Parameter synthesis for algebraic problems with a Boolean structure

Master of Science Thesis

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RWTH Aachen University
LuFG Theory of Hybrid Systems

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Outline

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1. Preliminaries
2. Parameter Synthesis
   - Base Algorithm
   - Sampling Heuristics
   - Splitting Heuristics
   - Incremental Solving
3. Experimental Evaluation

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Preliminaries

Nonlinear Real Arithmetic

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- first-order logic over the reals ($\mathbb{R}$)

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Example

$$\varphi(x, y) := (x \leq 0) \vee (y \geq x^3)$$

Preliminaries

SMT-Solving

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- logical formula $\varphi(x_1, ..., x_n)$

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Output

- $\varphi(0, 0)$

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Example

Input

- $\varphi(x, y) := (x \leq 0) \vee (y \geq x^3)$

Output

- $\varphi(0, 0) = (0 \leq 0) \vee (0 \geq 0^3) = \mathit{true} \vee \mathit{true} = \mathit{true}$

Preliminaries

SMT-Solving

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- Is $\varphi(x_1, ..., x_n)$ satisfiable?

Example

Input

- $\varphi(x, y) := (x \leq 0) \vee (y \geq x^3)$

Output

- $\varphi(0, 0) = (0 \leq 0) \vee (0 \geq 0^3) = \mathit{true} \vee \mathit{true} = \mathit{true}$
- φ is satisfiable!

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Parameter Synthesis Problem

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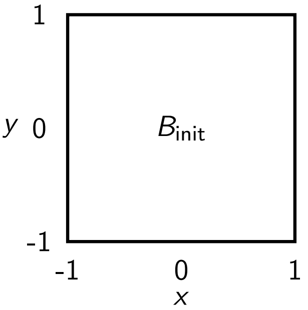

Preliminaries

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Input

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- find S_+, S_-
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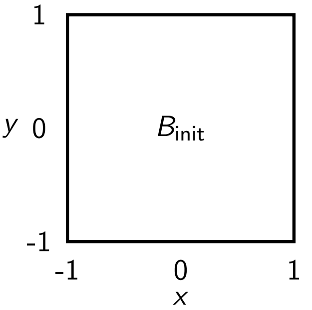

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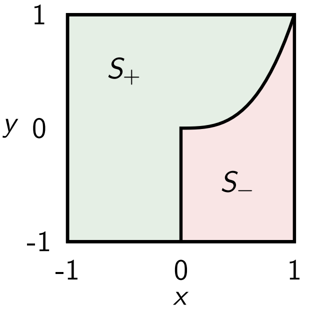

Preliminaries

Relaxed Parameter Synthesis Problem

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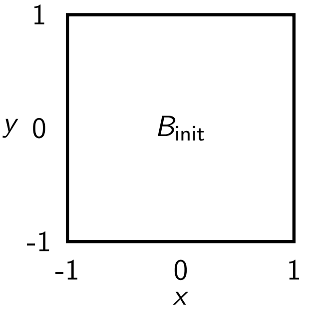

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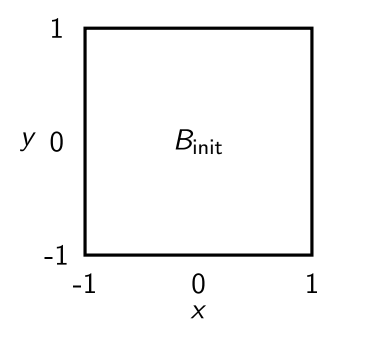

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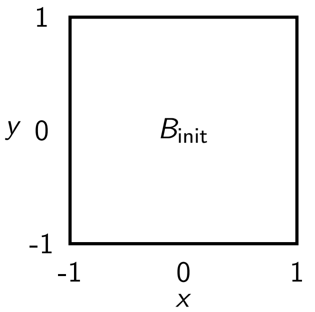

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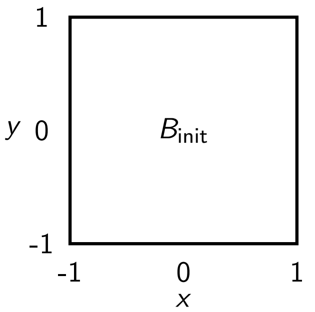

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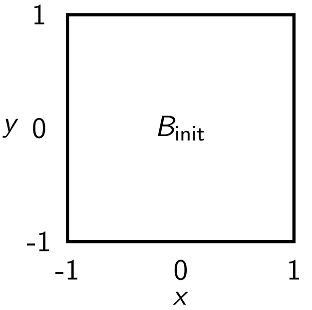

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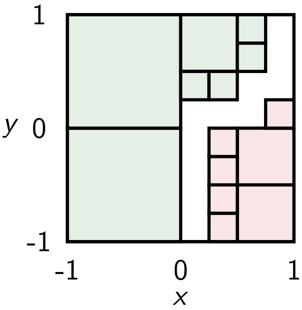

Preliminaries

Satisfying Box

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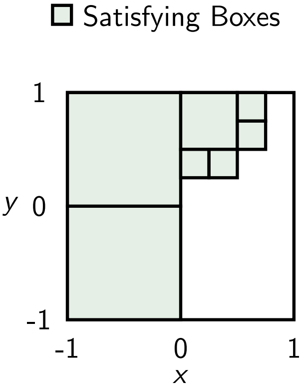


Preliminaries

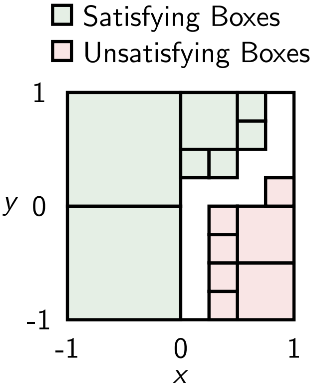
Satisfying Box

- $\forall x(B(x) \rightarrow \varphi(x))$

Unsatisfying Box

- $\forall x(B(x) \rightarrow \neg\varphi(x))$

Example



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Overview

$\boxed{B_{\text{init}}, \varphi}$

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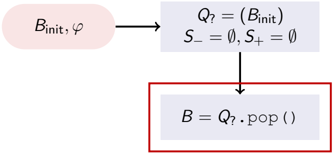
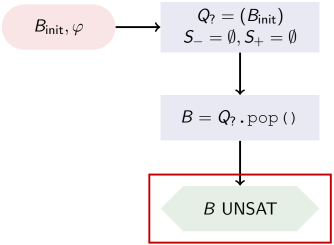
B_{init}, φ $\longrightarrow$ $\boxed{\begin{array}{c} Q_? = (B_{\text{init}}) \\ S_- = \emptyset, S_+ = \emptyset \end{array}}$

Overview


B_{init}, φ
$Q_? = (B_{\text{init}})$
$S_- = \emptyset, S_+ = \emptyset$
$B = Q_?.\texttt{pop()}$

Overview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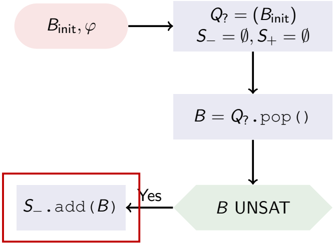
B UNSAT

Overview



B_{init}, φ
$Q_? = (B_{\text{init}})$
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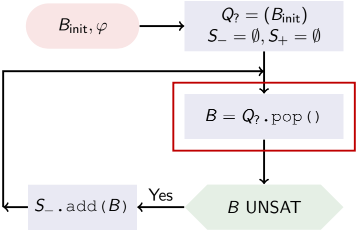
B UNSAT
Yes
$S_-.\texttt{add}(B)$

Overview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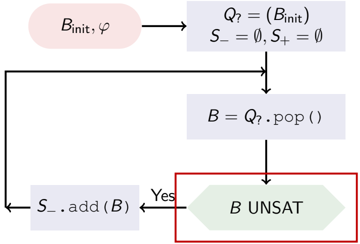

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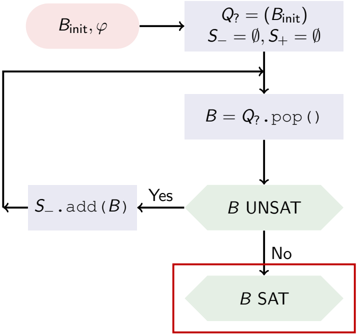

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$Q_? = (B_{\text{init}})$
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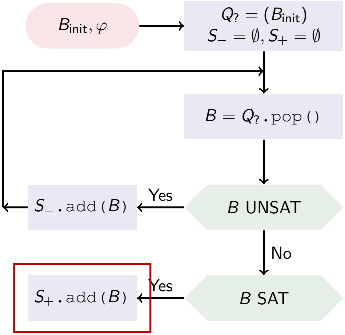
B UNSAT
Yes
$S_-.\texttt{add}(B)$
No
B SAT

Overview



B_{init}, φ
$Q_? = (B_{\text{init}})$
$S_- = \emptyset, S_+ = \emptyset$
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B UNSAT
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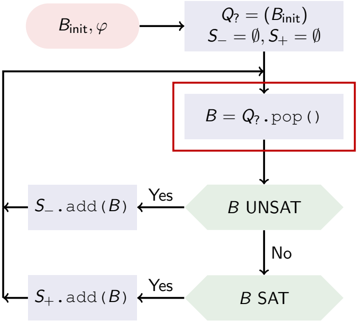
B SAT

Overview



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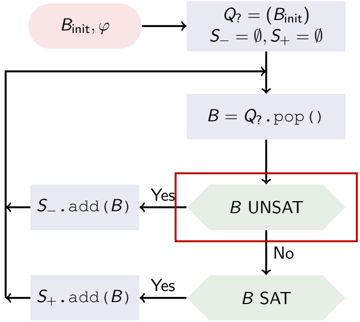

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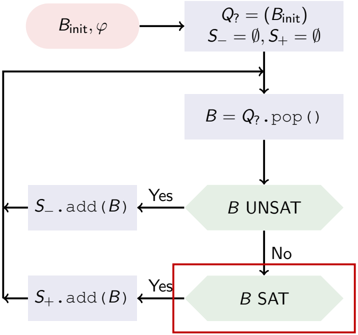

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$Q_? = (B_{\text{init}})$
$S_- = \emptyset, S_+ = \emptyset$
$B = Q_?$.pop()
S_-.add(B)
Yes
B UNSAT
No
S_+.add(B)
Yes
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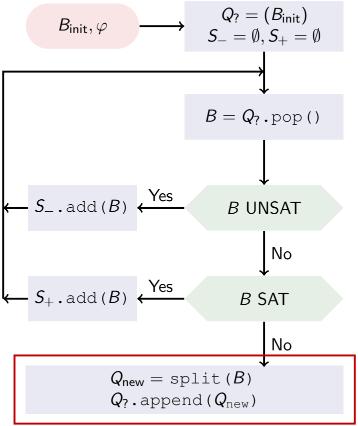

B_{init}, φ
$Q_? = (B_{\text{init}})$
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B UNSAT
Yes
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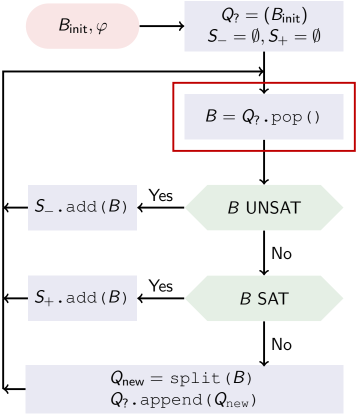
$Q_{\text{new}} = \texttt{split}(B)$
$Q_?.\texttt{append}(Q_{\text{new}})$

Overview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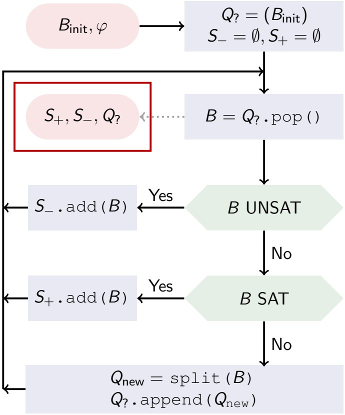
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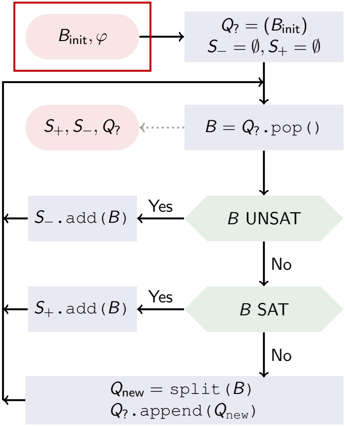
Base Algorithm

Example



Base Algorithm

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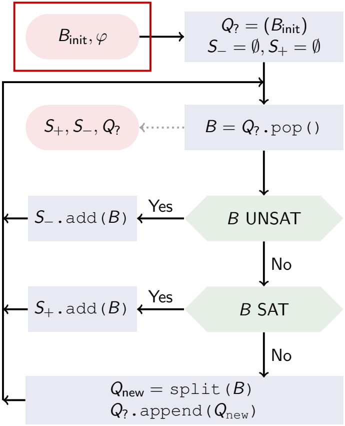
- $\varphi(x, y) := (x \leq 0) \vee (y \geq x^3)$



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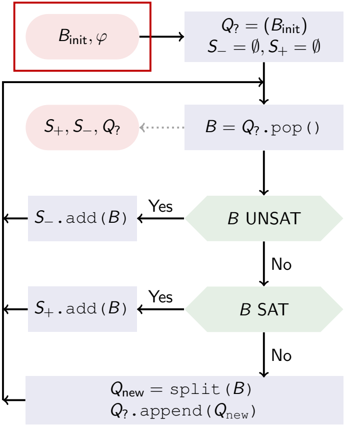

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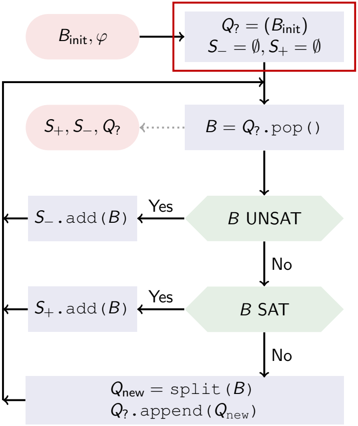

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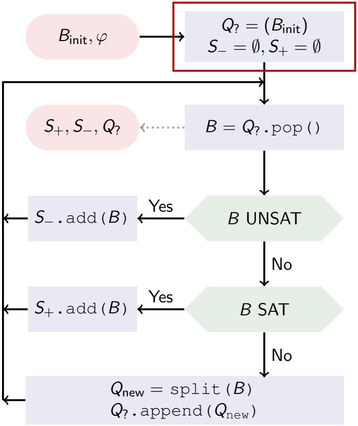

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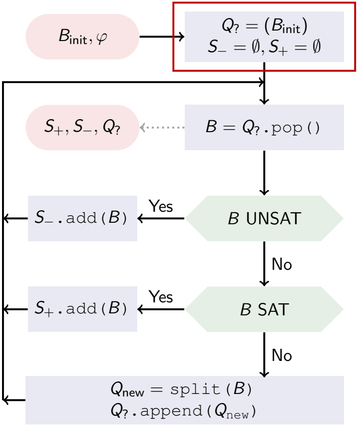

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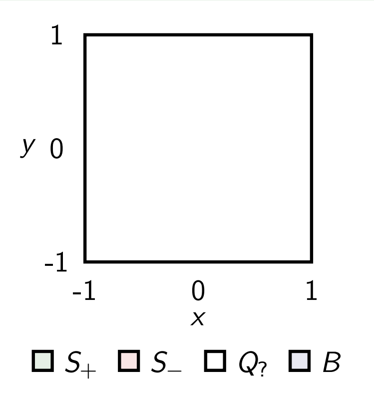

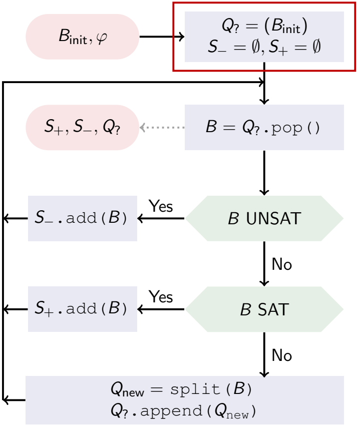

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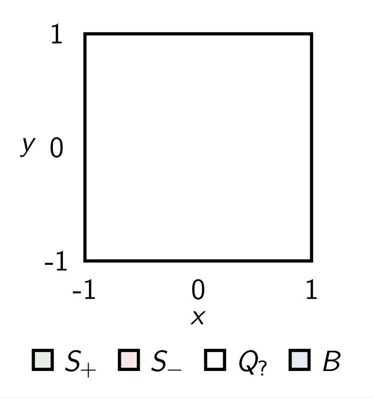

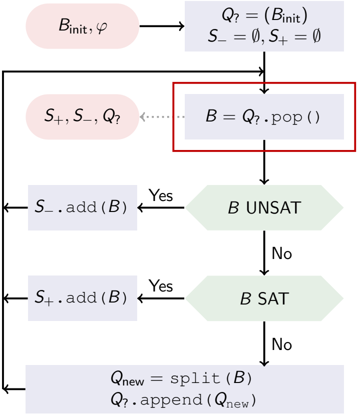

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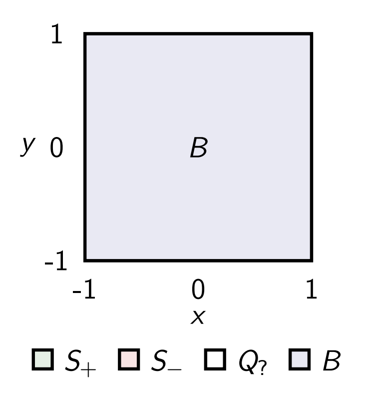
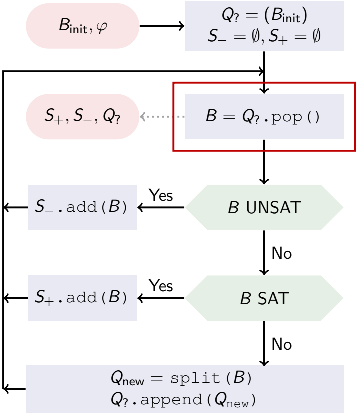
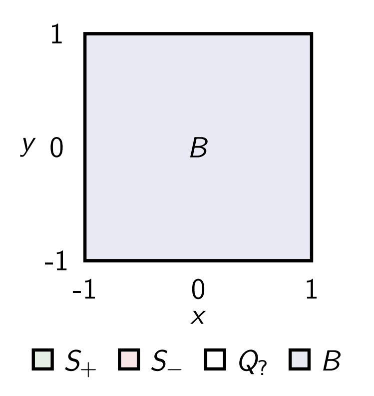


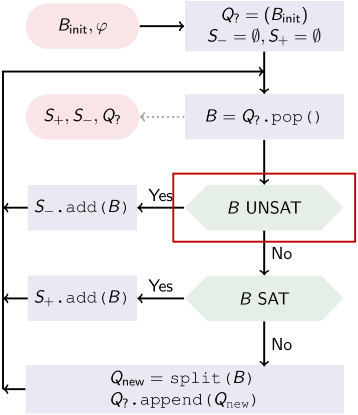


Base Algorithm

Example

- $\varphi(x,y) := (x \leq 0) \vee (y \geq x^3)$

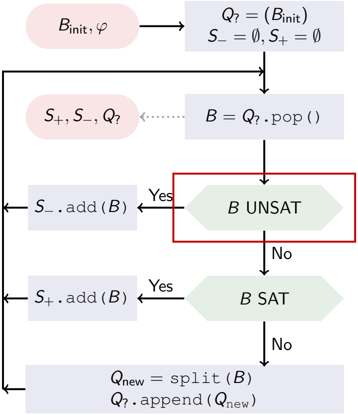




Base Algorithm

Unsatisfying Box

- $\forall x(B(x) \rightarrow \neg\varphi(x))$



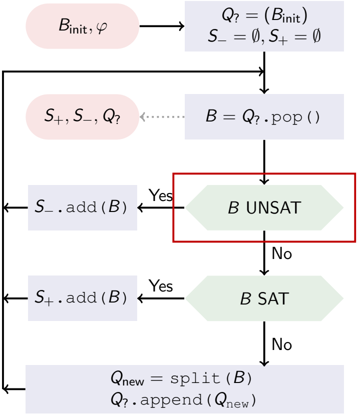
Base Algorithm

Unsatisfying Box

- $\forall x (B(x) \rightarrow \neg\varphi(x))$

Problem

Solvers cannot handle quantifiers.



Base Algorithm

Unsatisfying Box

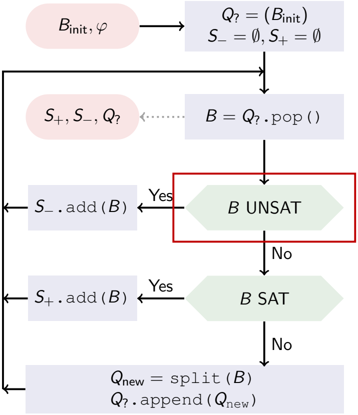
- $\forall x(B(x) \rightarrow \neg\varphi(x))$

Problem

Solvers cannot handle quantifiers.

Solution

$$\begin{aligned} & \forall x(B(x) \rightarrow \neg\varphi(x)) \\ \equiv\ & \neg\exists x\ \neg(\neg B(x) \vee \neg\varphi(x)) \\ \equiv\ & \neg\exists x\ (B(x) \wedge \varphi(x)) \\ \equiv\ & B(x) \wedge \varphi(x) \text{ is UNSAT} \\ \equiv\ & \text{'No satisfying } x \text{ exists in B'} \end{aligned}$$



Base Algorithm

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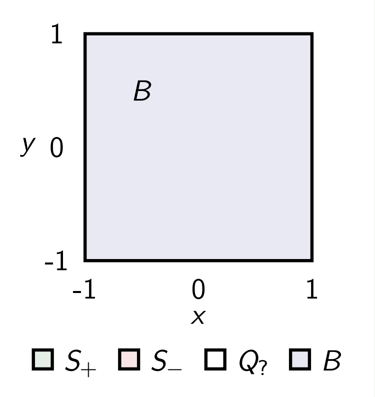

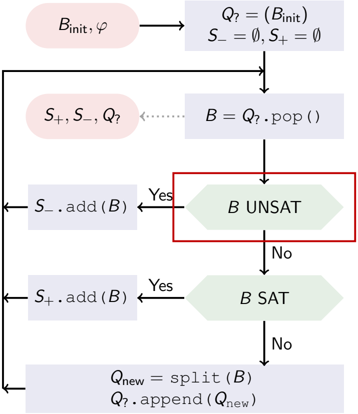

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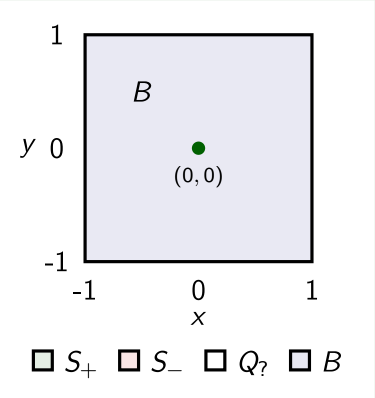

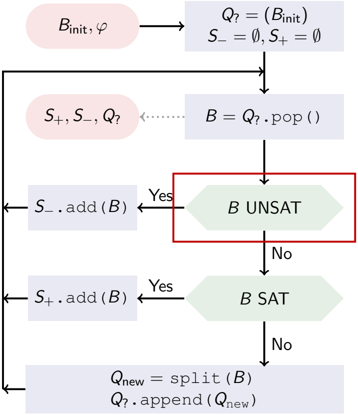

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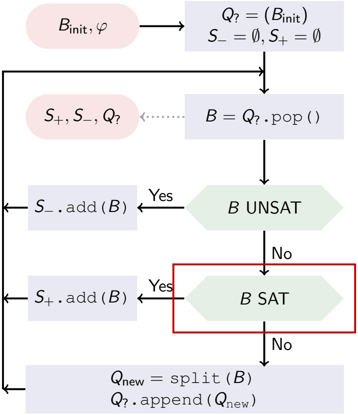

Base Algorithm

Satisfying Box

- $\forall x(B(x) \to \varphi(x))$

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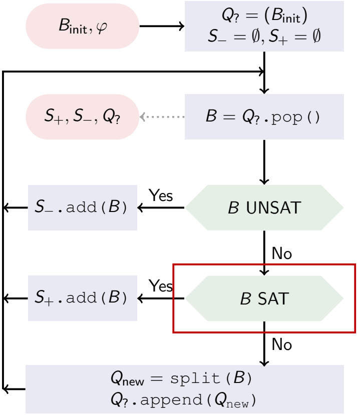
Solvers cannot handle quantifiers.



Base Algorithm

Satisfying Box

- $\forall x(B(x) \to \varphi(x))$

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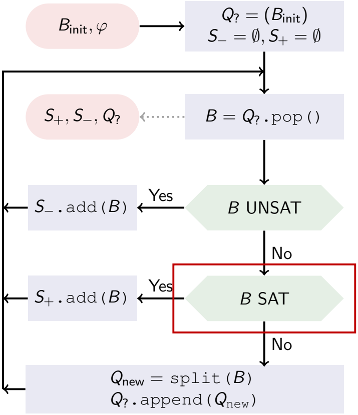
Solvers cannot handle quantifiers.

Solution

$$\begin{aligned} & \forall x(B(x) \to \varphi(x)) \\ \equiv\ & \neg\exists x\ \neg(\neg B(x) \vee \varphi(x)) \\ \equiv\ & \neg\exists x\ (B(x) \wedge \neg\varphi(x)) \\ \equiv\ & B(x) \wedge \neg\varphi(x) \text{ is UNSAT} \\ \equiv\ & \text{'No unsatisfying } x \text{ exists in B'} \end{aligned}$$



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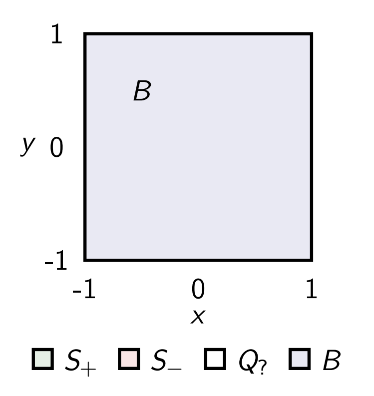

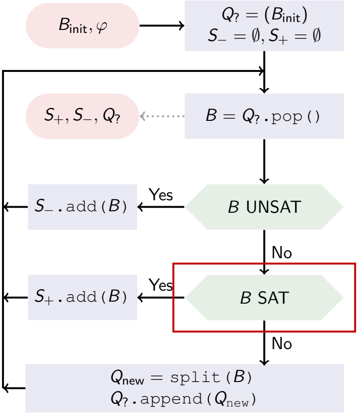

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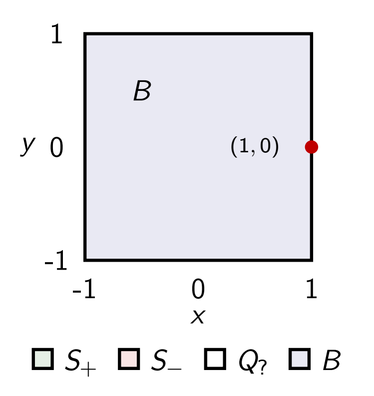

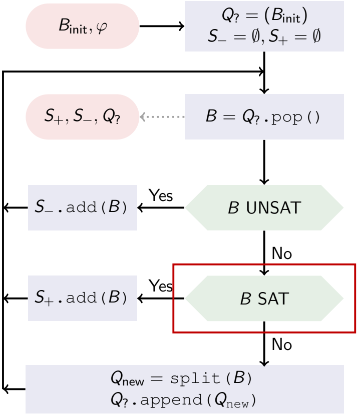

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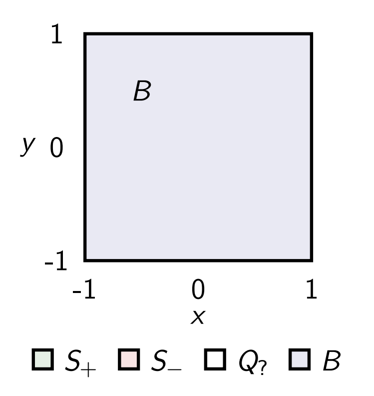

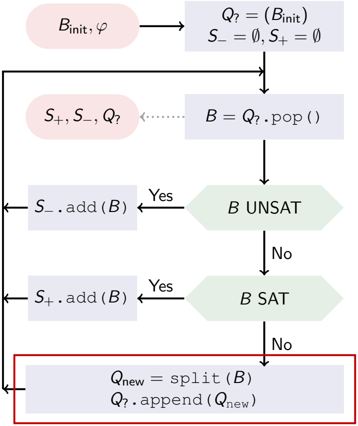

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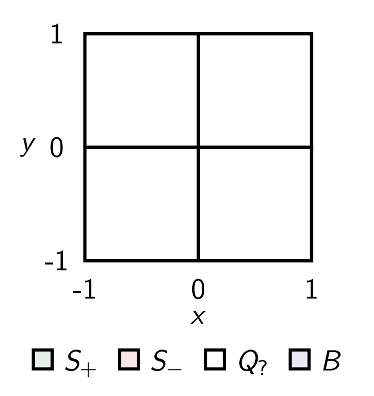

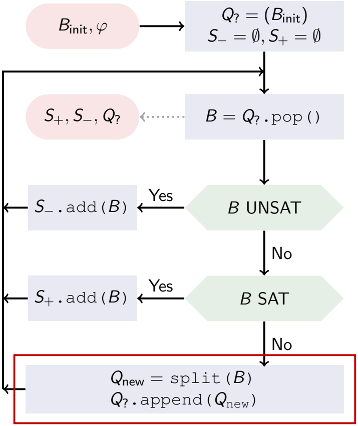

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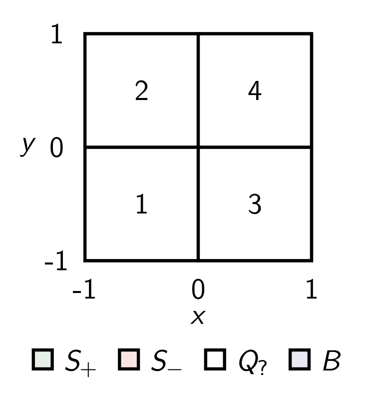

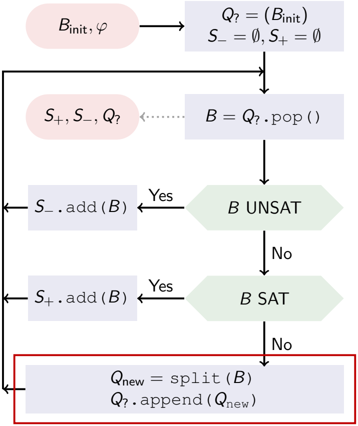

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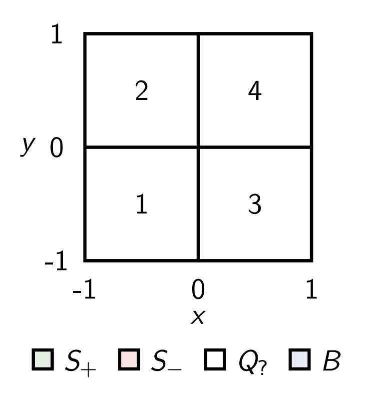

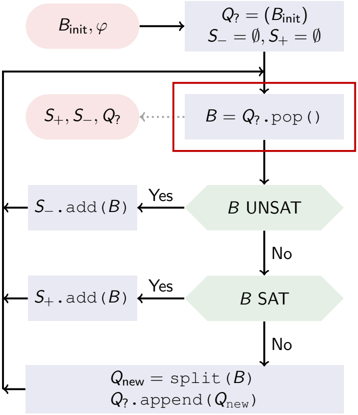

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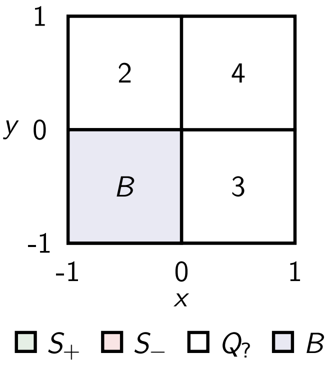

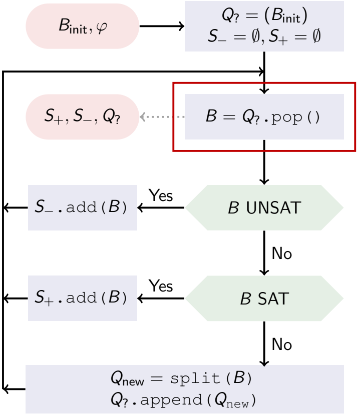

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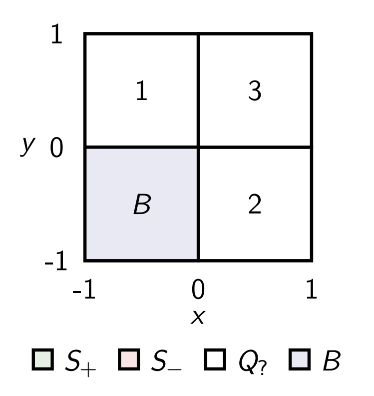

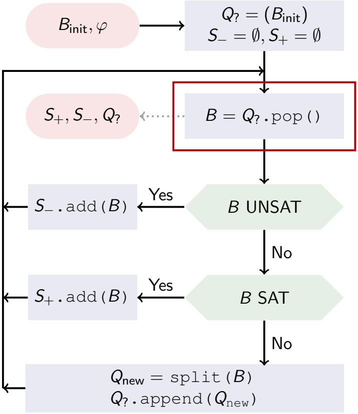

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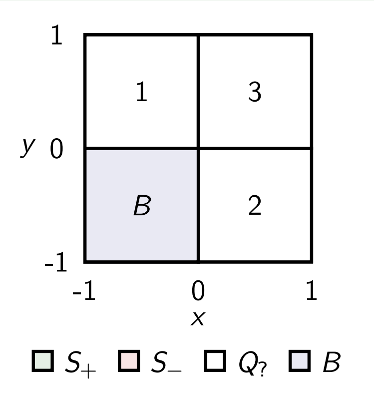

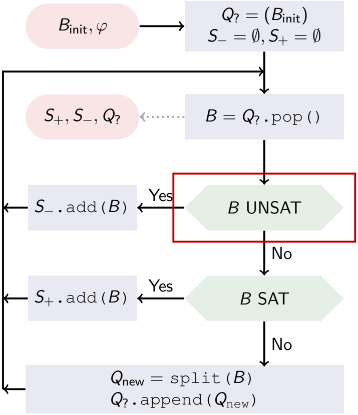

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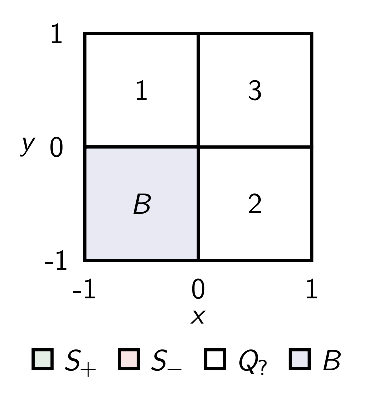

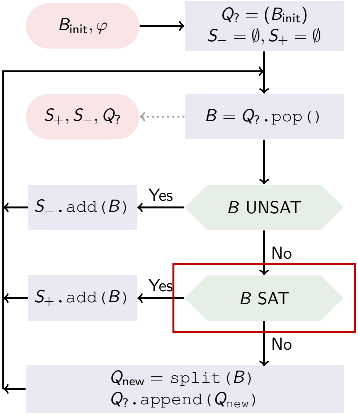

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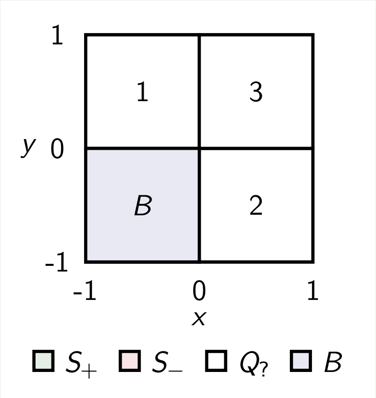

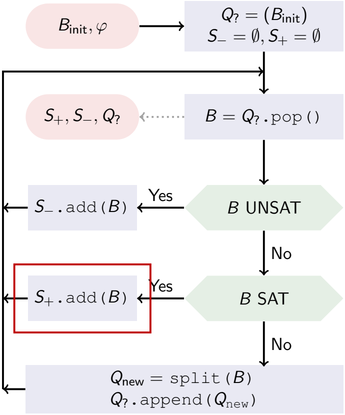

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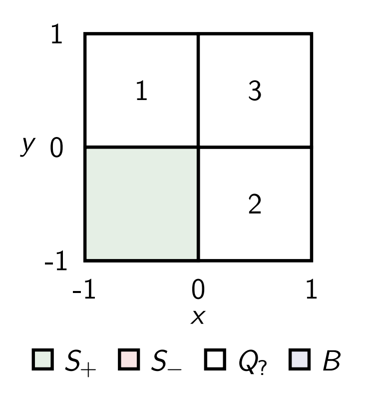

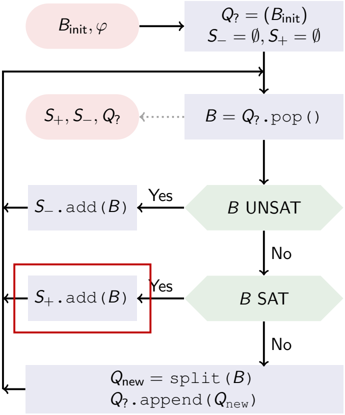

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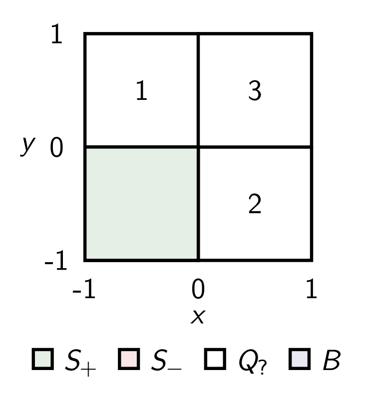

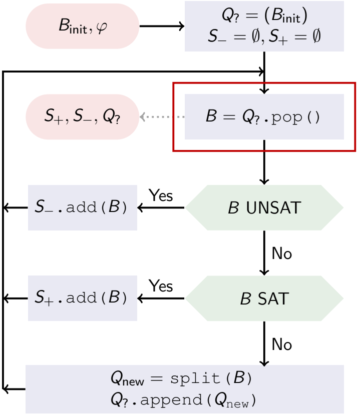

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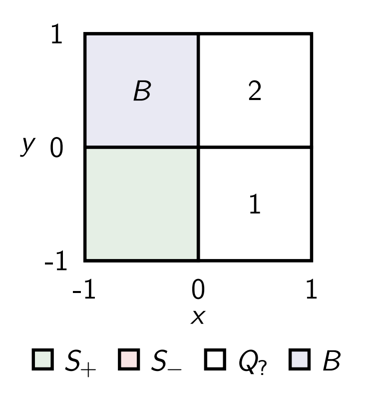

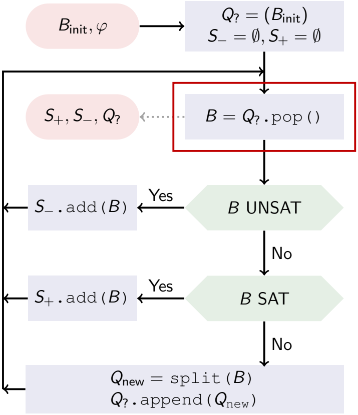

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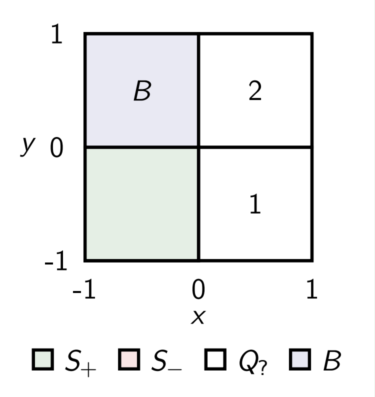

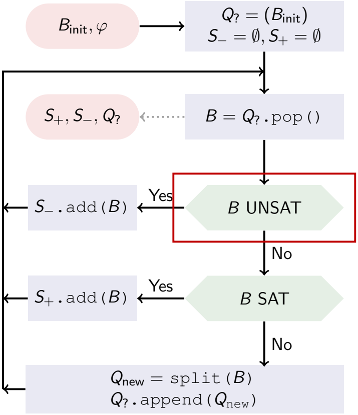

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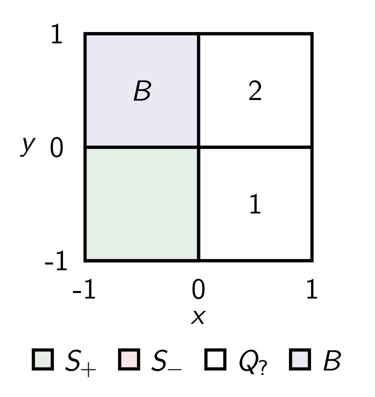

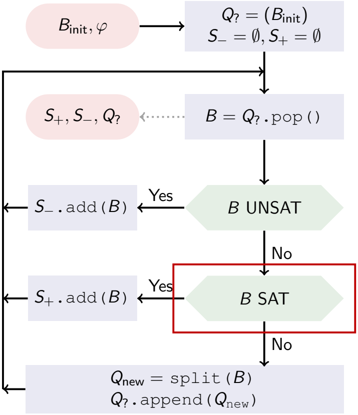

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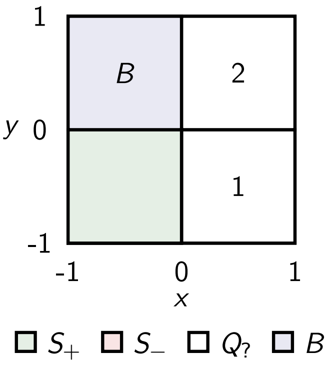

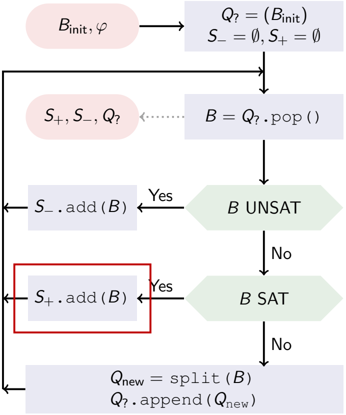

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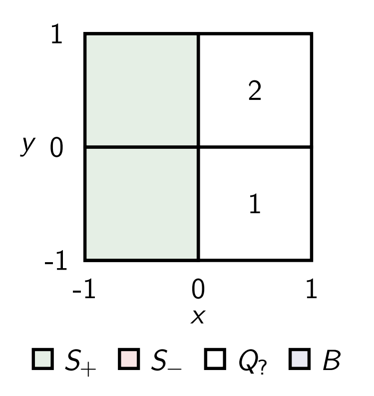

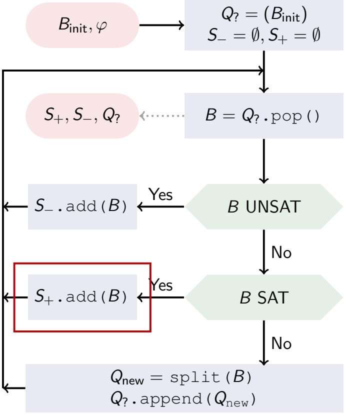

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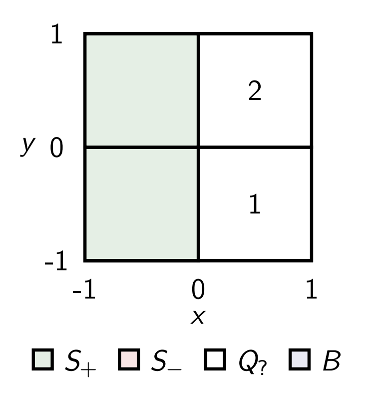

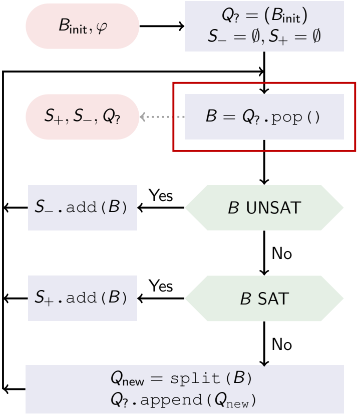

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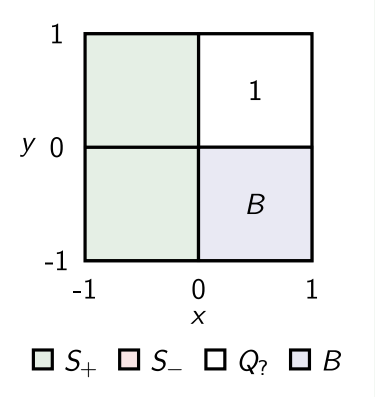

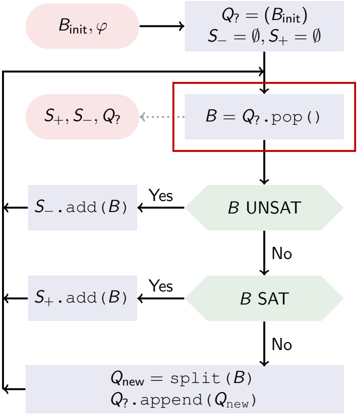

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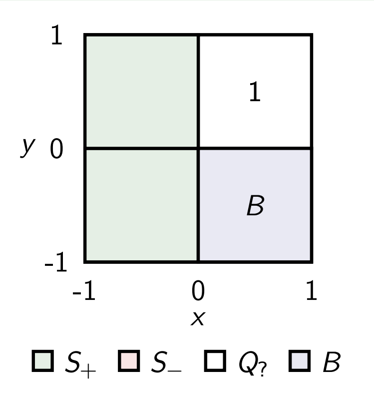

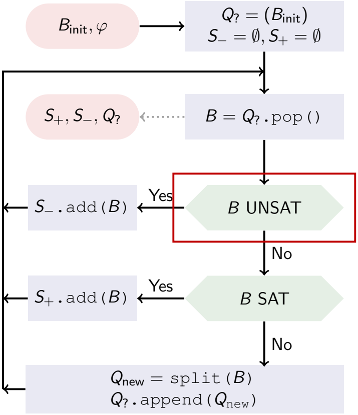

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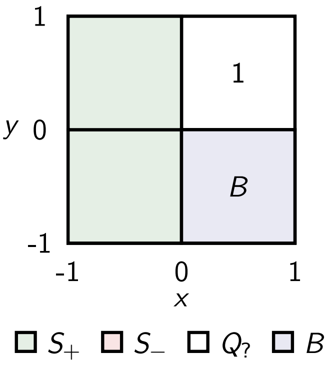

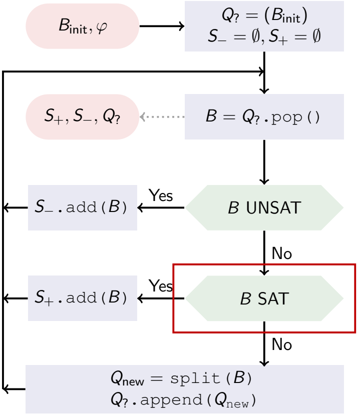

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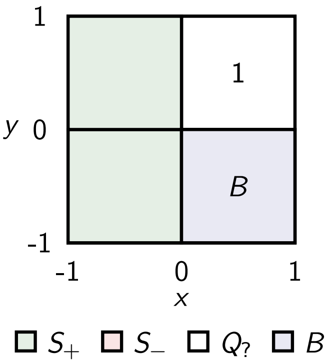

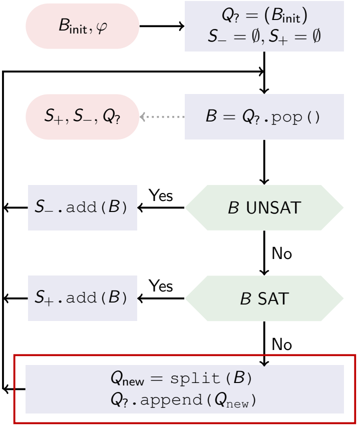

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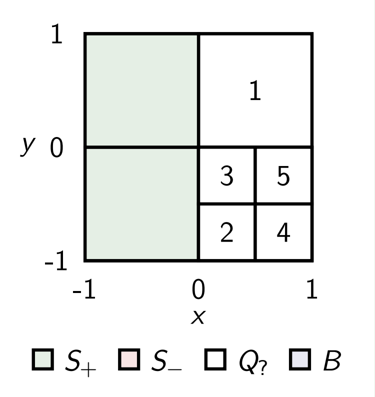

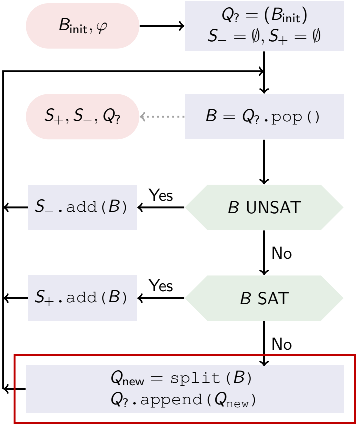

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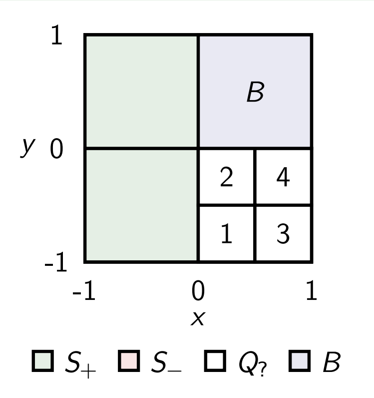

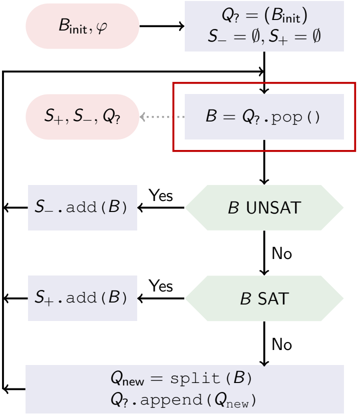

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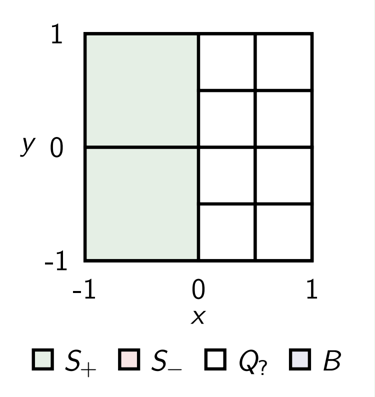
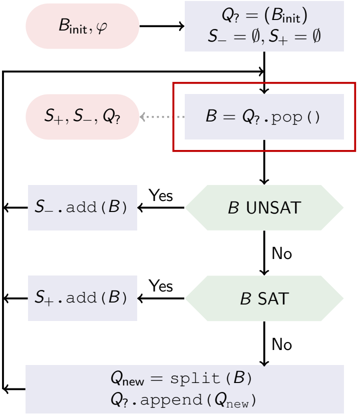
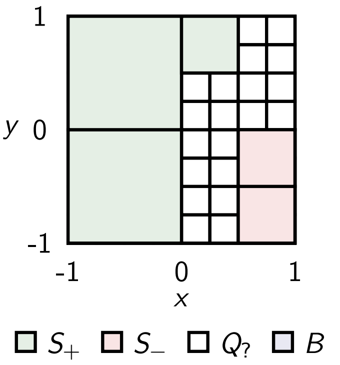


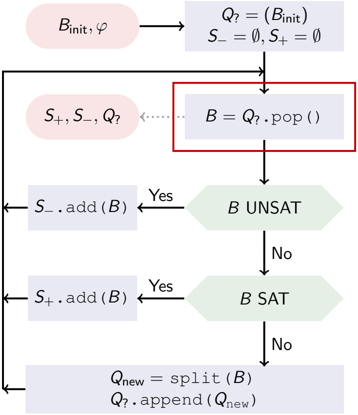


Base Algorithm

Example

- $\varphi(x, y) := (x \leq 0) \lor (y \geq x^3)$





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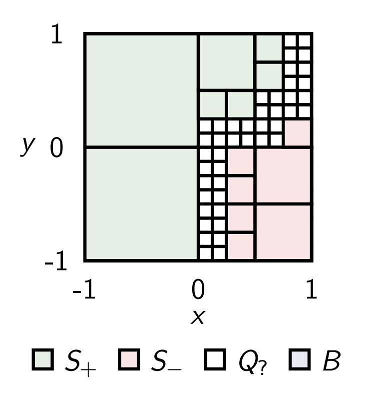

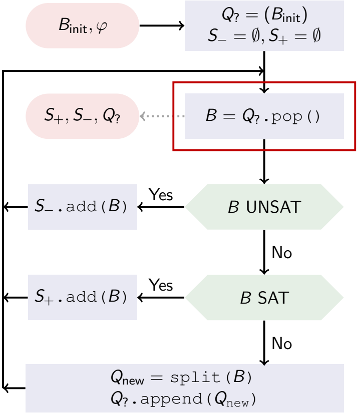

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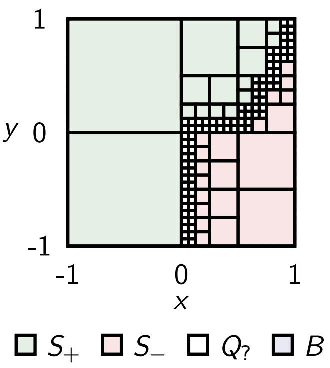

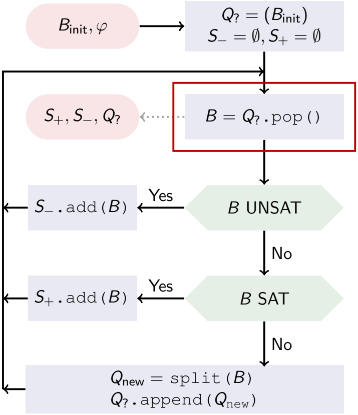

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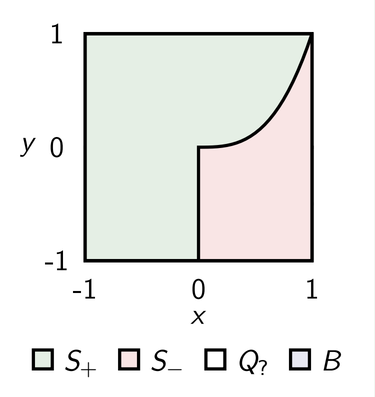
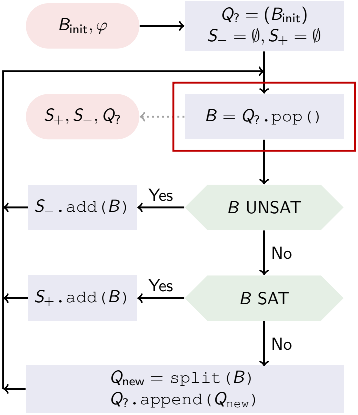
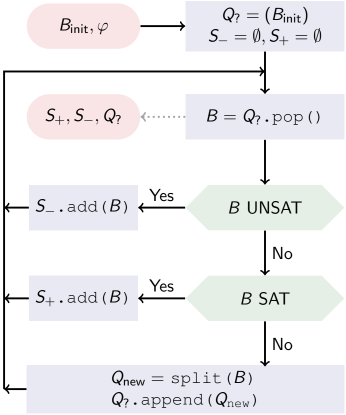


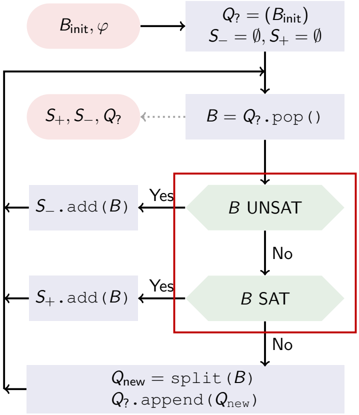


Sampling



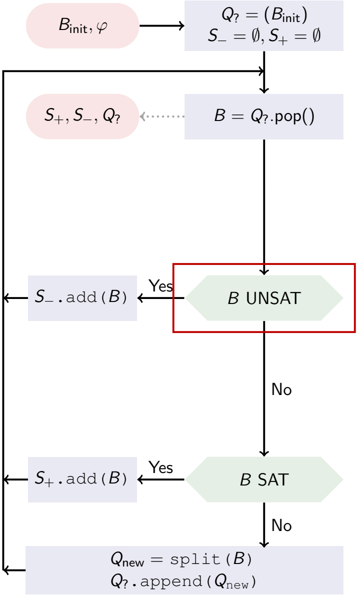
B_{init}, φ
$Q_? = (B_{\text{init}})$
$S_- = \emptyset, S_+ = \emptyset$
$S_+, S_-, Q_?$
$B = Q_?.\texttt{pop()}$
$S_-.\texttt{add}(B)$
Yes
B UNSAT
No
$S_+.\texttt{add}(B)$
Yes
B SAT
No
$Q_{\text{new}} = \texttt{split}(B)$
$Q_?.\texttt{append}(Q_{\text{new}})$

Sampling



$>$ 80% of the running time

Sampling: Recall



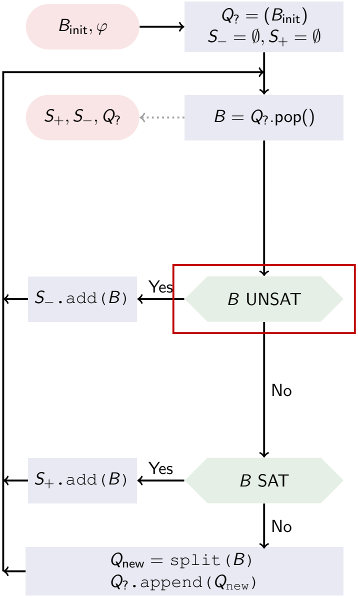
Sampling: Recall

Unsatisfying Box

- $\forall x(B(x) \rightarrow \neg\varphi(x))$

Problem

Solvers cannot handle quantifiers.



Sampling: Recall

Unsatisfying Box

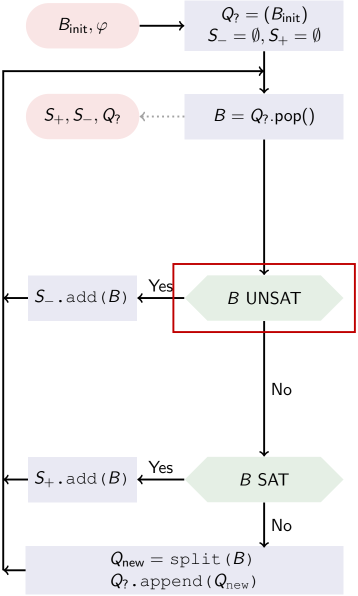
- $\forall x(B(x) \to \neg\varphi(x))$

Problem

Solvers cannot handle quantifiers.

Solution

$$
\begin{aligned}
& \forall x(B(x) \to \neg\varphi(x)) \\
\equiv\ & \neg\exists x\ \neg(\neg B(x) \vee \neg\varphi(x)) \\
\equiv\ & \neg\exists x\ (B(x) \wedge \varphi(x)) \\
\equiv\ & B(x) \wedge \varphi(x) \text{ is UNSAT} \\
\equiv\ & \text{'no satisfying } x \text{ exists in B'}
\end{aligned}
$$



Sampling: Idea

Solution

$$\forall x(B(x) \to \neg\varphi(x))$$
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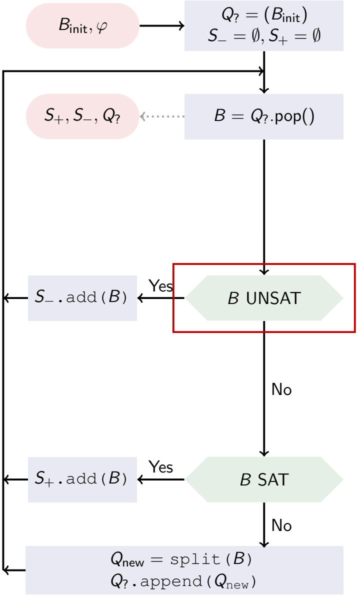

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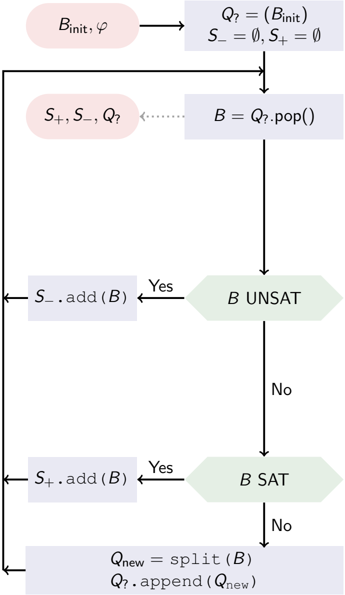
Sampling

- take $x \in B$, check $\varphi(x)$



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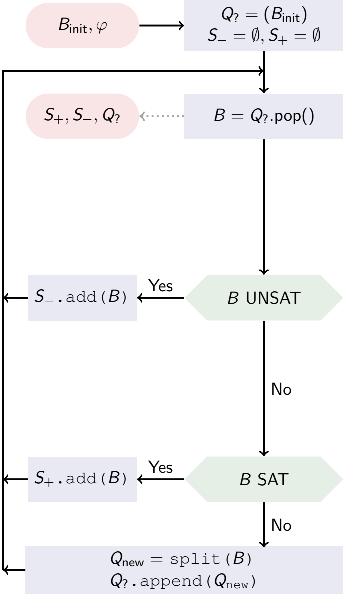
- $\varphi(x)$ holds: B can not be unsatisfying



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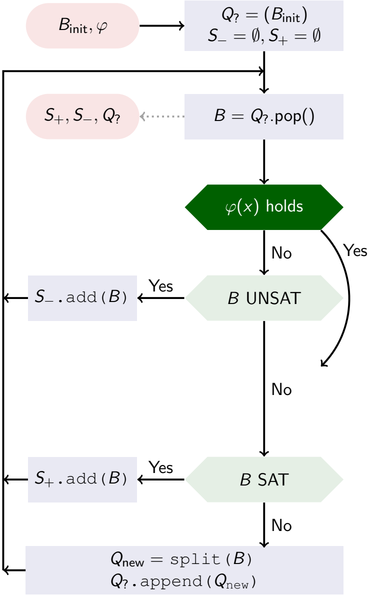

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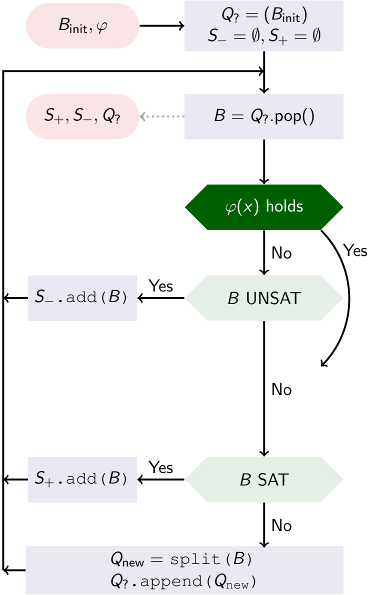

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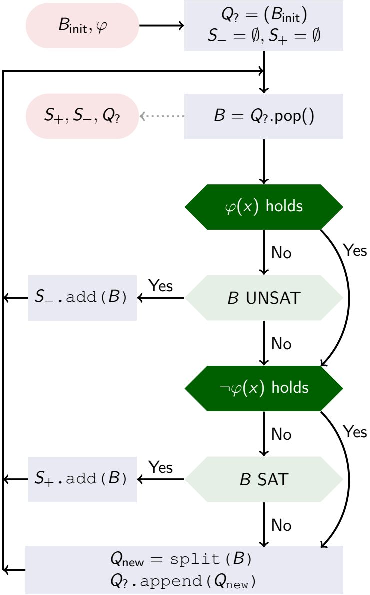

Sampling: Options

Sampling: Options

Normal

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Normal

- sample 'arbitrary' point

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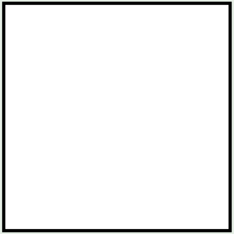

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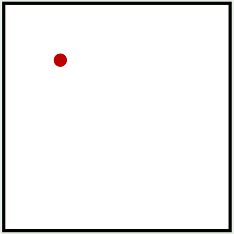

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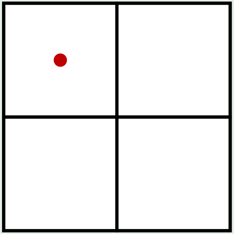
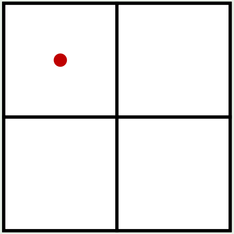


Sampling: Options

Normal

- sample 'arbitrary' point
- optionally carry samples when splitting the box



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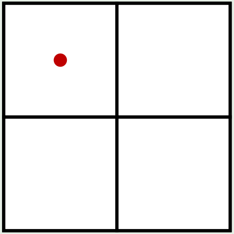

– carrying is time consuming

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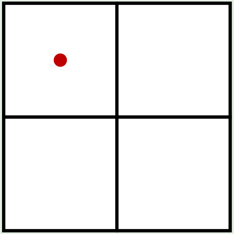

– carrying is time consuming

Clever

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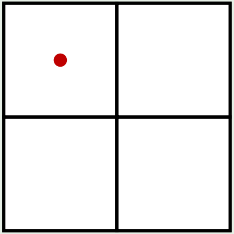

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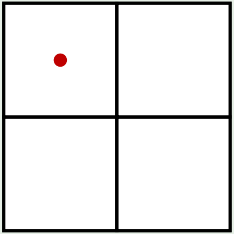

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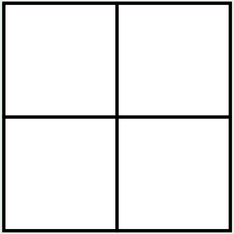

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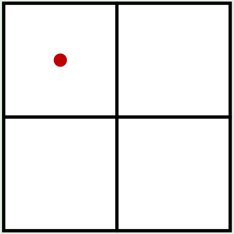

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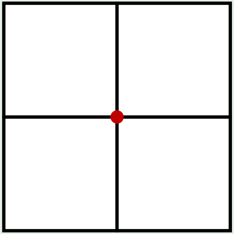

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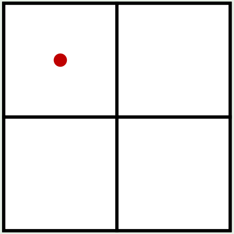
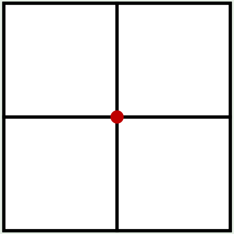


– carrying is time consuming

Clever

- take sample on cut
- automatically assign the sample to adjacent boxes



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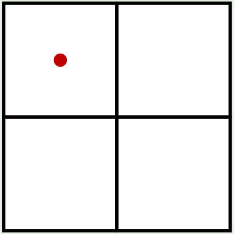

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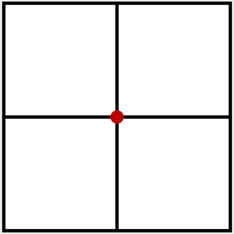

\+ sample only every 2nd split

Sampling: Options

Normal

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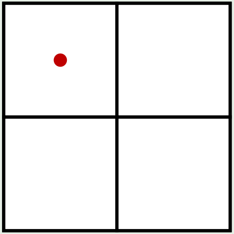

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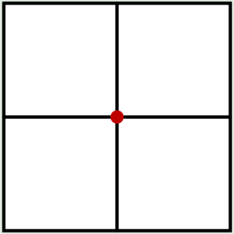
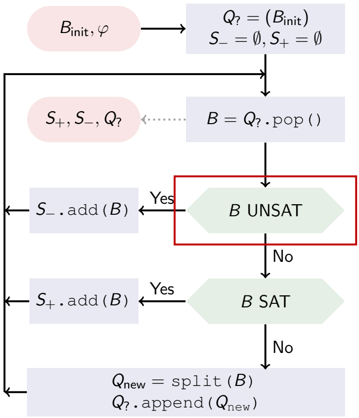


+ sample only every 2nd split
– only works on cuts

Model Saving



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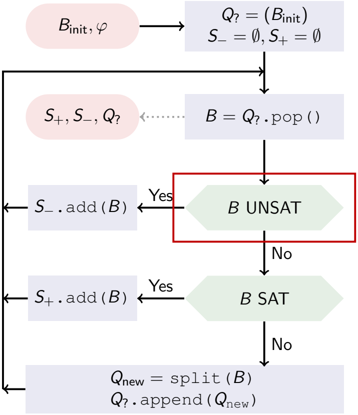
Idea

- if the answer is 'No', the solver has found a model



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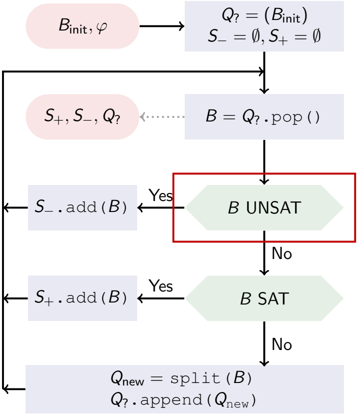
- if the answer is 'No', the solver has found a model
- use it this model as sample



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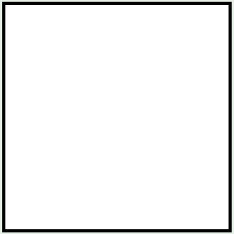

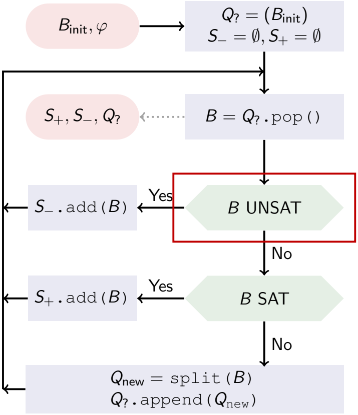

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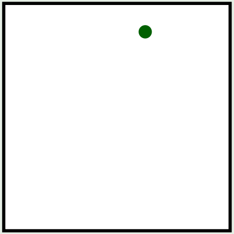

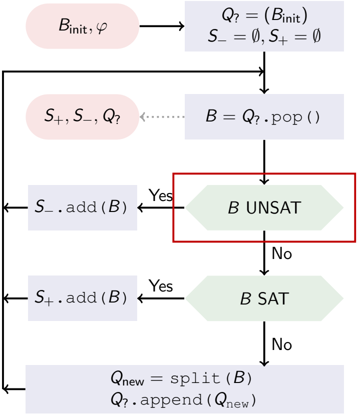

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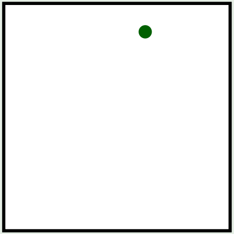

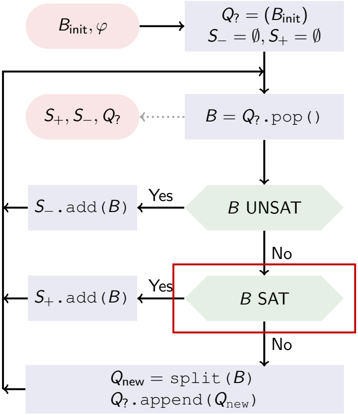

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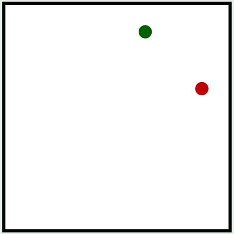

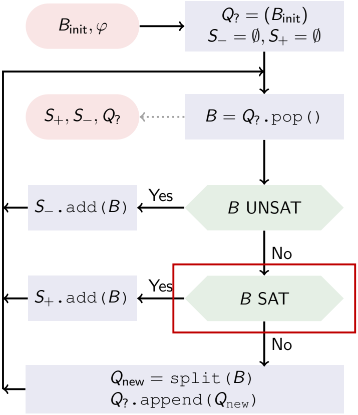

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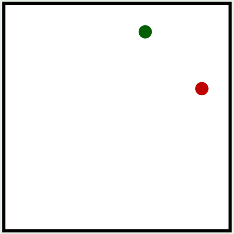

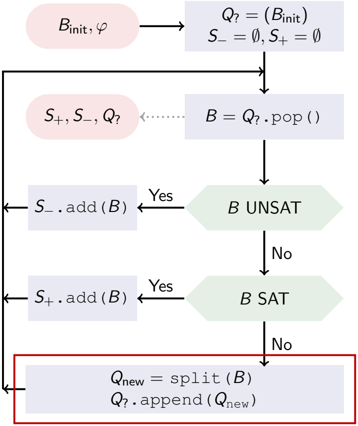

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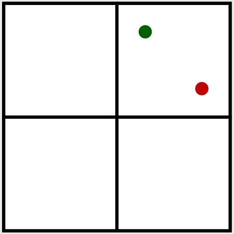

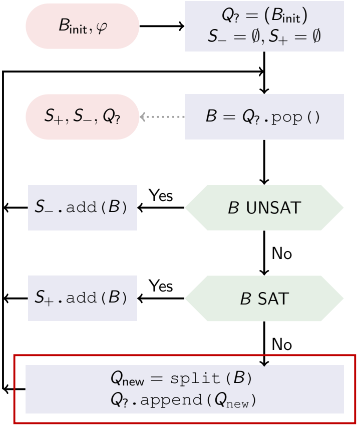

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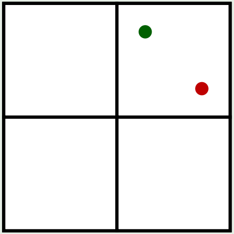

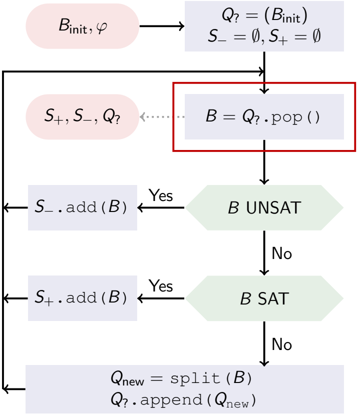

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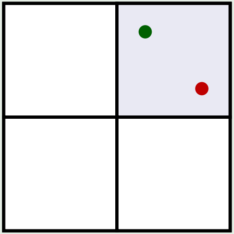

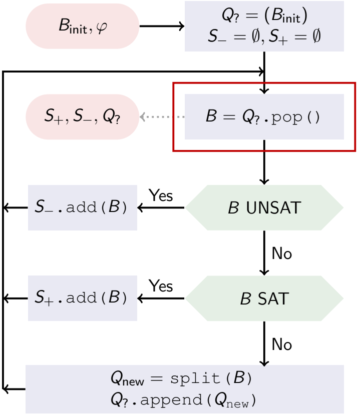

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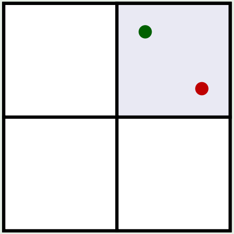
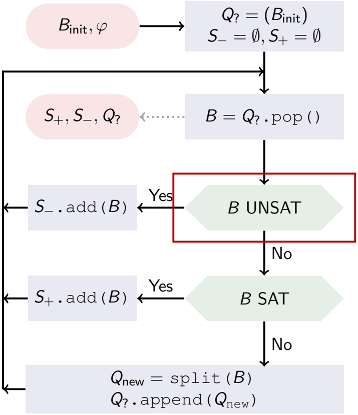


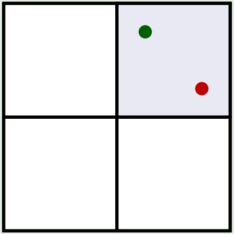


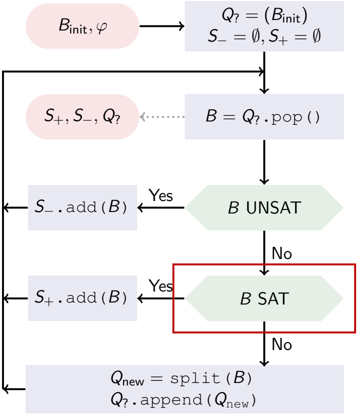
Model Saving

Idea

- if the answer is 'No', the solver has found a model
- use it this model as sample

Example





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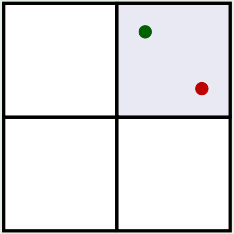

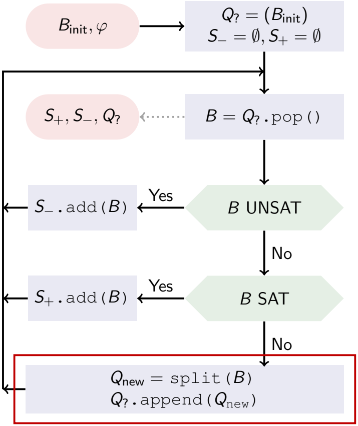

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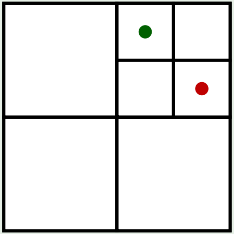
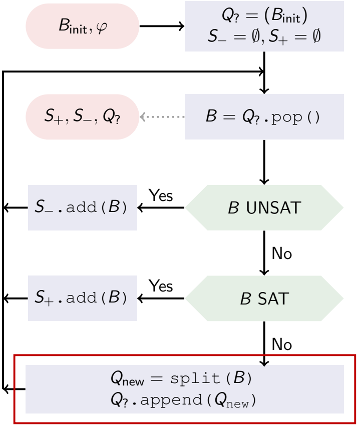
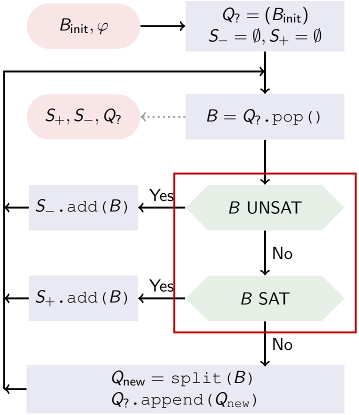




Splitting



$>$ 80% of the running time

Splitting: Options

Bisect All Dimensions

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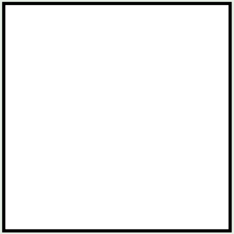

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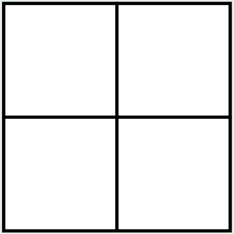

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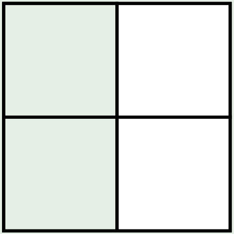

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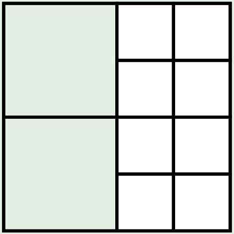

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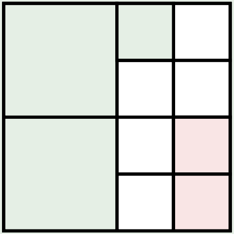

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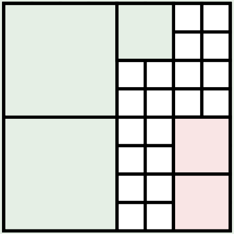

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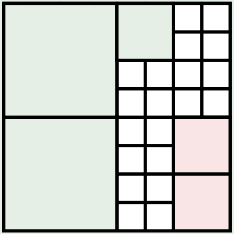

Bisect Single Dimension

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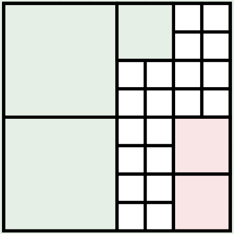

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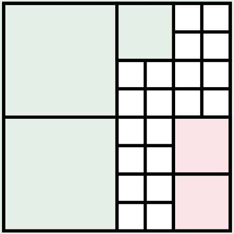

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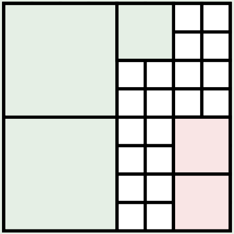

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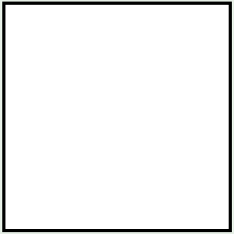

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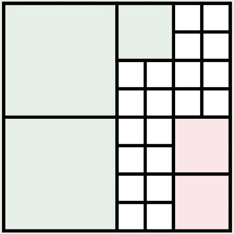

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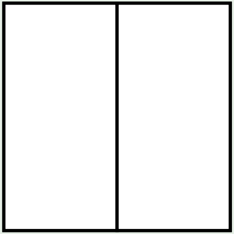

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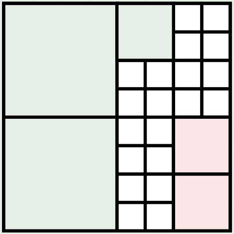

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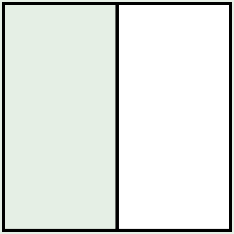

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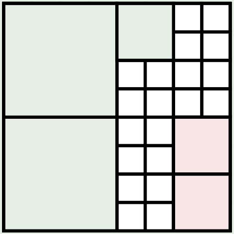

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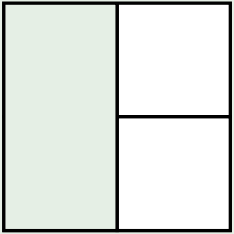

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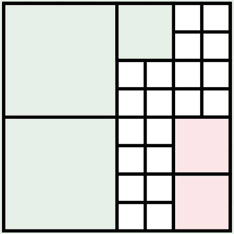

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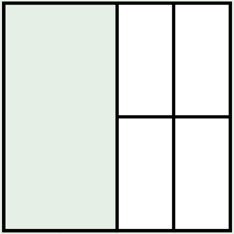

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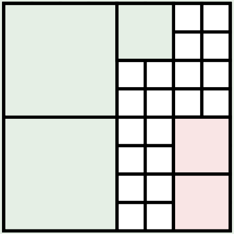

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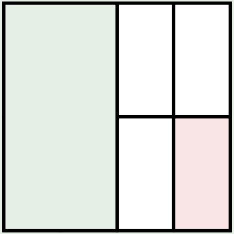

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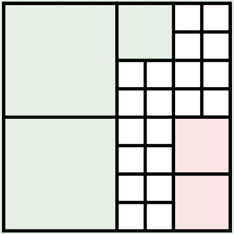

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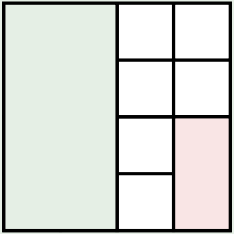

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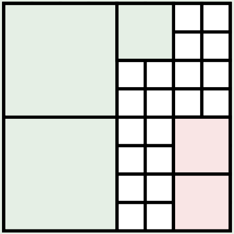

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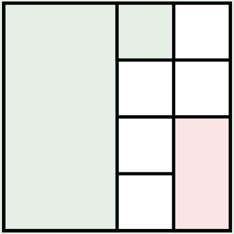

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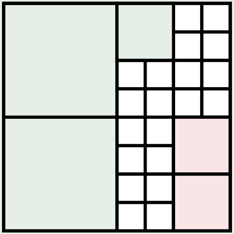

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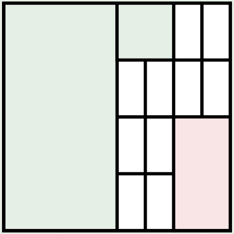

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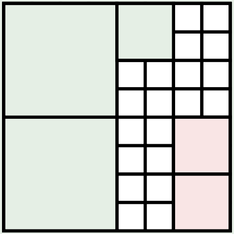

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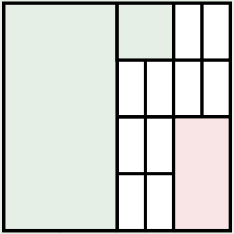

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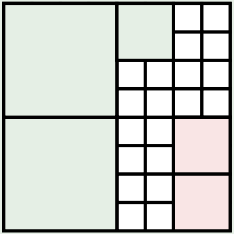

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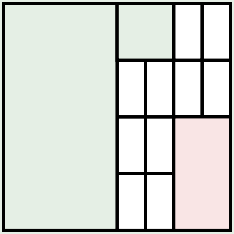

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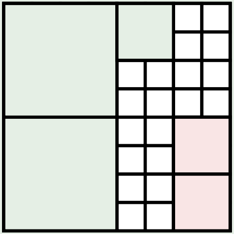

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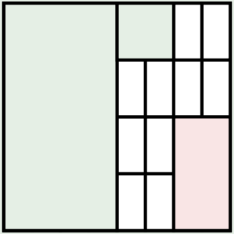

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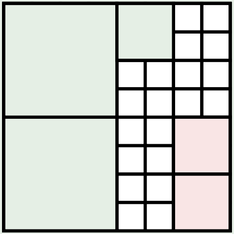

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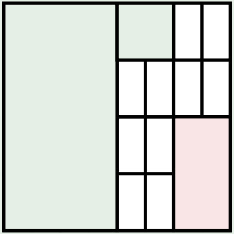

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Incremental Solving

Idea

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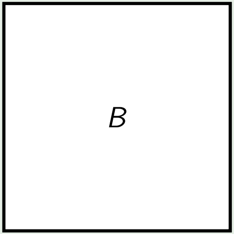

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- $B = [-1, 1] \times [-1, 1]$
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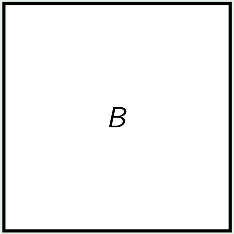

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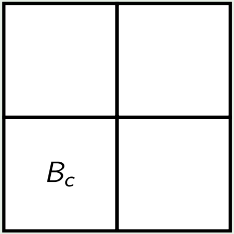

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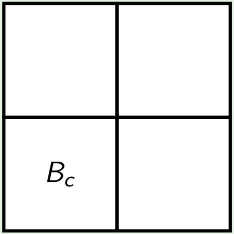

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- $B = [-1, 1] \times [-1, 1]$
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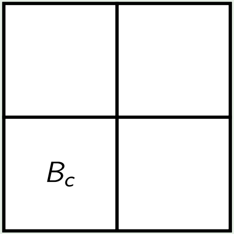

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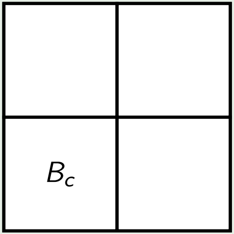

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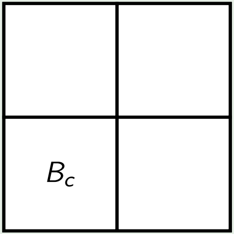

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Implementation

- single context for a box and its children

Example

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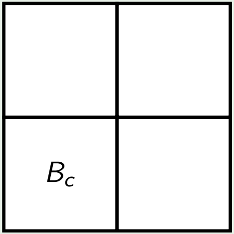

GUI Projection

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Graphical User Interface

- The GUI is able to depict only two dimensions.

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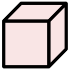

GUI Projection

Graphical User Interface

- The GUI is able to depict only two dimensions.

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- The algorithm can handle more than two dimensions.

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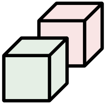

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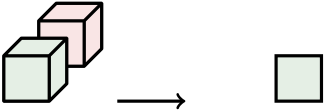

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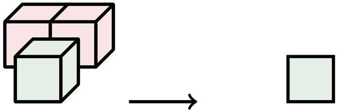

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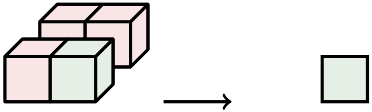

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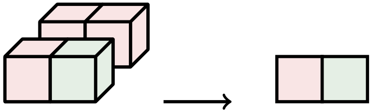

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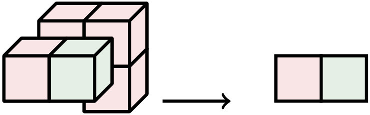

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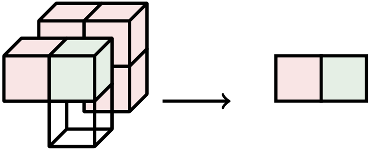

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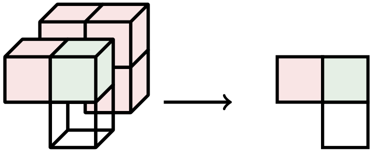

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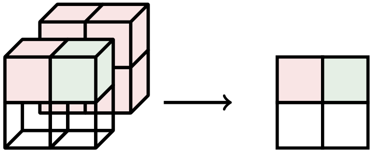

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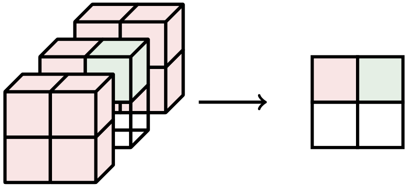

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Speed-Up Factors

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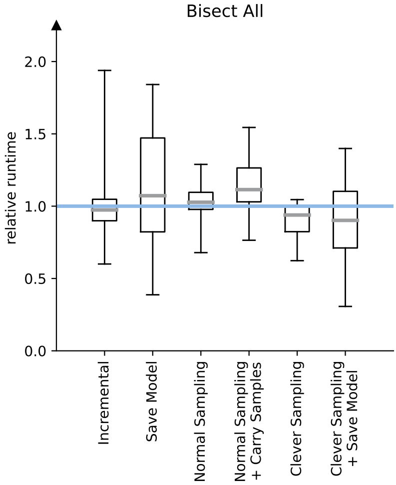

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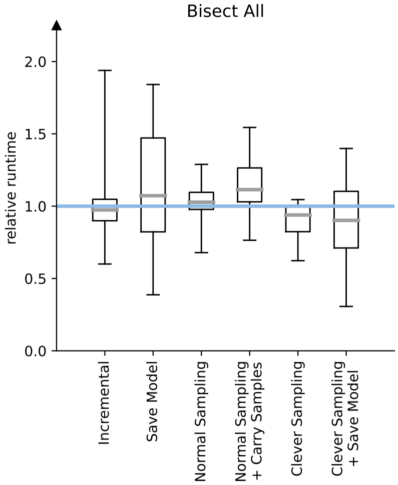
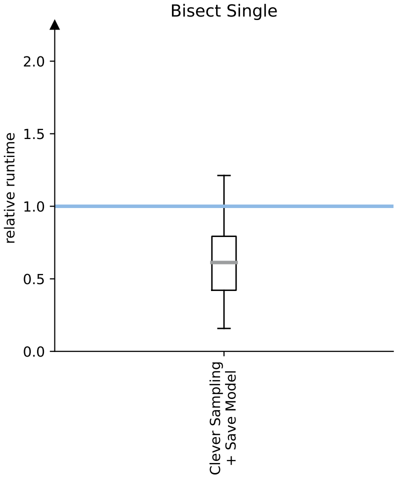




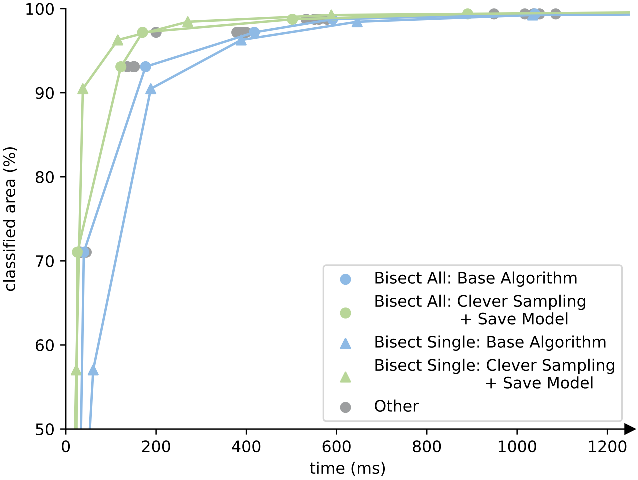
Solver Call Prevention

Solver Call Prevention

	Bisect All		Bisect Single	
	Preventable	Prevented	Preventable	Prevented
Base Algorithm	0.55	0.00	0.83	0.00
Incremental	0.55	0.00	-	-
Save Model	0.55	0.17	-	-
Normal Sampling	0.55	0.29	-	-
+ Carry Samples	0.55	0.31	-	-
Clever Sampling	0.55	0.29	-	-
+ Save Model	0.55	0.39	0.83	0.72

Splitting Heuristic Comparison

Splitting Heuristic Comparison



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Conclusion

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Topic

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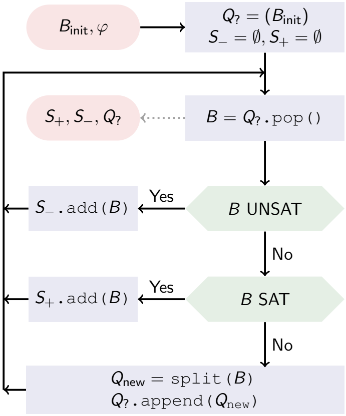
- parameter synthesis



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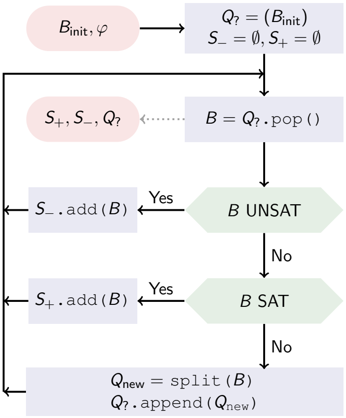

Conclusion

Topic

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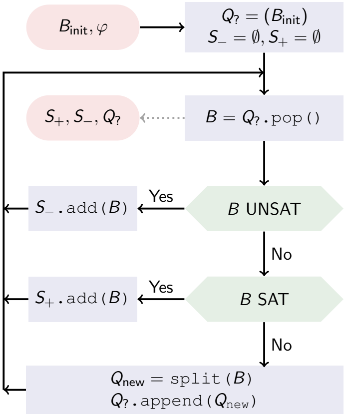
Performance

- good compared to PaSyPy



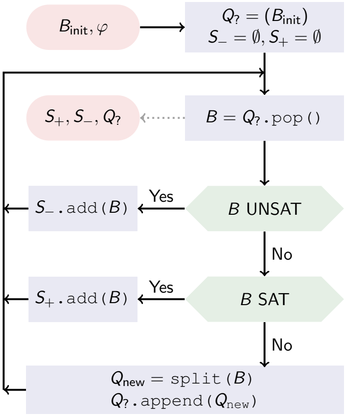
Conclusion

Topic

- parameter synthesis

Performance

- good compared to PaSyPy
- dependent on Z3 performance



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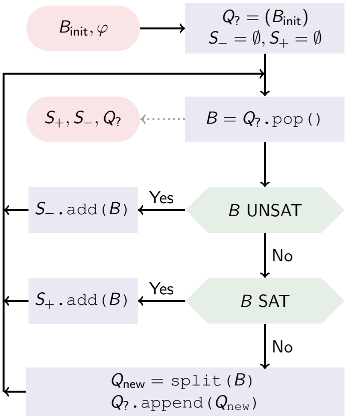
Performance

- good compared to PaSyPy
- dependent on Z3 performance
- implemented heuristics is very benchmark dependent



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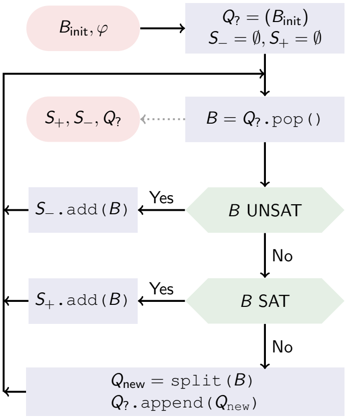
Open Questions & Future Work



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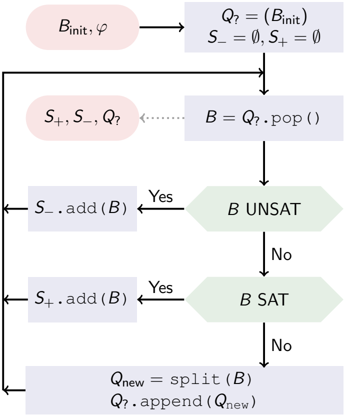
Open Questions & Future Work

- reason for low correlation of solving time and number of solver calls



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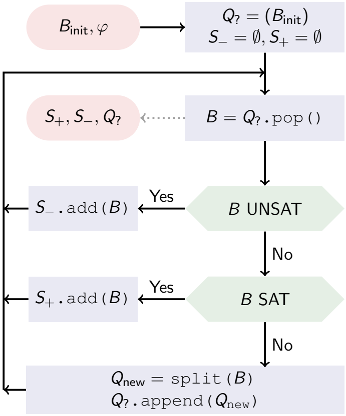
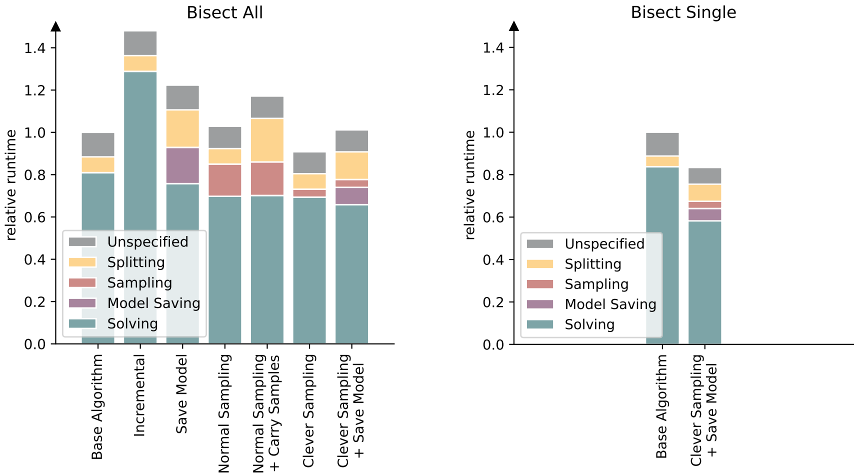
Open Questions & Future Work

- reason for low correlation of solving time and number of solver calls
- advanced splitting heuristics



Experimental Evaluation: Time Distribution



Evaluation: Average Speed-Up Factors

Bisect All

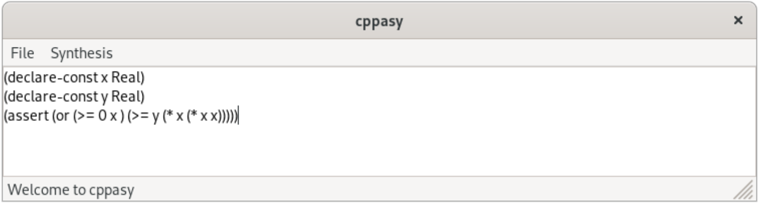
	4^{1910}	5^{1910}	6^{1910}	7^{1843}	8^{1319}	9^{922}	10^{732}
Clever Sampling	0.85	0.96	0.91	0.98	0.91	1.04	0.95
Base Algorithm	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Clever Sampling +Save Model	1.57	1.33	1.01	1.03	0.93	1.04	0.91
Normal Sampling	1.02	1.01	1.03	1.03	1.05	1.10	1.10
Normal Sampling +Carry Samples	1.12	1.12	1.17	1.15	1.17	1.23	1.23
Save Model	1.92	1.39	1.22	1.19	1.16	1.13	1.09
Incremental	1.39	-	1.48	-	1.1	-	1.07

Evaluation: Average Speed-Up Factors

Bisect Single

	4^{1910}	5^{1910}	6^{1910}	7^{1578}	8^{1008}	9^{763}	10^{568}
Clever Sampling +Save Model	0.79	0.77	0.83	0.71	0.64	0.61	0.59
Base Algorithm	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Implementation: GUI



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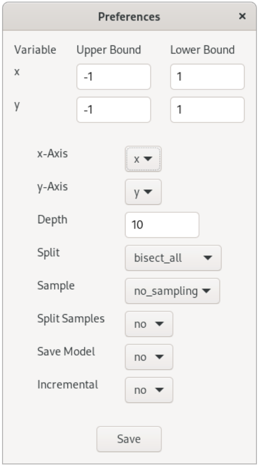

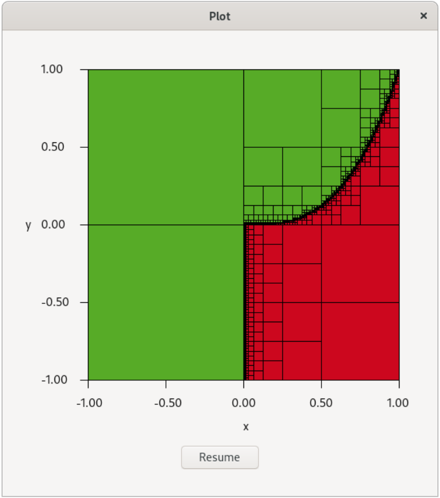

Implementation: CLI

```
$ ./build/cli --help
Allowed options:
  -h [ --help ]          produce help message
  --boundaries-file arg Text file containing a list of all variables and their
                         boundaries. The file should contain lines of the from
                         '<variable-name> <lower-bound> <upper-bound>'.
  --default-boundaries arg (=10)
                         Set default 'radius' of the inital orthotope.
  --splitting-heuristic arg (=bisect_all)
                         Select a splitting heuristic. Options are 'bisect_all'
                         and 'bisect_single'
  --sampling-heuristic arg (=no_sampling)
                         Select a sampling heuristic. Options are 'no_sampling',
                         'center', and clever'
  --max-depth arg (=10) maximal depth.
  --save-model           Save models found by solver. Only useful if
                         'split-samples' enabled.
  --incremental          Enable incremental solving.
  --split-samples        Also carry samples when splitting orthotopes.
  --splits-needed        Returns true if splits are needed to process this
                         formula.
  --print-orthotopes     Prints all (SAFE, UNSAFE and UNKNOWN) resulting
                         orthotopes.
```