# Comparing HyperPCTL, HyperPCTL*, and PHL

## Bachelor's Thesis Final Talk

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LuFG Theory of Hybrid Systems

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# Outline

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# Markov Chains

## Definition

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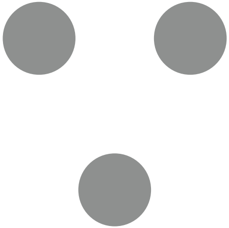

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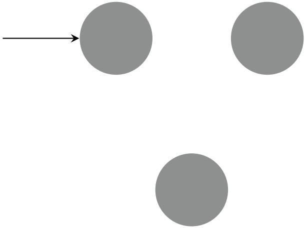

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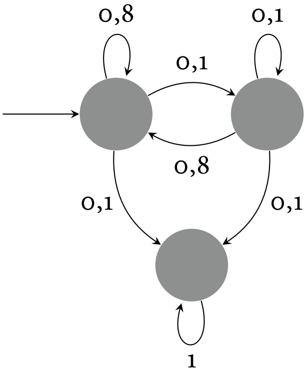

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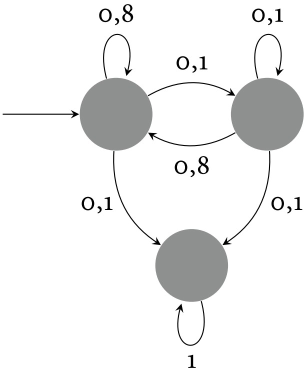

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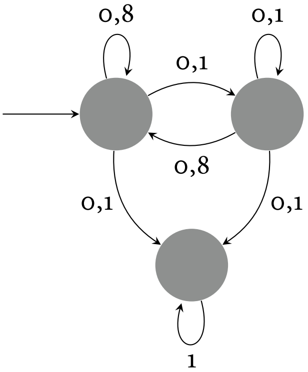

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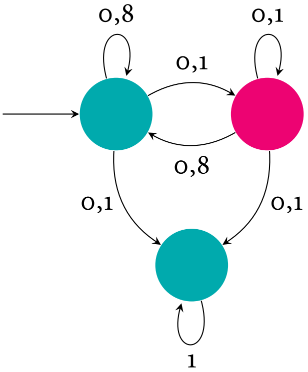

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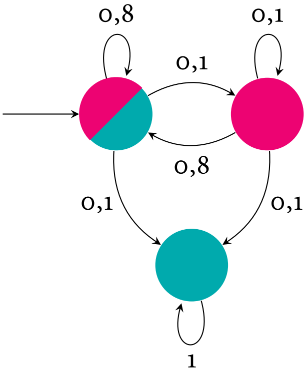

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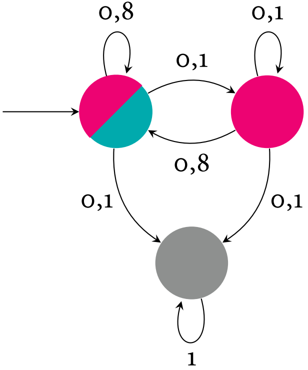

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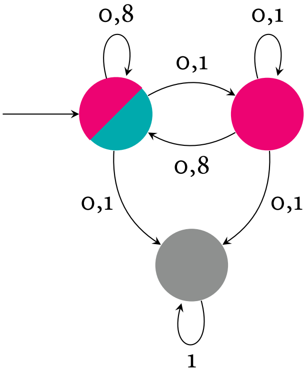
$$\sum_{t \in S} p(s,t) = 1, \quad \forall s \in S$$

For all $s \in S$, the function $p_s$, defined by $p_s(t) := p(s,t)$, is a probability distribution.



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#### Path Expressions

$$\vartheta ::= \bigcirc\varphi \mid \varphi\,\mathsf{U}\,\varphi \mid \varphi\,\mathsf{U}^{\leq k}\,\varphi$$

# HyperPCTL*

## Example

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$$\mathbb{P}_{\hat{\pi}_1 \leftarrow \varepsilon}\left(\bullet_{\hat{\pi}_1} \mathbin{\mathsf{U}} \left(\mathbb{P}_{\hat{\pi}_2 \leftarrow \hat{\pi}_1, \hat{\pi}_3 \leftarrow \varepsilon}\left(\bullet_{\hat{\pi}_2} \mathbin{\mathsf{U}} \bullet_{\hat{\pi}_3}\right) > c_1\right)\right) > c_2$$

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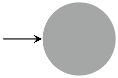

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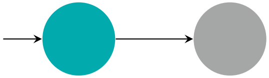

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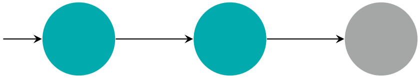

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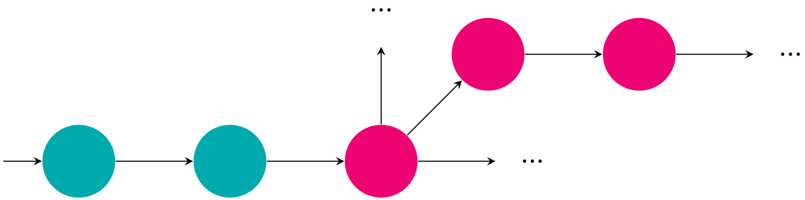

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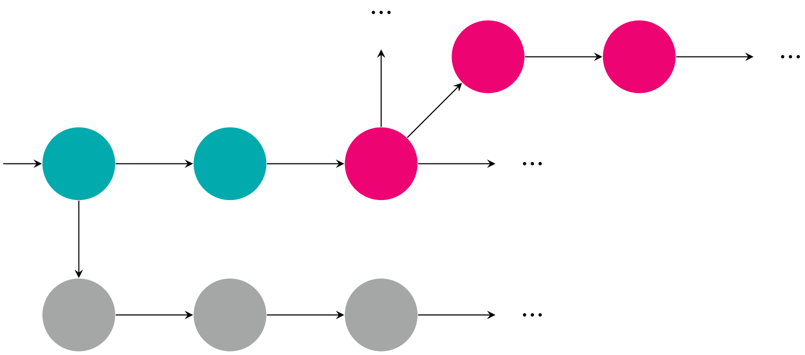

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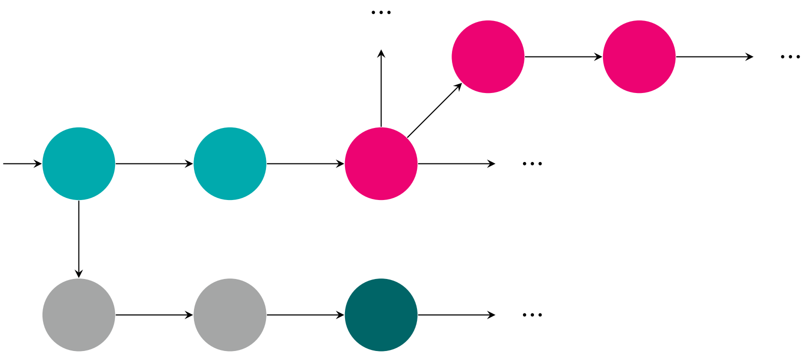

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Taking out all $\vartheta$ rules from PHL, we are left with

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- Map $\mathbb{P}$ to $\mathbb{P}_{\hat{\pi} \leftarrow \varepsilon}$

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**Theorem**

There is no PHL probabilistic expression that evaluates equivalently to $\mathbb{P}_{\hat{\pi}_1,\hat{\pi}_2}(a_{\hat{\pi}_1} \mathsf{U}\, b_{\hat{\pi}_2})$ on DTMCs.

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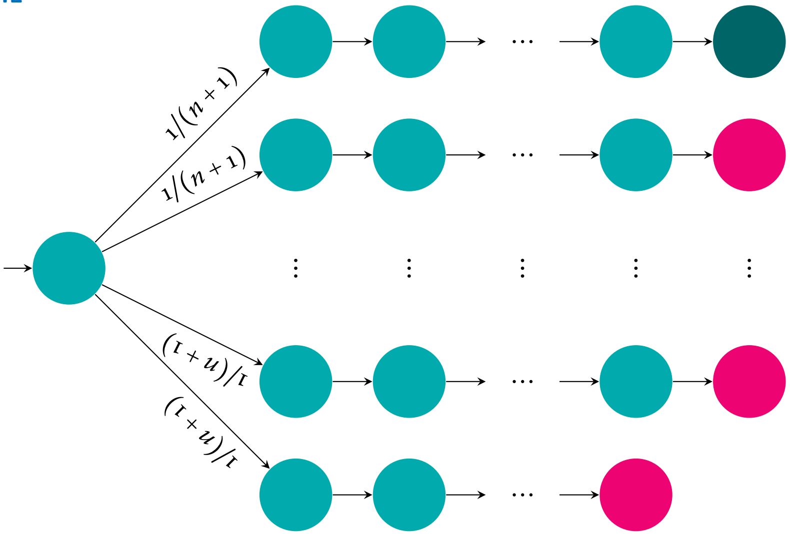

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What does not work:

$$\forall \hat{\pi}.\Diamond a_{\hat{\pi}} \models \mathbb{P}(\Diamond a) = 1 \quad \text{but} \quad \mathbb{P}(\Diamond a) = 1 \not\models \forall \hat{\pi}.\Diamond a_{\hat{\pi}}$$

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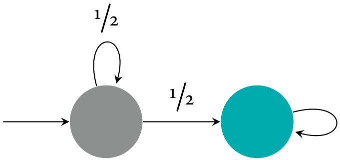

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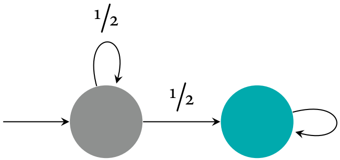


$\forall \hat{\pi}.\Diamond\bullet_{\hat{\pi}}$ is a divergent property.

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Idea: Isolate properties that can be consistently checked against path prefixes, since counterexamples are probabilistically 'detectable' $\Longrightarrow$ nondivergent properties.

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## Cylinder Sets, Measures, and Divergence

### Definition

A property $P$ is called **non-divergent** iff to each DTMC $\mathcal{D}$ and path $\pi$ on $\mathcal{D}$ with

$$\mathcal{D}, \pi \models P$$

there exists a finite prefix $\pi_{\mathrm{PRE}} \sqsubseteq \pi$ with

$$\mathcal{D}, \pi' \models P,$$

for all $\pi' \in \mathsf{Cyl}(\pi_{\mathrm{PRE}})$.

# Comparison: HyperPCTL* vs. PHL

## Identifying a non-divergent fragment of HyperCTL*

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- Path-positive: Only strictly propositional subformulae can be negated:
  - $\mathcal{D}, \pi \vDash \neg(a \,\mathsf{U}\, b) \iff \neg\Big(\exists i < \omega : b \in l\big(\pi(i)\big) \;\wedge\; \forall j < i : a \in l\big(\pi(j)\big)\Big)$
    $\iff \forall i < \omega : b \notin l\big(\pi(i)\big) \;\vee\; \exists j < i : a \notin l\big(\pi(j)\big)$

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  - $\mathcal{D}, \pi \models \neg(a \,\mathsf{U}\, b) \iff \neg\Big(\exists i < \omega : b \in l\big(\pi(i)\big) \wedge \forall j < i : a \in l\big(\pi(j)\big)\Big)$
    $\iff \forall i < \omega : b \notin l\big(\pi(i)\big) \vee \exists j < i : a \notin l\big(\pi(j)\big)$

# Comparison: HyperPCTL* vs. PHL

## Identifying a non-divergent fragment of HyperCTL*

$$\vartheta ::= a_{\hat{\pi}} \mid \mathsf{true} \mid \vartheta \wedge \vartheta \mid \neg\vartheta \mid \bigcirc\vartheta \mid \vartheta \,\mathsf{U}\, \vartheta \mid \forall\hat{\pi}.\vartheta \qquad \text{(HyperCTL}^*\text{)}$$

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    $\iff \forall i < \omega : b \notin l\big(\pi(i)\big) \;\vee\; \exists j < i : a \notin l\big(\pi(j)\big)$
  - Negation of $\bigcirc$ implicitly allowed via $\neg\bigcirc a \equiv \bigcirc\neg a$.

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$\rightsquigarrow$ the recursively-existential path-positive fragment of HyperCTL*: $[\downarrow\exists^*|\pi^+]$

# Comparison: HyperPCTL* vs. PHL

## Rec.-exist. path-positive HyperCTL* to HyperPCTL*

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**Theorem**

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**Theorem**

$$[\downarrow\exists^*|\pi^+]\text{-HyperCTL}^* \preccurlyeq \text{HyperPCTL}^*$$

Proof: Nondivergence & Explicit transformation:

- Map '$\exists\hat{\pi}.\ \ldots$' to '$\mathbb{P}_{\hat{\pi}\leftarrow\mathsf{last}}(\ldots) > 0$' recursively.
- Take over all other syntactic elements.

master@090e759

# Comparison: HyperPCTL* vs. PHL

## Rec.-exist. path-positive HyperCTL* to HyperPCTL*

$$\exists \hat{\pi}_1.\ \exists \hat{\pi}_2.\ \left(\exists \hat{\pi}_3.\ \Diamond \bullet_{\hat{\pi}_3}\right) \mathsf{U} \bullet_{\hat{\pi}_1} \mathsf{U} \Diamond \bullet_{\hat{\pi}_2}$$

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$$\mathbb{P}_{\hat{\pi}_1\leftarrow\varepsilon,\hat{\pi}_2\leftarrow\varepsilon}\Big( \qquad\qquad\qquad\qquad \Big) > 0$$

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# Outline

# HyperPCTL

## Syntax

### Syntax

#### State Formulae

$$\varphi ::= \forall \hat{s}.\varphi \mid \exists \hat{s}.\varphi \mid \varphi \wedge \varphi \mid \neg\varphi \mid \texttt{true} \mid a_{\hat{s}} \mid \rho < \rho$$

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$$\varphi ::= \forall \hat{s}.\varphi \mid \exists \hat{s}.\varphi \mid \varphi \land \varphi \mid \neg\varphi \mid \mathtt{true} \mid a_{\hat{s}} \mid \rho < \rho$$

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#### Path Formulae

$$\vartheta ::= \bigcirc\varphi \mid \varphi \,\mathsf{U}\, \varphi \mid \varphi \,\mathsf{U}^{[k_1,k_2]}\, \varphi$$

# PHL Syntax Recap

## Definition for Markov Chains

### Syntax

#### Top-Level Formulae

$$\varphi ::= \varphi \wedge \varphi \mid \neg\varphi \mid \rho < \rho \mid \vartheta$$

#### Probabistic Expressions & **Unmarked** LTL

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## Incompatibilities

$\mathcal{D} = (S, s_\iota, p, \mathsf{AP}, l)$

PHL relies on the presence of an initial state, HyperPCTL does not.

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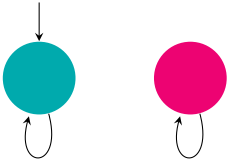

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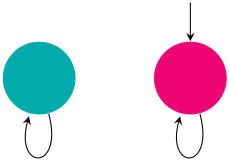

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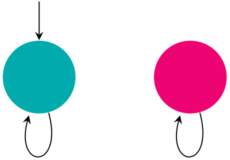

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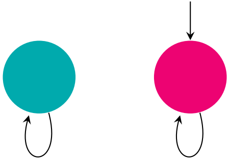

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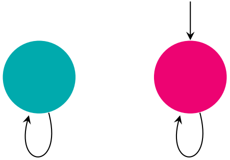

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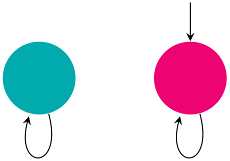

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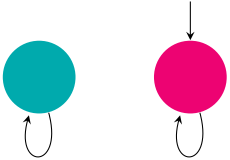

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Partial solution: Assume $s_\iota$ is uniquely marked with `init`.

# Comparison: HyperPCTL vs. PHL

## HyperCTL*-less PHL to HyperPCTL

### Top-Level Formulae

$$\varphi ::= \varphi \wedge \varphi \mid \neg\varphi \mid \rho < \rho$$

### Probabilistic Expressions & LTL

$$\rho ::= \mathbb{P}(\eta) \mid \rho + \rho \mid c \cdot \rho \mid c$$

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  - Allow $\bigcirc^k\eta$, as it can be emulated via $\mathsf{true}\,\mathsf{U}^{[k,k]}\,\eta$.

# Comparison: HyperPCTL vs. PHL

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  - Allow $\bigcirc^k\eta$, as it can be emulated via $\mathsf{true}\,\mathsf{U}^{[k,k]}\,\eta$.
- Special cases
  - $\mathbb{P}(\Diamond\Box\eta) \sim c$.
  - $\mathbb{P}(\Box\Diamond\eta) \sim c$.

# Comparison: HyperPCTL vs. PHL

## Focusing on HyperCTL*

$$\vartheta ::= a_{\hat{\pi}} \mid \mathtt{true} \mid \vartheta \wedge \vartheta \mid \neg\vartheta \mid \bigcirc\vartheta \mid \vartheta \,\mathsf{U}\, \vartheta \mid \forall\hat{\pi}.\vartheta \qquad (\text{HyperCTL}^*)$$

First, carry over what we can from HyperPCTL* vs. PHL.

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- Modification:
  - HyperPCTL quantifies over states, has no intrinsics to continue paths where previously drawn ones start $\rightsquigarrow$ restrict HyperCTL* to PNF.

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$\rightsquigarrow$ PNF-existential LTL-shallow HyperCTL*: $[\exists^*|\mathrm{LTL}^s]$

# Comparison: HyperPCTL vs. PHL

## PNF-existential LTL-shallow HyperCTL* to HyperPCTL

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Proof: Nondivergence & Explicit transformation:

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- Map '$\exists \hat{\pi}_0 \cdots \exists \hat{\pi}_n. \ \ldots$' to '$\exists \hat{s}_0 \cdots \exists \hat{s}_n. \bigwedge_{i \le n} \mathtt{init}_{\hat{s}_i} \wedge \mathbb{P}(\ldots) > 0$'.

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- Map $a_{\hat{\pi}_i}$ to $a_{\hat{s}_i}$.

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- Take over all other syntactic elements.

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

**Conjecture**

$$[\downarrow\exists^*|\text{LTL}^s]\text{-HyperCTL}^* \preccurlyeq \text{HyperPCTL}$$

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Idea: Bind the behaviour of each path variable to that of the one that comes before it.

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists \hat{\pi}_1.\ a_{\hat{\pi}_1} \mathsf{U} \exists \hat{\pi}_2.\ \left(b_{\hat{\pi}_2} \mathsf{U} \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\right)$.

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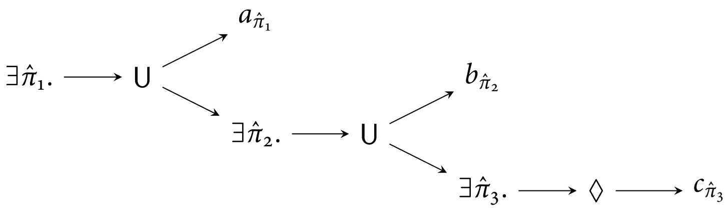

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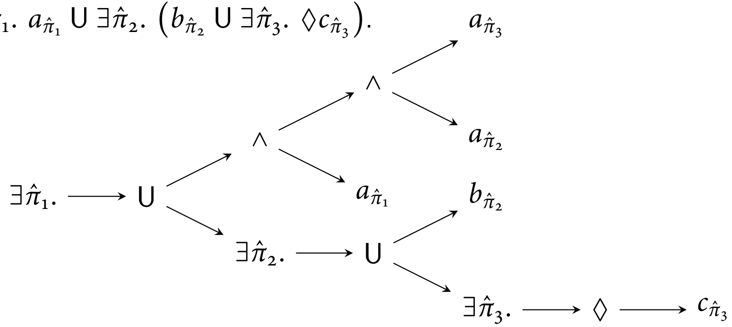

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## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

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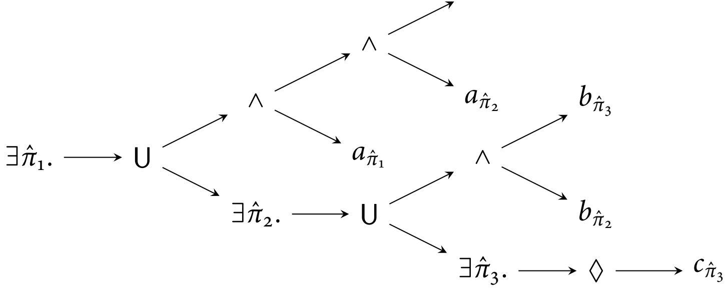
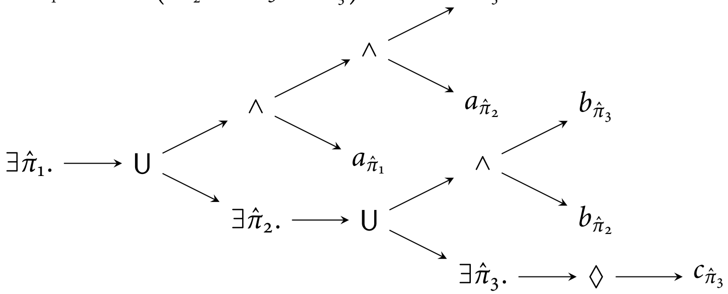


# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists \hat{\pi}_1.\ a_{\hat{\pi}_1}\ \mathsf{U}\ \exists \hat{\pi}_2.\ \big(b_{\hat{\pi}_2}\ \mathsf{U}\ \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$

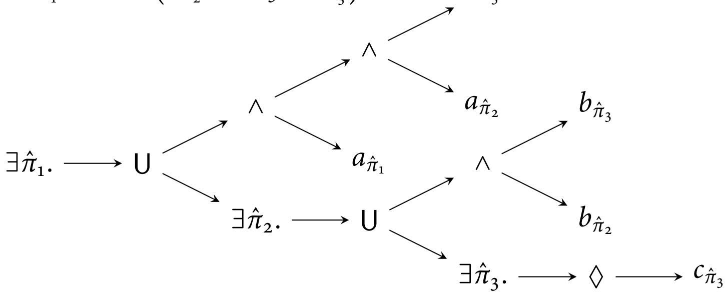



$$\exists \hat{\pi}_1.\ (a_{\hat{\pi}_1} \wedge a_{\hat{\pi}_2} \wedge a_{\hat{\pi}_3})\ \mathsf{U}\ \exists \hat{\pi}_2.\ \big((b_{\hat{\pi}_2} \wedge b_{\hat{\pi}_3})\ \mathsf{U}\ \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big)$$

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists \hat{\pi}_1.\ a_{\hat{\pi}_1} \,\mathsf{U}\, \exists \hat{\pi}_2.\ \big(b_{\hat{\pi}_2} \,\mathsf{U}\, \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$

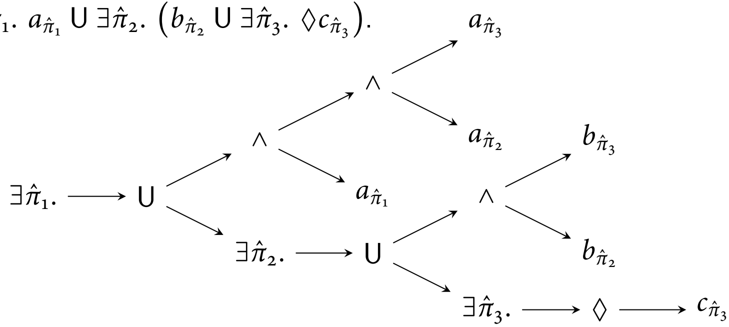



$$\mathbb{P}\Big( (a_{\hat{\pi}_1} \wedge a_{\hat{\pi}_2} \wedge a_{\hat{\pi}_3}) \,\mathsf{U}\, \exists \hat{\pi}_2.\ \big((b_{\hat{\pi}_2} \wedge b_{\hat{\pi}_3}) \,\mathsf{U}\, \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big)\Big) > 0$$

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists\hat{\pi}_1.\ a_{\hat{\pi}_1}\ \mathsf{U}\ \exists\hat{\pi}_2.\ \big(b_{\hat{\pi}_2}\ \mathsf{U}\ \exists\hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$

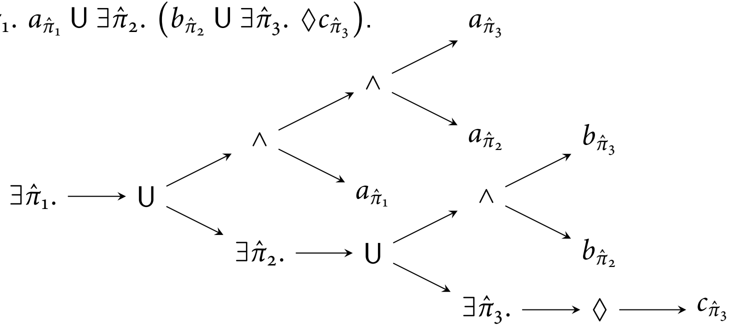



$$\mathbb{P}\Bigg(\big(a_{\hat{\pi}_1} \wedge a_{\hat{\pi}_2} \wedge a_{\hat{\pi}_3}\big)\ \mathsf{U}\ \mathbb{P}\Big(\big(b_{\hat{\pi}_2} \wedge b_{\hat{\pi}_3}\big)\ \mathsf{U}\ \exists\hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\Big) > 0\Bigg) > 0$$

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists\hat{\pi}_1.\ a_{\hat{\pi}_1} \,\mathsf{U}\, \exists\hat{\pi}_2.\ \big(b_{\hat{\pi}_2} \,\mathsf{U}\, \exists\hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$

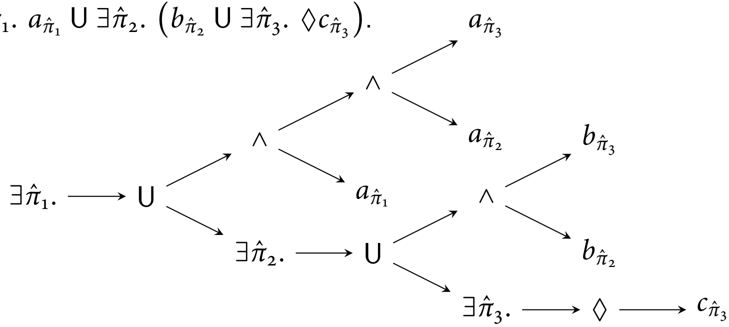



$$\mathbb{P}\Big(\big(a_{\hat{\pi}_1} \wedge a_{\hat{\pi}_2} \wedge a_{\hat{\pi}_3}\big) \,\mathsf{U}\, \mathbb{P}\Big(\big(b_{\hat{\pi}_2} \wedge b_{\hat{\pi}_3}\big) \,\mathsf{U}\, \mathbb{P}(\Diamond c_{\hat{\pi}_3}) > 0\Big) > 0\Big) > 0$$

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists \hat{\pi}_1.\ a_{\hat{\pi}_1} \mathsf{U}\, \exists \hat{\pi}_2.\ \big(b_{\hat{\pi}_2} \mathsf{U}\, \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$

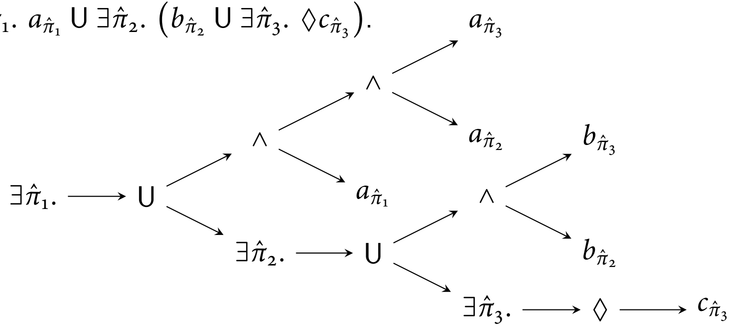



$$\mathbb{P}\Big((a_{\hat{s}_1} \wedge a_{\hat{s}_2} \wedge a_{\hat{s}_3}) \,\mathsf{U}\, \mathbb{P}\big((b_{\hat{s}_2} \wedge b_{\hat{s}_3}) \,\mathsf{U}\, \mathbb{P}(\Diamond c_{\hat{s}_3}) > 0\big) > 0\Big) > 0$$

# Comparison: HyperPCTL vs. PHL

## Rec.-exist. LTL-shallow HyperCTL* to HyperPCTL

Example: $\exists \hat{\pi}_1.\ a_{\hat{\pi}_1} \mathsf{U}\ \exists \hat{\pi}_2.\ \big(b_{\hat{\pi}_2} \mathsf{U}\ \exists \hat{\pi}_3.\ \Diamond c_{\hat{\pi}_3}\big).$




$$\exists \hat{s}_1.\exists \hat{s}_2.\exists \hat{s}_3.\ \bigwedge_{1\leq i\leq 3} \mathtt{init}_{\hat{s}_i} \wedge \mathbb{P}\Bigg( (a_{\hat{s}_1} \wedge a_{\hat{s}_2} \wedge a_{\hat{s}_3})\ \mathsf{U}\ \mathbb{P}\Big( (b_{\hat{s}_2} \wedge b_{\hat{s}_3})\ \mathsf{U}\ \mathbb{P}(\Diamond c_{\hat{s}_3}) > 0\Big) > 0\Bigg) > 0$$

# Outline

# Conclusion

**HyperPCTL* vs. PHL.**

# Conclusion

**HyperPCTL$^*$ vs. PHL.**

- PHL$^{\text{no}\vartheta} \prec$ HyperPCTL$^*$.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\mathsf{PHL}^{\mathsf{no}\vartheta} \prec \mathsf{HyperPCTL}^*$.

  $\mathsf{PHL}^{\mathsf{no}\vartheta} \not\succ \mathsf{HyperPCTL}^*$.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\mathsf{PHL}^{\mathsf{no}\vartheta} \prec \mathsf{HyperPCTL}^*$.

  $\mathsf{PHL}^{\mathsf{no}\vartheta} \not\succ \mathsf{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL* $\prec$ HyperPCTL*.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\mathrm{PHL}^{\mathrm{no}\vartheta} \prec$ HyperPCTL*.

  $\mathrm{PHL}^{\mathrm{no}\vartheta} \not\succ$ HyperPCTL*.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL* $\prec$ HyperPCTL*.

**HyperPCTL vs. PHL.**

# Conclusion

**HyperPCTL$^*$ vs. PHL.**

- $\mathrm{PHL}^{\mathrm{no}\vartheta} \prec \mathrm{HyperPCTL}^*$.

  $\mathrm{PHL}^{\mathrm{no}\vartheta} \not\succ \mathrm{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL$^*$ $\prec$ HyperPCTL$^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.

  $\text{PHL}^{\text{no}\vartheta} \nsucc \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}$.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.

  $\text{PHL}^{\text{no}\vartheta} \not\succcurlyeq \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}$.
- $[\exists^*|\text{LTL}^s]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}$.

# Conclusion

**HyperPCTL$^*$ vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.

  $\text{PHL}^{\text{no}\vartheta} \not\succ \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL$^*$ $\prec$ HyperPCTL$^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\text{LTL}^s]$-HyperCTL$^*$ $\prec$ HyperPCTL.
- $[\downarrow\exists^*|\text{LTL}^s]$-HyperCTL$^*$ $\prec$ HyperPCTL.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.

  $\text{PHL}^{\text{no}\vartheta} \not\succcurlyeq \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}$.
- $[\exists^*|\text{LTL}^s]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}$.
- $[\downarrow\exists^*|\text{LTL}^s]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}$.

**Also covered in the thesis:**

# Conclusion

**$\text{HyperPCTL}^*$ vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.
  $\text{PHL}^{\text{no}\vartheta} \not\succ \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}$.
- $[\exists^*|\text{LTL}^s]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}$.
- $[\downarrow\exists^*|\text{LTL}^s]$-$\text{HyperCTL}^* \prec \text{HyperPCTL}$.

**Also covered in the thesis:**

- Details of the downscaled PHL.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\mathrm{PHL}^{\mathrm{no}\vartheta} \prec \mathrm{HyperPCTL}^*$.
  $\mathrm{PHL}^{\mathrm{no}\vartheta} \nsucc \mathrm{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL* $\prec$ HyperPCTL*.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\mathrm{LTL}^s]$-$\mathrm{PHL}^{\mathrm{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\mathrm{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.
- $[\downarrow\exists^*|\mathrm{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.

**Also covered in the thesis:**

- Details of the downscaled PHL.
- Reverse directions.

# Conclusion

**HyperPCTL$^*$ vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.
  $\text{PHL}^{\text{no}\vartheta} \not\succeq \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL$^*$ $\prec$ HyperPCTL$^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\text{LTL}^s]$-HyperCTL$^*$ $\prec$ HyperPCTL.
- $[\downarrow\exists^*|\text{LTL}^s]$-HyperCTL$^*$ $\prec$ HyperPCTL.

**Also covered in the thesis:**

- Details of the downscaled PHL.
- Reverse directions.
- Path-positiveness ($\pi^+$) cannot be lifted.

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# Conclusion

**HyperPCTL$^*$ vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.

  $\text{PHL}^{\text{no}\vartheta} \not\succ \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL$^* \prec$ HyperPCTL$^*$.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\text{LTL}^s]$-HyperCTL$^* \prec$ HyperPCTL.
- $[\downarrow\exists^*|\text{LTL}^s]$-HyperCTL$^* \prec$ HyperPCTL.

**Also covered in the thesis:**

- Details of the downscaled PHL.
- Reverse directions.
- Path-positiveness $(\pi^+)$ cannot be lifted.
- Excursus on HyperPCTL and HyperPCTL$^*$.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\mathrm{PHL}^{\mathrm{no}\vartheta} \prec \mathrm{HyperPCTL}^*$.
  $\mathrm{PHL}^{\mathrm{no}\vartheta} \not\succcurlyeq \mathrm{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL* $\prec$ HyperPCTL*.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\mathrm{LTL}^s]$-$\mathrm{PHL}^{\mathrm{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\mathrm{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.
- $[\downarrow\exists^*|\mathrm{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.

**Also covered in the thesis:**

- Details of the downscaled PHL.
- Reverse directions.
- Path-positiveness $(\pi^+)$ cannot be lifted.
- Excursus on HyperPCTL and HyperPCTL*.
- Upscaling results for MDPs.

# Conclusion

**HyperPCTL* vs. PHL.**

- $\text{PHL}^{\text{no}\vartheta} \prec \text{HyperPCTL}^*$.
  $\text{PHL}^{\text{no}\vartheta} \not\succ \text{HyperPCTL}^*$.
- $[\downarrow\exists^*|\pi^+]$-HyperCTL* $\prec$ HyperPCTL*.

**HyperPCTL vs. PHL.**

- Incompatibilities.
  - Initial states.
  - Nested paths.
- $[\text{LTL}^s]$-$\text{PHL}^{\text{no}\vartheta} \prec$ HyperPCTL.
- $[\exists^*|\text{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.
- $[\downarrow\exists^*|\text{LTL}^s]$-HyperCTL* $\prec$ HyperPCTL.

**Also covered in the thesis:**

- Details of the downscaled PHL.
- Reverse directions.
- Path-positiveness $(\pi^+)$ cannot be lifted.
- Excursus on HyperPCTL and HyperPCTL*.
- Upscaling results for MDPs.
- Modifications to HyperPCTL*.