_1(85)$	1	-1	-1				
$s_5(85)$	1	-1	-1				
$p_2(5)$	0	1	0				
$p_3(10)$	0	0	1				
$s_8(145)$	1	1	4				
$s_9(275)$	3	-1	-2				

Less lazy theory solving

Backtracking: remove $s_6 \geq 5$, Incrementality: add $s_5 \geq 5$

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$s_6(5)$	$s_7(10)$		$s_4(100)$	$s_2(0)$	$s_7(10)$
$s_1(85)$	1	-1	-1	$s_1(90)$	1	-1	-1
$s_2(5)$	0	1	0	$s_6(0)$	0	1	0
$s_3(10)$	0	0	1	$s_3(10)$	0	0	1
$p_1(85)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_5(85)$	1	-1	-1	$s_5(90)$	1	-1	-1
$p_2(5)$	0	1	0	$p_2(0)$	0	1	0
$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_8(145)$	1	1	4	$s_8(140)$	1	1	4
$s_9(275)$	3	-1	-2	$s_9(280)$	3	-1	-2

Less lazy theory solving

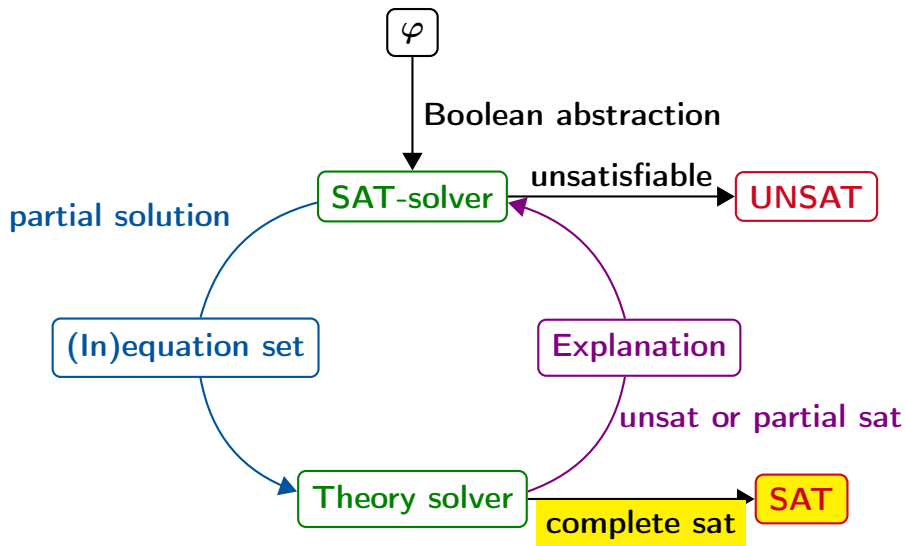
Backtracking: remove $s_6 \geq 5$, Incrementality: add $s_5 \geq 5$

$$\begin{array}{llll}
 p_1 = 0 & \rightarrow & s_1 = & p_1 & s_1 = 0 \\
 p_2 = 0 & \rightarrow & s_2 = & p_2 & s_2 = 0 \\
 p_3 = 0 & \rightarrow & s_3 = & p_3 & s_3 = 0 \\
 p_1 + p_2 + p_3 \geq 100 & \rightarrow & s_4 = & p_1 + p_2 + p_3 & s_4 \geq 100 \\
 p_1 \geq 5 & \rightarrow & s_5 = & p_1 & s_5 \geq 5 \\
 p_2 \geq 5 & \rightarrow & s_6 = & p_2 & s_6 \geq 5 \\
 p_3 \geq 10 & \rightarrow & s_7 = & p_3 & s_7 \geq 10 \\
 p_1 + 2p_2 + 5p_3 \leq 180 & \rightarrow & s_8 = & p_1 + 2p_2 + 5p_3 & s_8 \leq 180 \\
 3p_1 + 2p_2 + p_3 \leq 300 & \rightarrow & s_9 = & 3p_1 + 2p_2 + p_3 & s_9 \leq 300
 \end{array}$$

	$s_4(100)$	$s_6(5)$	$s_7(10)$		$s_4(100)$	$s_2(0)$	$s_7(10)$
$s_1(85)$	1	-1	-1	$s_1(90)$	1	-1	-1
$s_2(5)$	0	1	0	$s_6(0)$	0	1	0
$s_3(10)$	0	0	1	$s_3(10)$	0	0	1
$p_1(85)$	1	-1	-1	$p_1(90)$	1	-1	-1
$s_5(85)$	1	-1	-1	$s_5(90)$	1	-1	-1
$p_2(5)$	0	1	0	$p_2(0)$	0	1	0
$p_3(10)$	0	0	1	$p_3(10)$	0	0	1
$s_8(145)$	1	1	4	$s_8(140)$	1	1	4
$s_9(275)$	3	-1	-2	$s_9(280)$	3	-1	-2

Since the assignment is complete, return SAT for the original problem.

Less lazy SMT-solving



What could also happen...

- **Problem:** When working in the less lazy modus, in the Simplex theory solver a bound of a **non-basic slack variable** s could be activated. If the current value of this non-basic variable now violates its newly activated bound, our invariant (all non-basic variable values are within the corresponding bounds) would not hold!
- **Solution:** Since the result in the previous solver state was SAT, all the basic variables satisfy their bounds. Thus pivoting with an arbitrary basic variable s' whose row has a non-zero coefficient for s solves the problem. Note: there is always such a row.
- **New problem:** Now also the bounds of s' could be activated, leading to a similar problem. However, now it can happen that all basic variables are assigned values outside their bounds!
- **Solution:** After activating a bound, first check satisfiability and activate further bounds afterwards one by one.