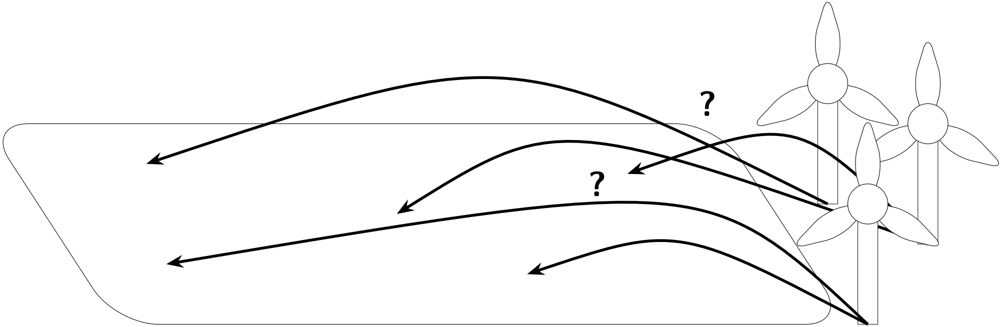
WindProof: A Validation Framework for Wind Farm Simulations

Arne Leon

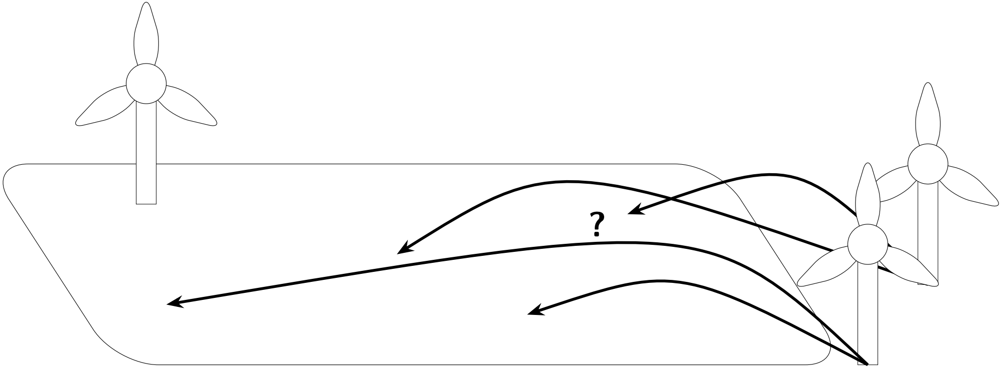
RWTH Aachen University

November 8, 2024

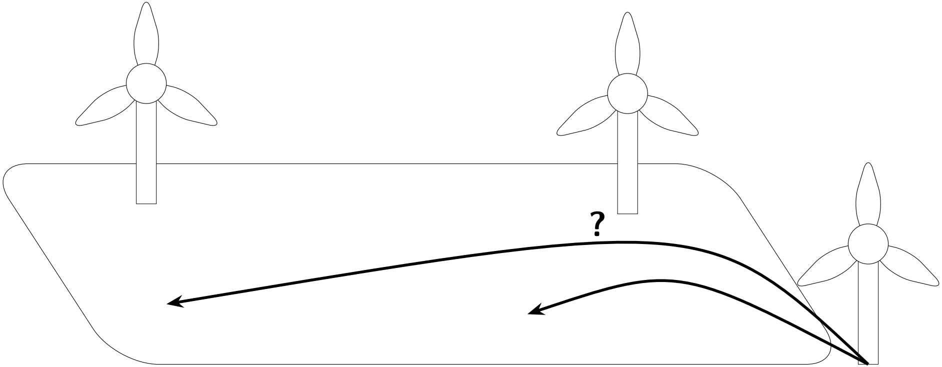
Optimization



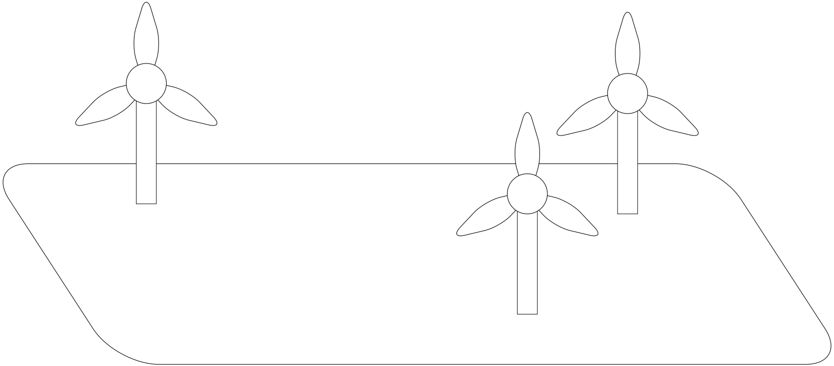
Optimization



Optimization

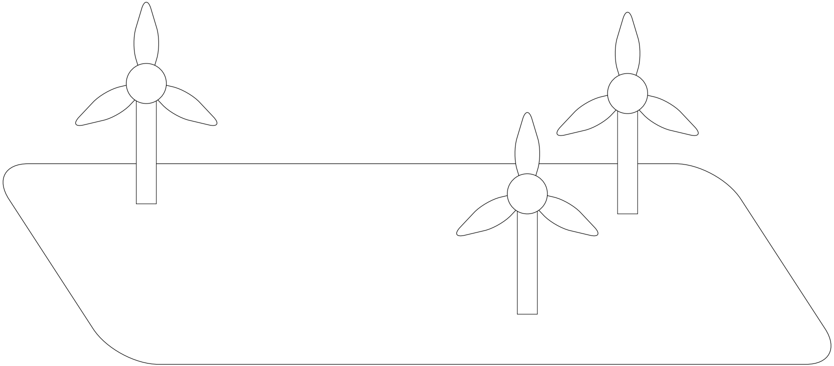


Optimization



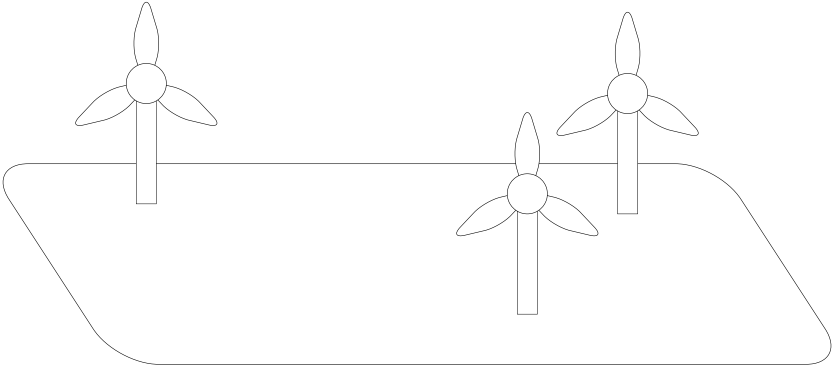
Optimization

Is this good?



Optimization

Is this good? Maybe.



Outline

Outline

- Motivation

Outline

- Motivation
- **Preliminaries**

Outline

- Motivation
- **Preliminaries**
  - Windroses

Outline

- Motivation
- **Preliminaries**
  - Windroses
  - Turbine Data

Outline

- Motivation
- **Preliminaries**
  - Windroses
  - Turbine Data
  - Jensen Wake Model

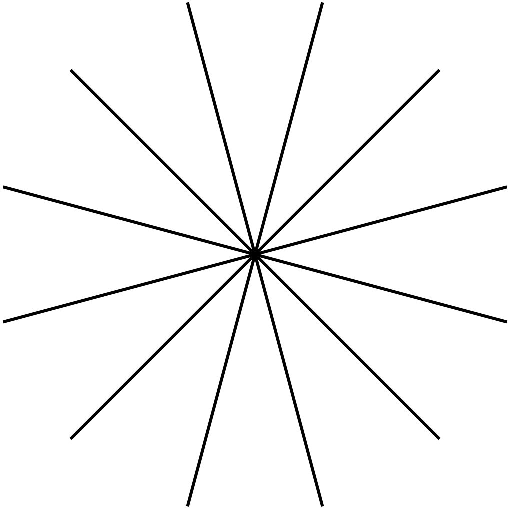
Outline

Outline

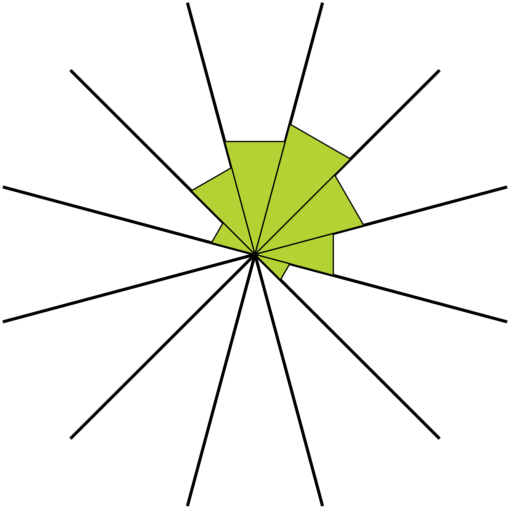
Outline

- Motivation
- **Preliminaries**
  - Windroses
  - Turbine Data
  - Jensen Wake Model
- Wind Proof's Design
- Evaluation & Example Application
- Conclusion

Windroses

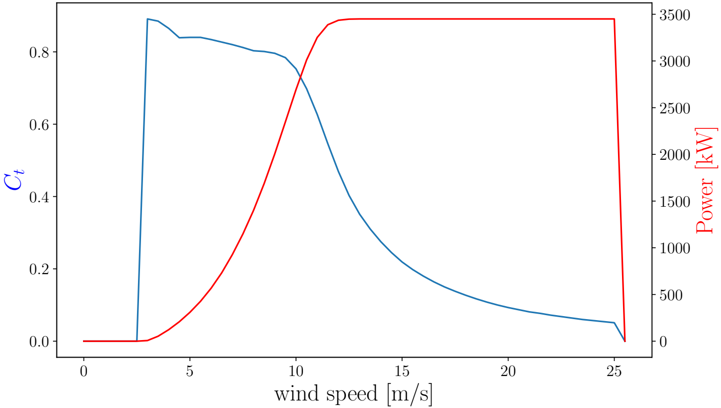


Windroses



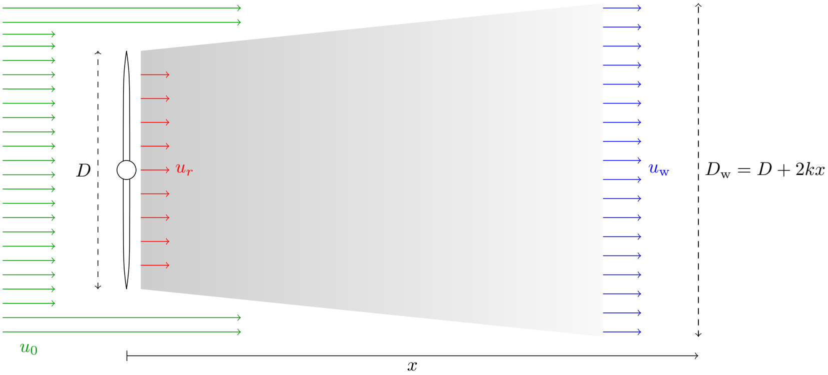
Turbine Data

Turbine Data



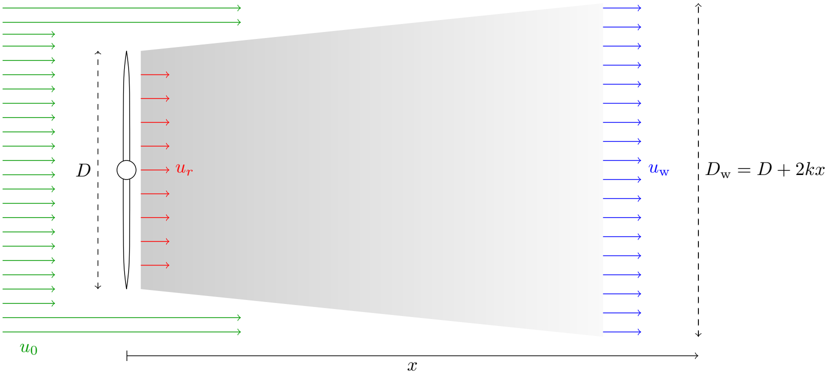
Velocity Deficit

Velocity Deficit



from [3]

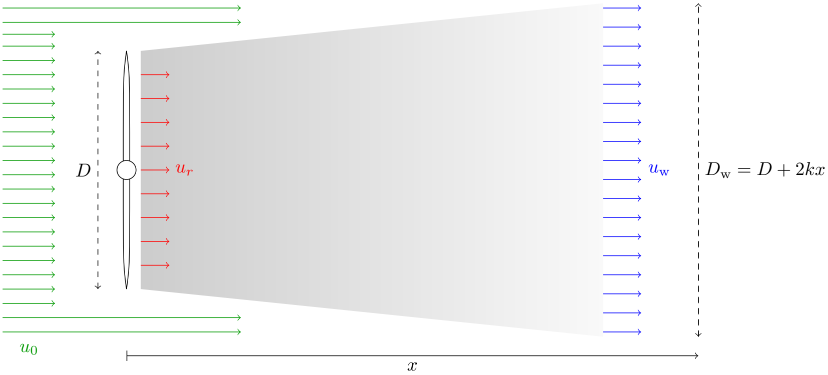
Velocity Deficit



from [3]

$$1 - \frac{u_w}{v_r} =$$

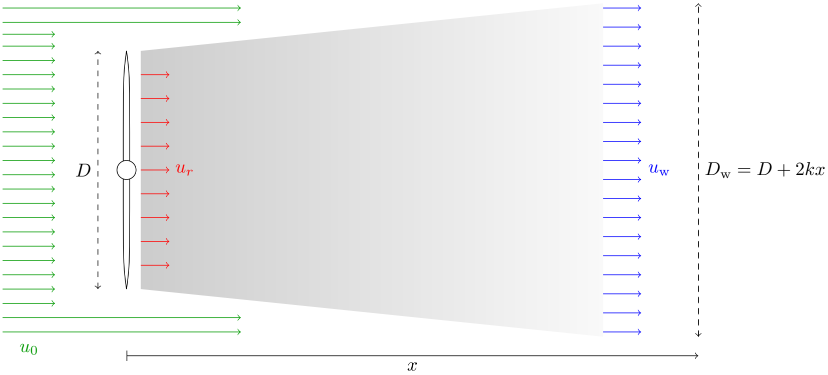
Velocity Deficit



from [3]

$$1 - \frac{u_w}{v_r} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot$$

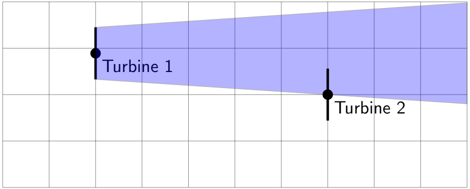
Velocity Deficit



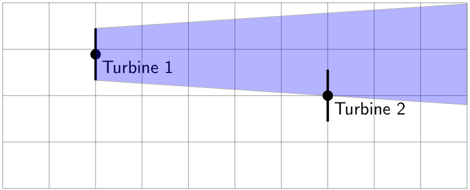
from [3]

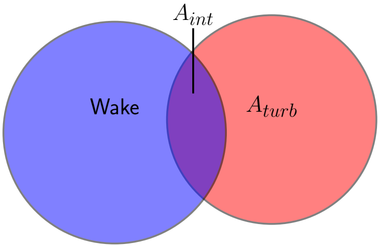
$$1 - \frac{u_w}{v_r} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2$$

Turbine Coverage

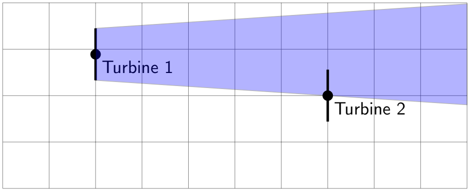


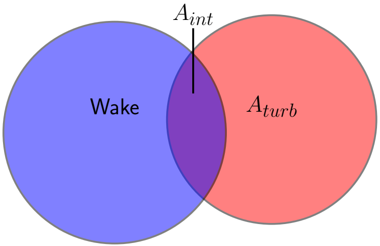
Turbine Coverage





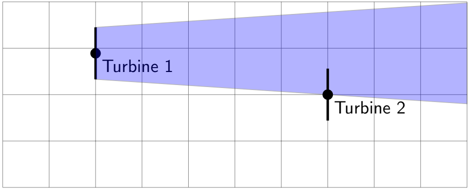
Turbine Coverage

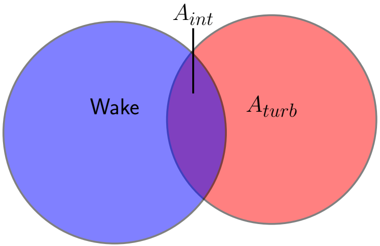




$$1 - \frac{u_w}{v_r} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2$$

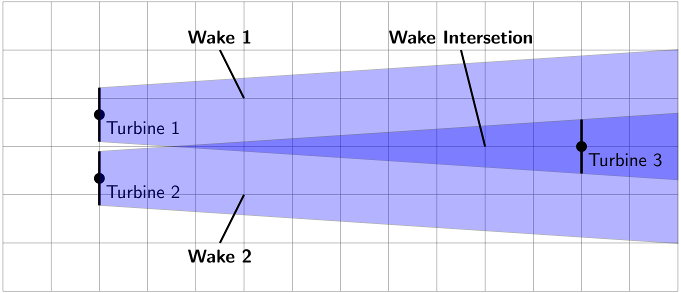
Turbine Coverage



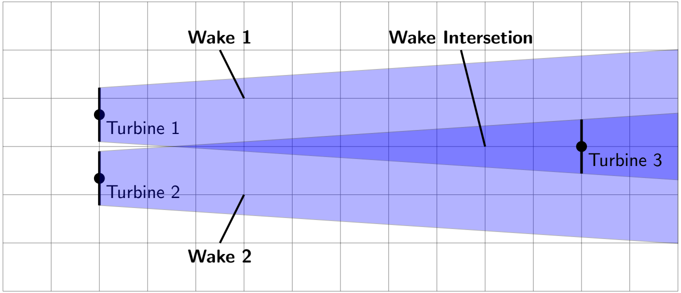


$$1 - \frac{u_w}{v_r} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2 \cdot \frac{A_{int}}{A_{turb}}$$

Wake Intersection



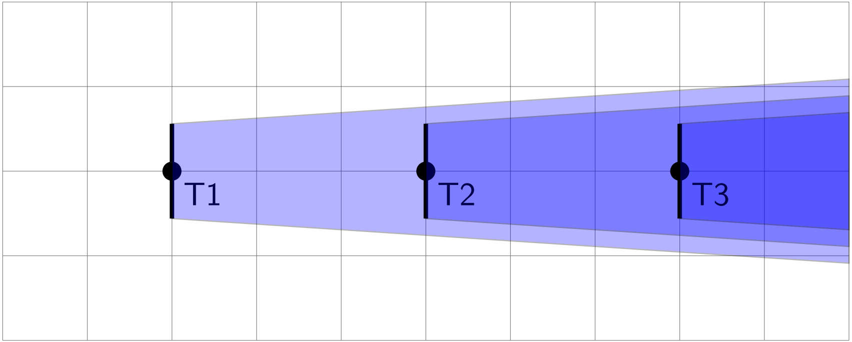
Wake Intersection



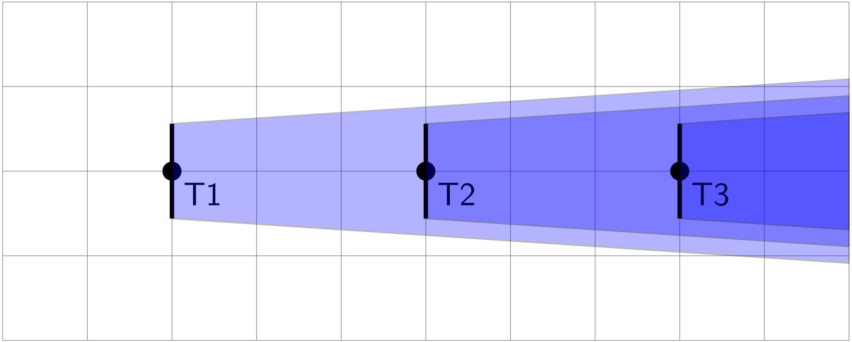
Wake Intersection

$$\delta_{total} = \sqrt{\delta_1^2 + \delta_2^2}$$

Speed Adaption Factor

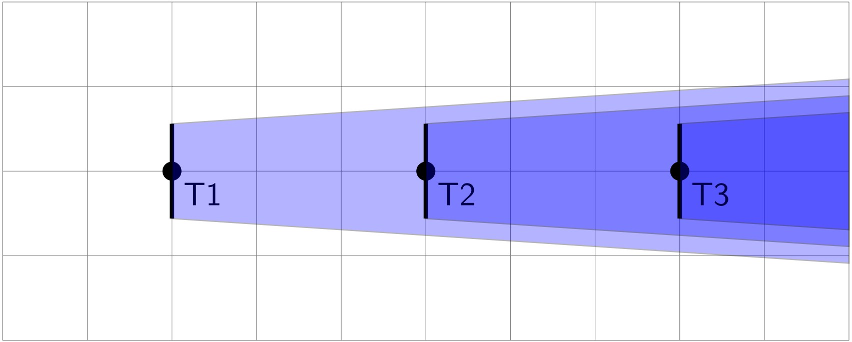


Speed Adaption Factor



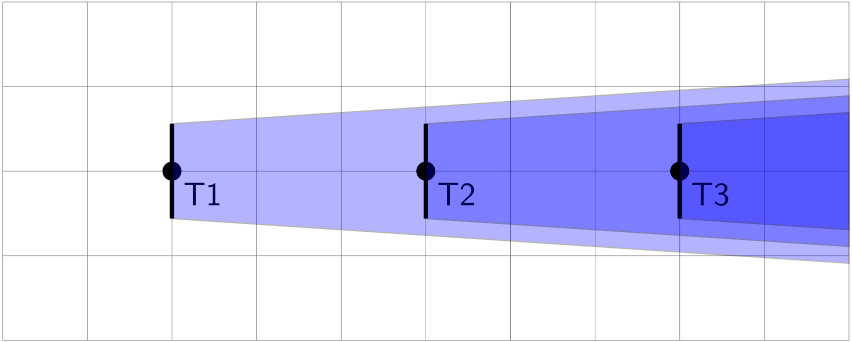
$$1 - \frac{u_w}{v_r} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2 \cdot \frac{A_{int}}{A_{turb}}$$

Speed Adaption Factor



$$1 - \frac{u_w}{u_0} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2 \cdot \frac{A_{int}}{A_{turb}}$$

Speed Adaption Factor



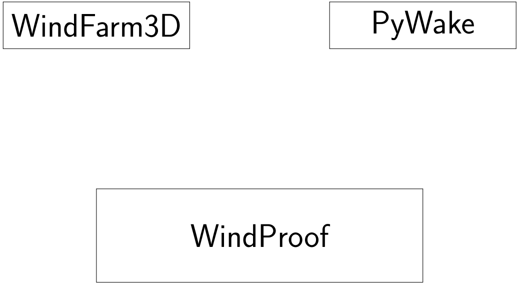
$$1 - \frac{u_w}{u_0} = \left(1 - \sqrt{1 - T_{ct}(v_r)}\right) \cdot \left(\frac{r}{r + x \cdot k}\right)^2 \cdot \frac{A_{int}}{A_{turb}} \cdot \frac{u_0}{v_r}$$

Outline

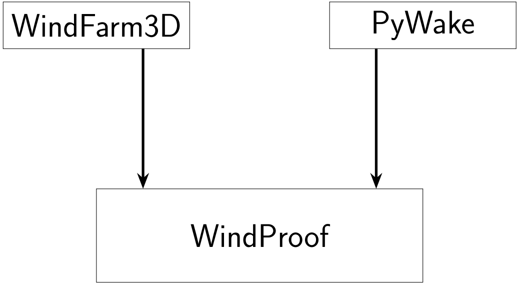
WindProof

WindProof

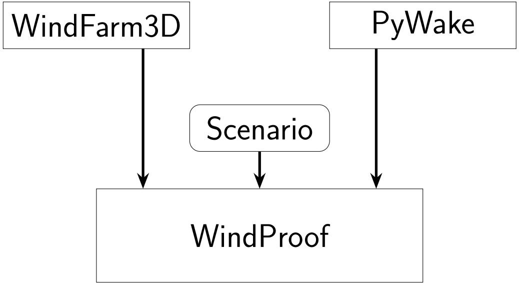
WindProof



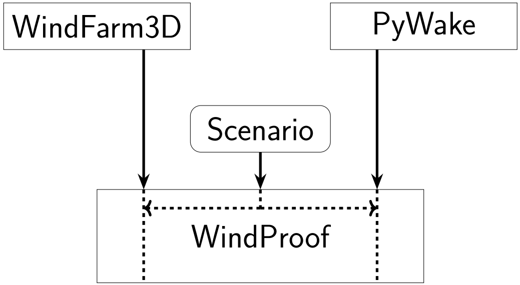
WindProof



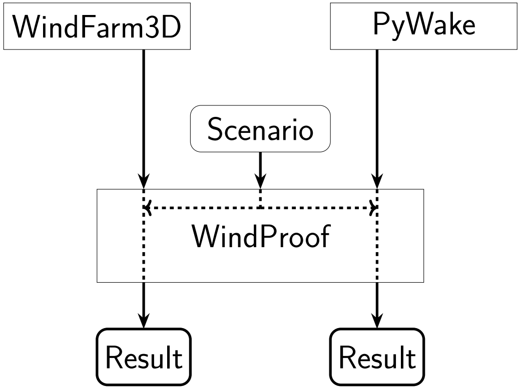
WindProof



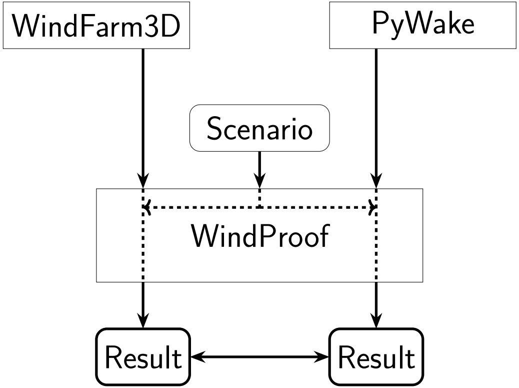
WindProof



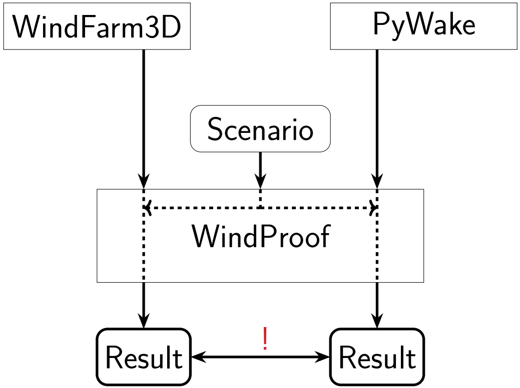
WindProof



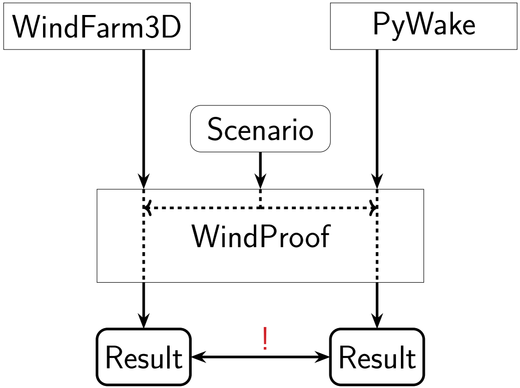
WindProof



WindProof

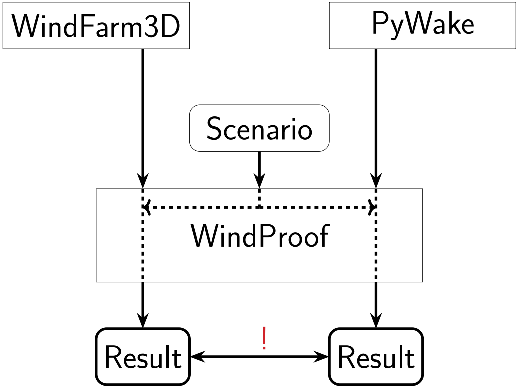


WindProof



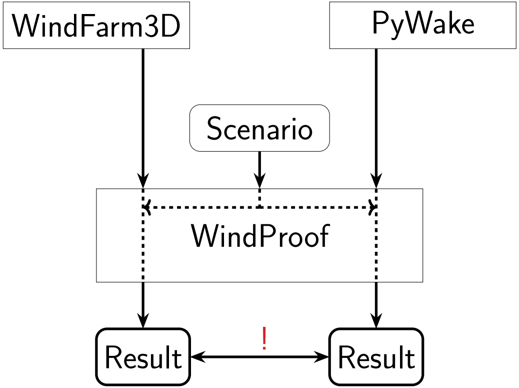
- 50+ Scenario Library

WindProof



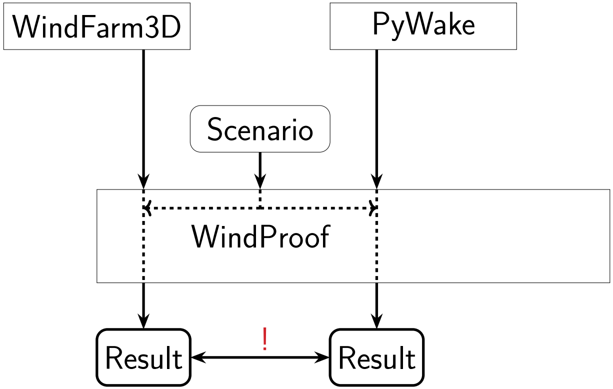
- 50+ Scenario Library
- Random Scenarios

WindProof



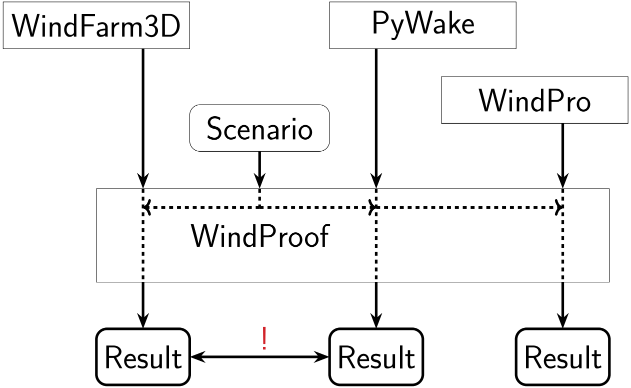
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes

WindProof



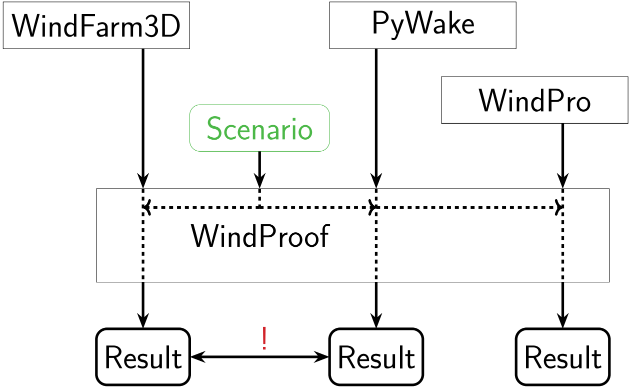
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:

WindProof



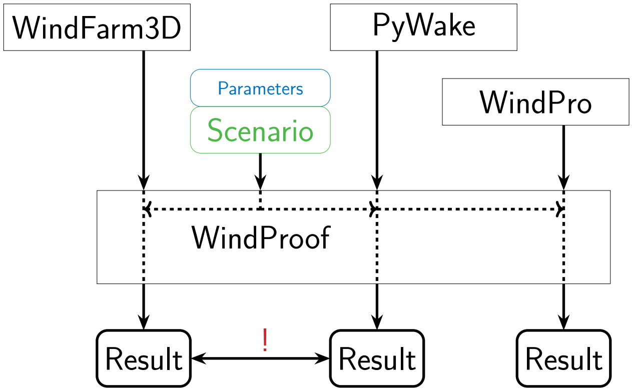
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools

WindProof



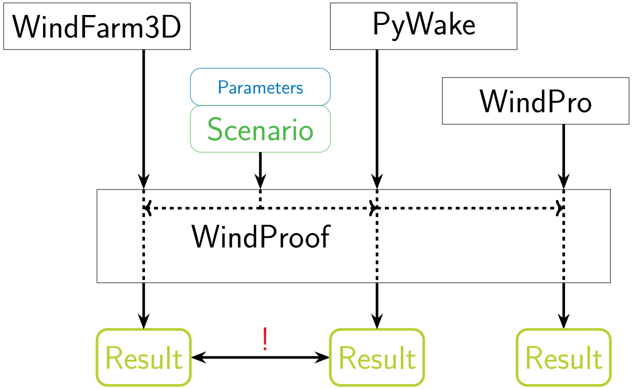
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios

WindProof



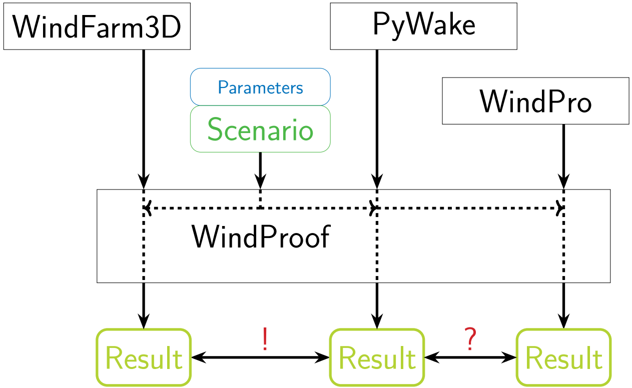
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios
  - more Parameters

WindProof



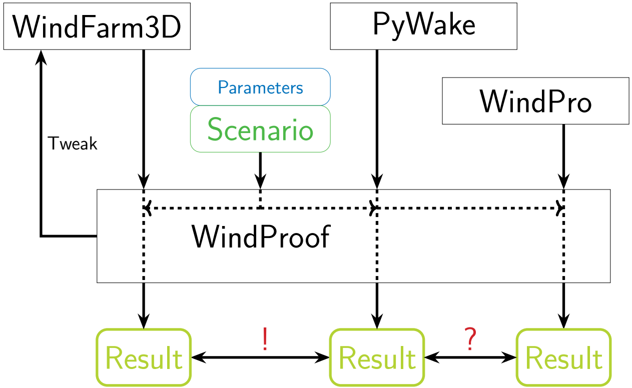
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios
  - more Parameters
  - more Calculations

WindProof



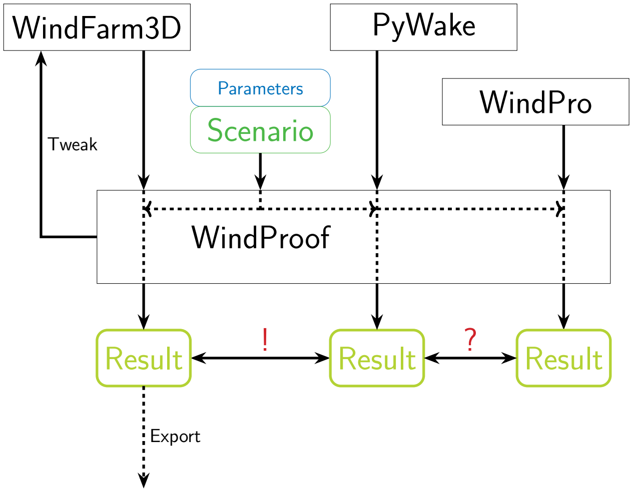
- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios
  - more Parameters
  - more Calculations
  - more Analytics

WindProof



- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios
  - more Parameters
  - more Calculations
  - more Analytics
- Interation over Hyperparameters

WindProof

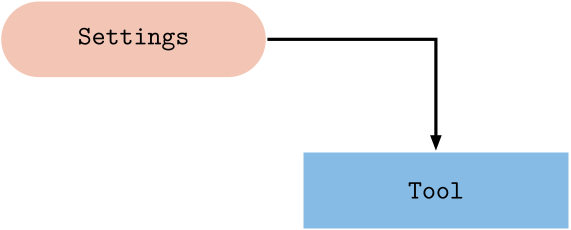


- 50+ Scenario Library
- Random Scenarios
- Generic; many Abstract classes
- Extendable with:
  - more Tools
  - more Scenarios
  - more Parameters
  - more Calculations
  - more Analytics
- Interation over Hyperparameters
- Uses Pandas

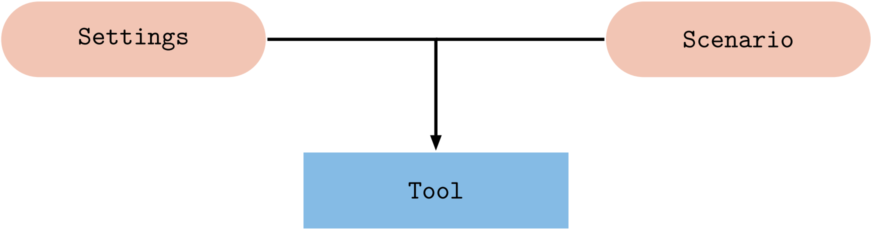
The `Tool` class

`Tool`

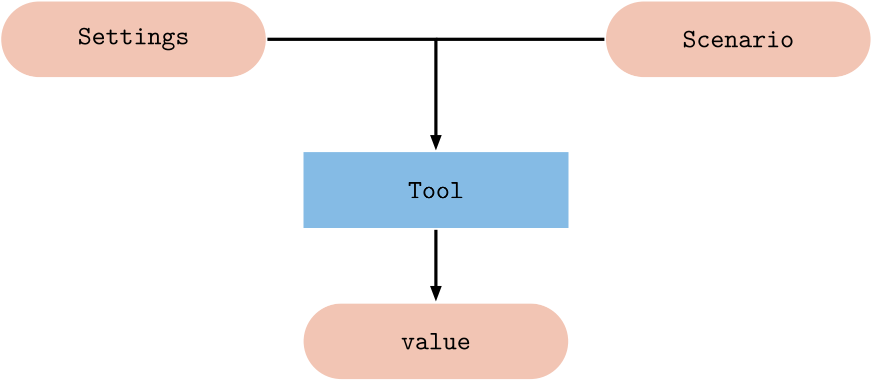
The `Tool` class



The `Tool` class



The `Tool` class



Settings vs. Scenario

Settings

Scenario

Settings vs. Scenario

Settings

Scenario

- General Information

Settings vs. Scenario

Settings

Scenario

- General Information
  - Binning Size

Settings vs. Scenario

Settings

Scenario

- General Information
  - Binning Size
  - Wake Intersection Model

Settings vs. Scenario

Settings

Scenario

- General Information
  - Binning Size
  - Wake Intersection Model
  - ...

Settings vs. Scenario

Settings

- General Information
  - Binning Size
  - Wake Intersection Model
  - ...

Scenario

- Case-specific Information

Settings vs. Scenario

Settings

Scenario

- General Information
  - Binning Size
  - Wake Intersection Model
  - ...
- Case-specific Information
  - Turbine Types

Settings vs. Scenario

Settings

- General Information
  - Binning Size
  - Wake Intersection Model
  - ...

Scenario

- Case-specific Information
  - Turbine Types
  - Turbine Positions

Settings vs. Scenario

Settings

- General Information
  - Binning Size
  - Wake Intersection Model
  - . . .

Scenario

- Case-specific Information
  - Turbine Types
  - Turbine Positions
  - Wind Data

Outline

- Motivation
- Preliminaries
- Wind Proof's Design
- **Evaluation & Example Application**
- Conclusion

Measure of Accuracy

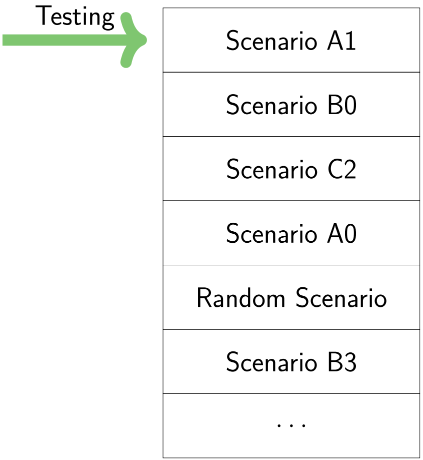
$$\mathrm{diff}(v_1, v_2) = \begin{cases} 0, & v_1 = v_2 = 0, \\ 100 \cdot \frac{|v_1 - v_2|}{(v_1 + v_2)}, & otherwise. \end{cases}$$

Adapted from [4]

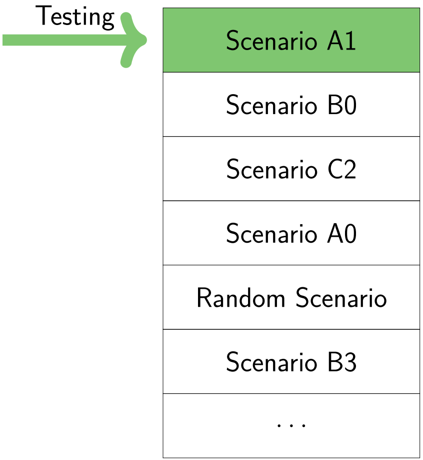
How to WindProof

Scenario A1
Scenario B0
Scenario C2
Scenario A0
Random Scenario
Scenario B3
...

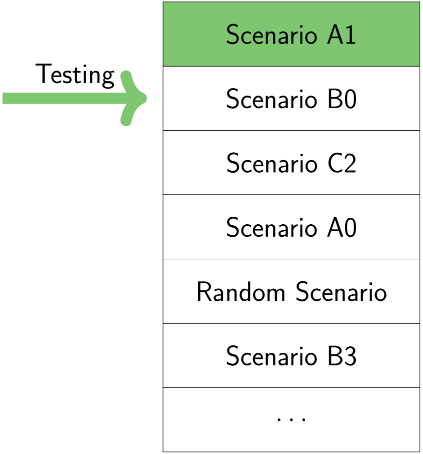
How to WindProof



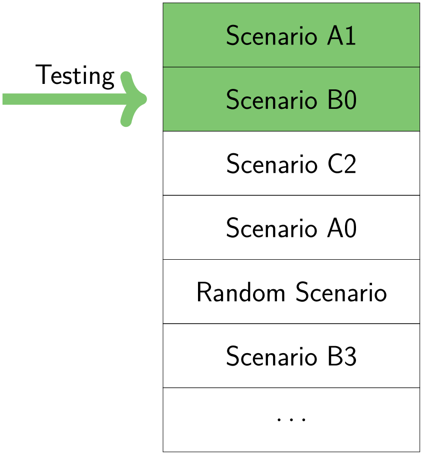
How to WindProof



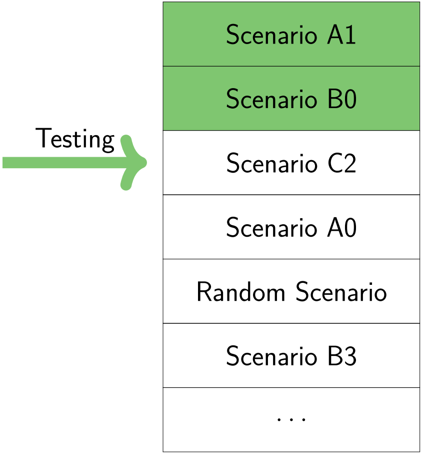
How to WindProof



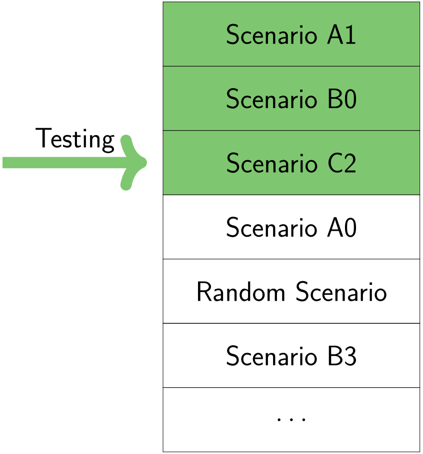
How to WindProof



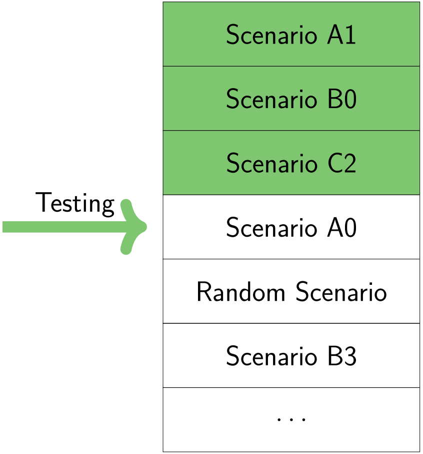
How to WindProof



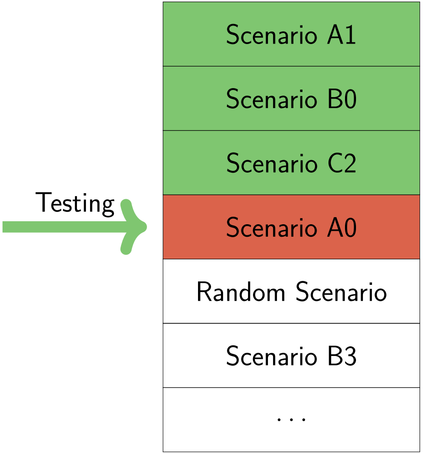
How to WindProof



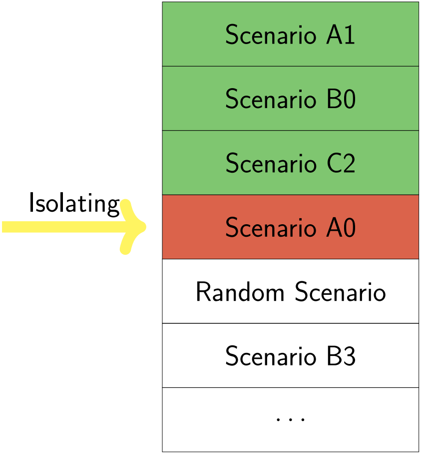
How to WindProof



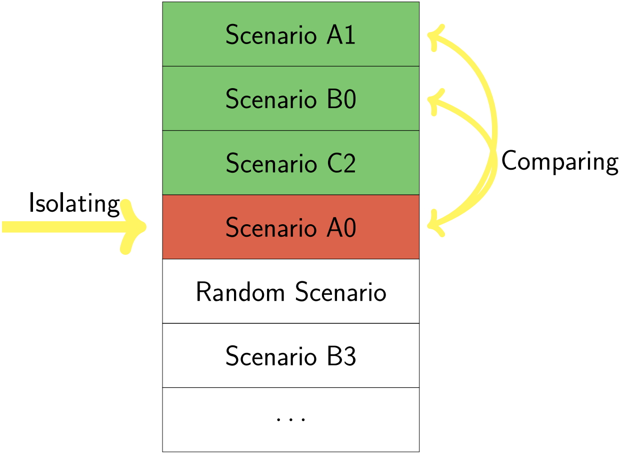
How to WindProof



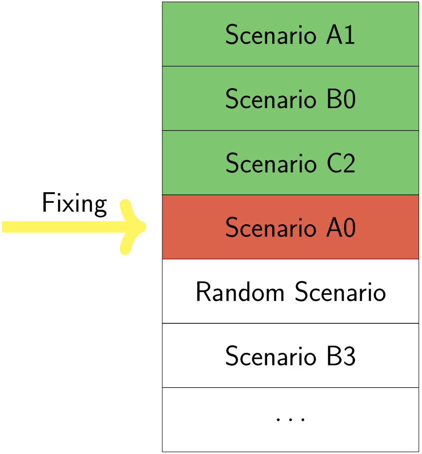
How to WindProof



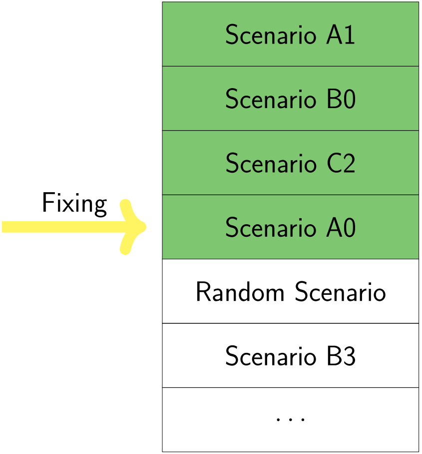
How to WindProof



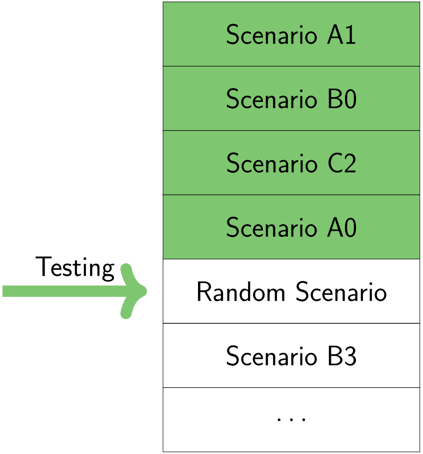
How to WindProof



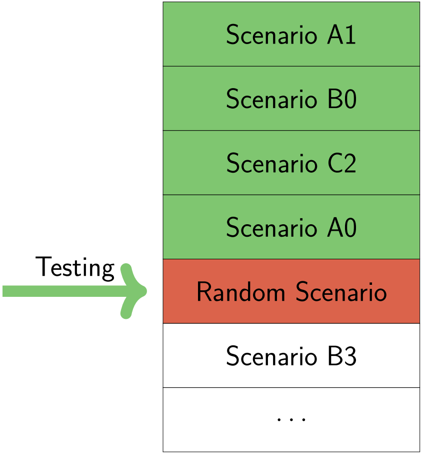
How to WindProof



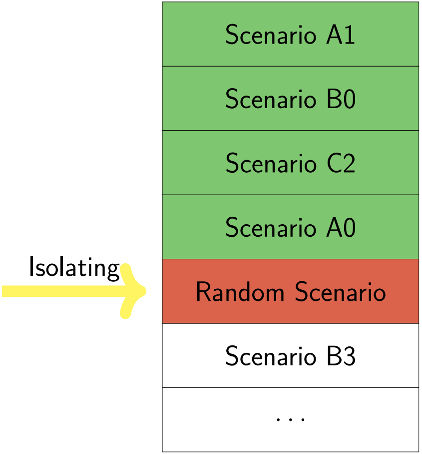
How to WindProof



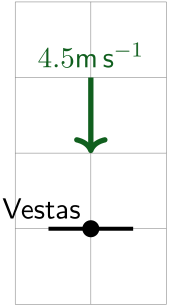
How to WindProof



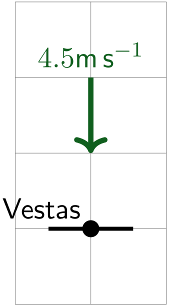
How to WindProof

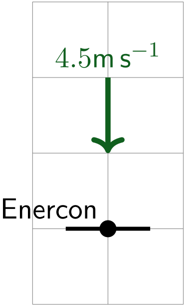


Difference 1: Power Curve Interpolation

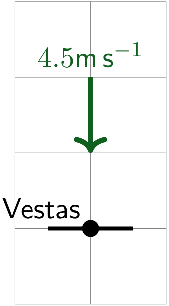


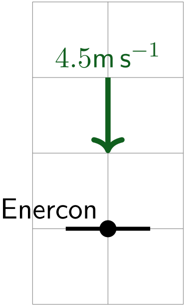
Difference 1: Power Curve Interpolation





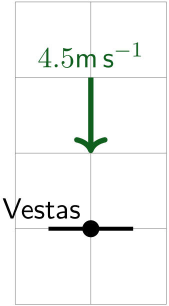
Difference 1: Power Curve Interpolation



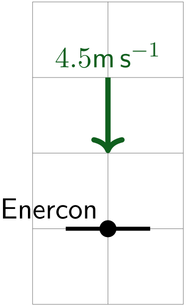


- Error

Difference 1: Power Curve Interpolation

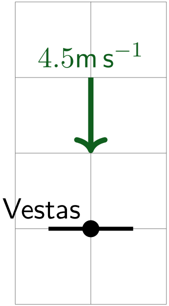


- Error

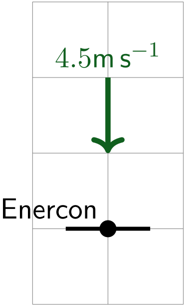


- No Error

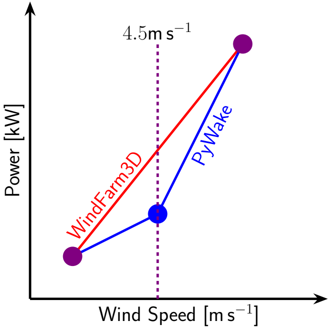
Difference 1: Power Curve Interpolation



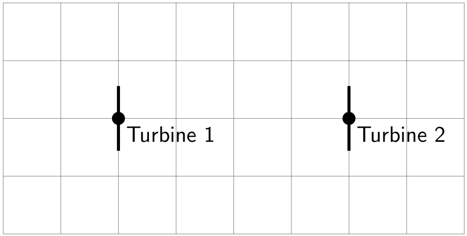
- Error



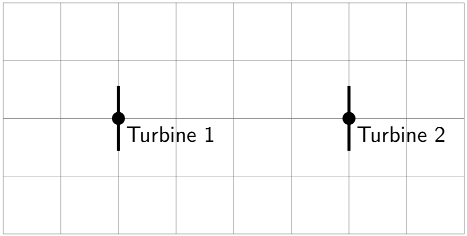
- No Error

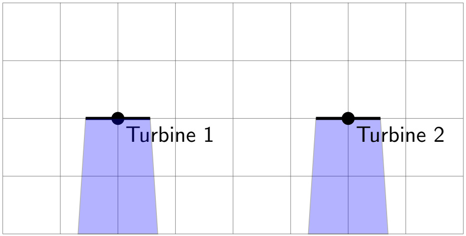


Difference 2: Wake Decay Factor

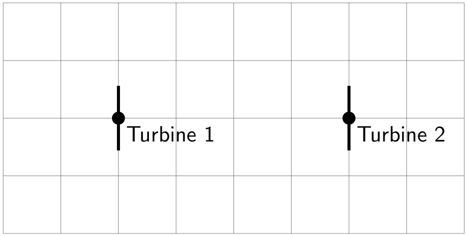


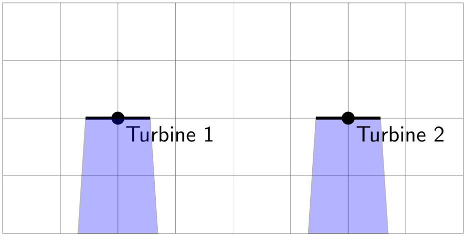
Difference 2: Wake Decay Factor

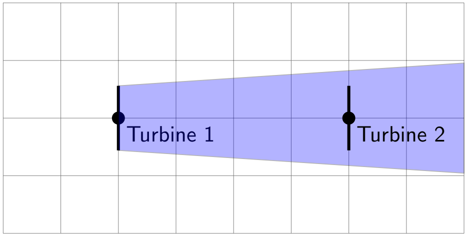




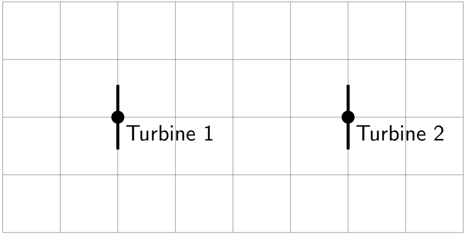
Difference 2: Wake Decay Factor

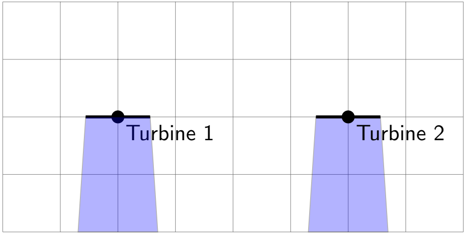


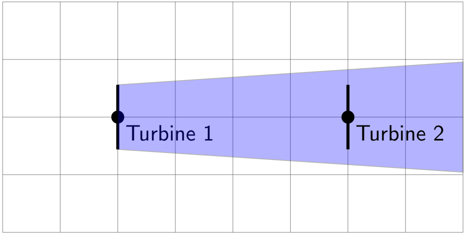




Difference 2: Wake Decay Factor







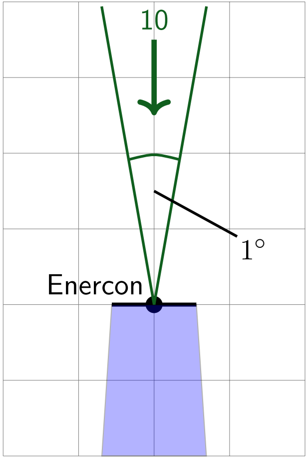
Wake Decay Factor k

Approximated through

$$k \approx \frac{0.5}{\ln \frac{z}{z_0}}$$

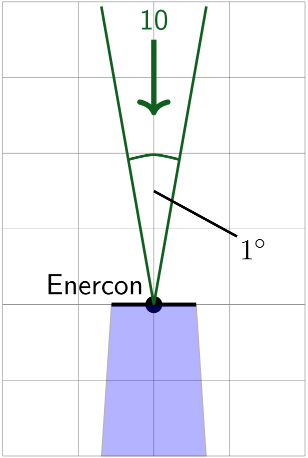
Difference 3: Sector Width

360 Sectors Condensed:

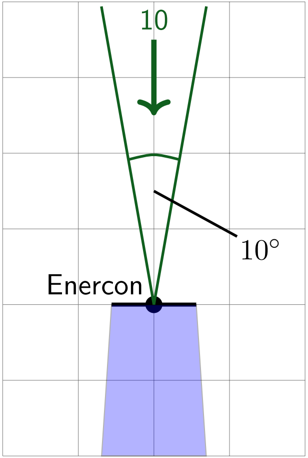


Difference 3: Sector Width

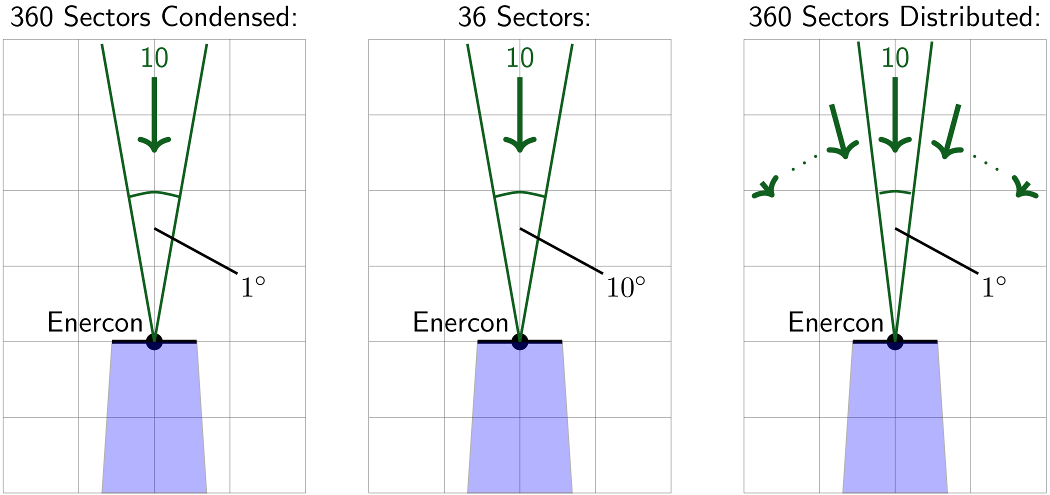
360 Sectors Condensed:



36 Sectors:



Difference 3: Sector Width

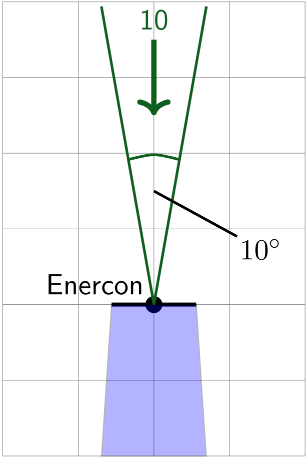


Difference 3: Sector Width

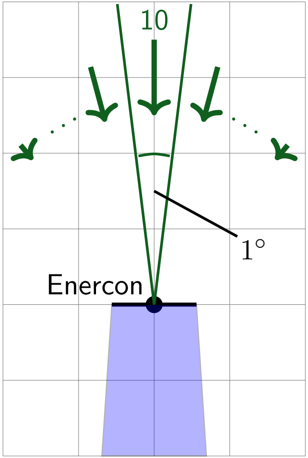
360 Sectors Condensed:

- No Error

36 Sectors:



360 Sectors Distributed:

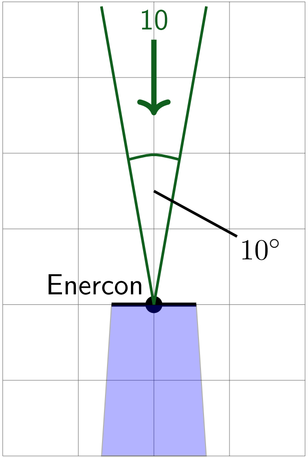


Difference 3: Sector Width

360 Sectors Condensed:

- No Error

36 Sectors:



360 Sectors Distributed:

- No Error

Difference 3: Sector Width

360 Sectors Condensed:

- No Error

36 Sectors:

- Significant Error

360 Sectors Distributed:

- No Error

Difference 3: Sector Width

360 Sectors Condensed:

- No Error

36 Sectors:

- Significant Error
- AEP equal to:

360 Sectors Distributed:

- No Error

Difference 3: Sector Width

360 Sectors Condensed:

- No Error

36 Sectors:

- Significant Error
- AEP equal to:

$\Leftarrow$ for `WindFarm3D`

360 Sectors Distributed:

- No Error

Difference 3: Sector Width

360 Sectors Condensed:

- No Error

36 Sectors:

- Significant Error
- AEP equal to:

$\Leftarrow$ for `WindFarm3D`

for `PyWake` $\Rightarrow$

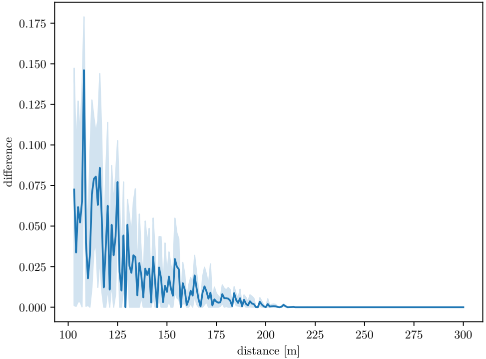
360 Sectors Distributed:

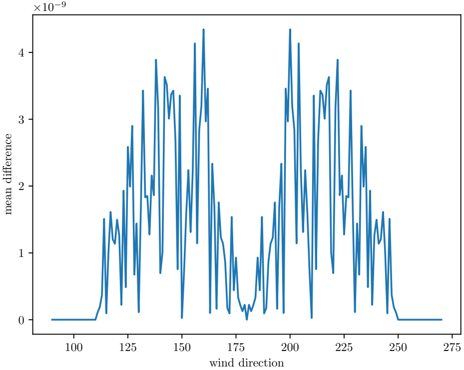
- No Error

Random 2 Turbine Scenarios

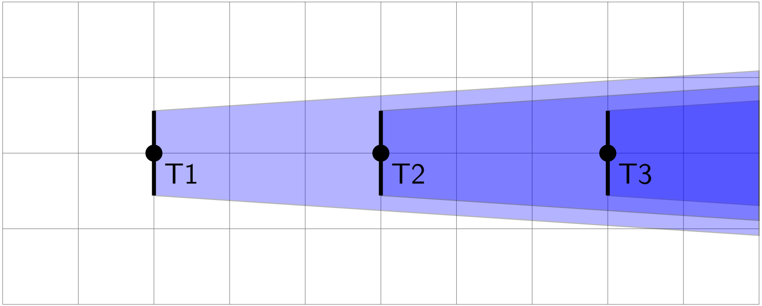
Random 2 Turbine Scenarios

Difference 4: Circle Intersection

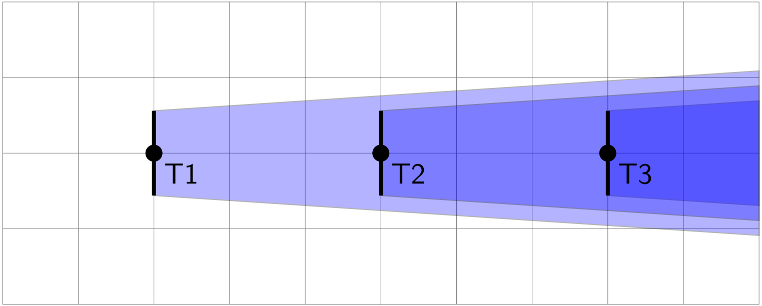




Difference 5: Speed Adaption Factor



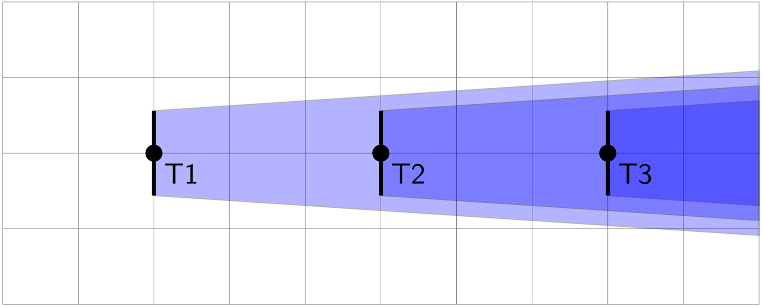
Difference 5: Speed Adaption Factor



(WF) $\delta_{\to 3} = 0.463935$

(PW) $\delta_{\to 3} = 0.356844$

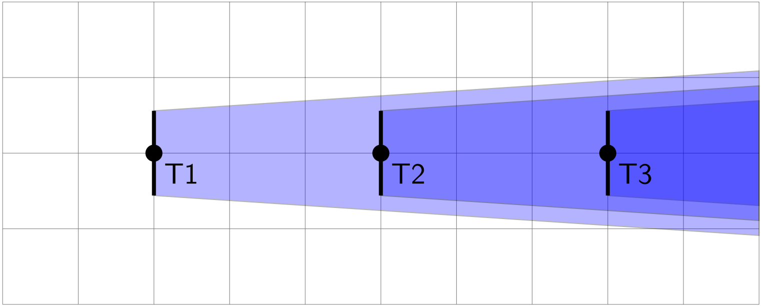
Difference 5: Speed Adaption Factor



$$\text{(WF)}\ \delta_{\rightarrow 3} = 0.463935 = \sqrt{0.173072^2 + \delta_{2\rightarrow 3}^2}$$

$$\text{(PW)}\ \delta_{\rightarrow 3} = 0.356844 = \sqrt{0.173072^2 + \delta_{2\rightarrow 3}^2}$$

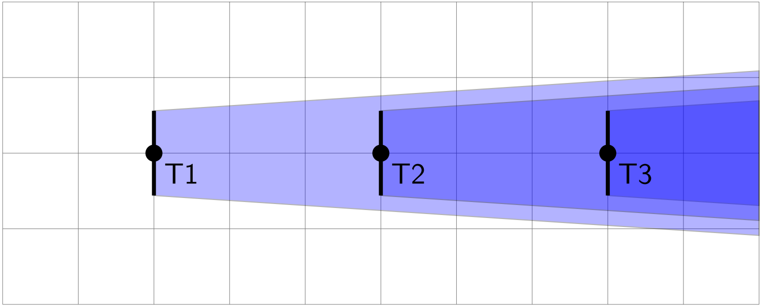
Difference 5: Speed Adaption Factor



$$\text{(WF) } \delta_{\rightarrow 3} = 0.463935 = \sqrt{0.173072^2 + \delta_{2\rightarrow 3}^2} \Rightarrow \delta_{2\rightarrow 3} = 0.430444$$

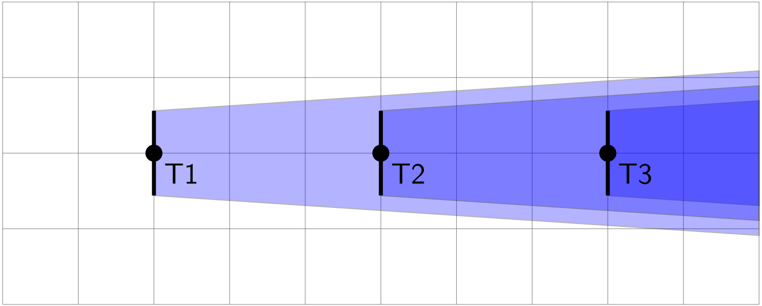
$$\text{(PW) } \delta_{\rightarrow 3} = 0.356844 = \sqrt{0.173072^2 + \delta_{2\rightarrow 3}^2} \Rightarrow \delta_{2\rightarrow 3} = 0.312064$$

Difference 5: Speed Adaption Factor



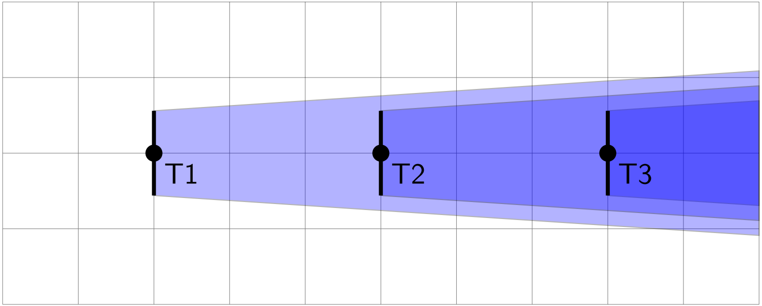
$$\frac{0.430444}{0.312064}$$

Difference 5: Speed Adaption Factor



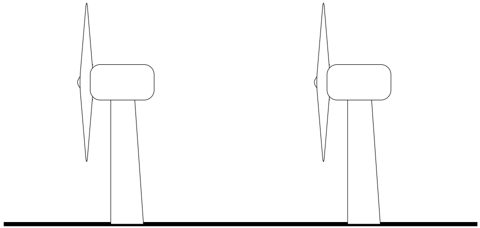
$$\frac{0.430444}{0.312064} \approx 1.376872$$

Difference 5: Speed Adaption Factor

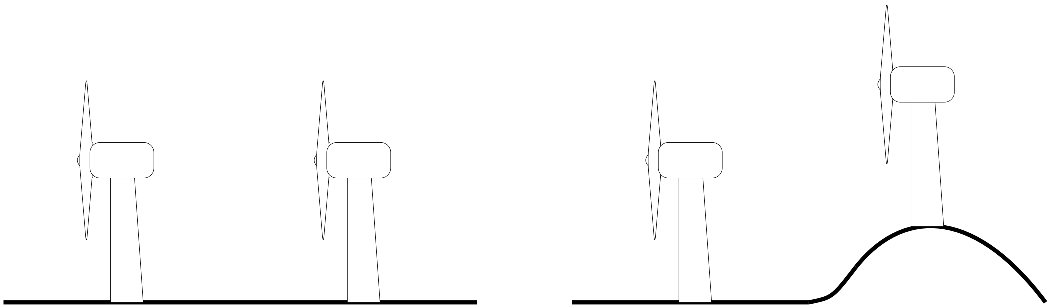


$$\frac{0.430444}{0.312064} \approx 1.376872 \approx \frac{10}{7.26284}$$

Difference 6: Elevation



Difference 6: Elevation



Conclusion

Conclusion

- WindProof; a Validation Framework

Conclusion

- WindProof; a Validation Framework
  - Easily extendable

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation
  - Wake Decay Factor

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation
  - Wake Decay Factor
  - Sector Width

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation
  - Wake Decay Factor
  - Sector Width
  - Circle Intersection Area

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation
  - Wake Decay Factor
  - Sector Width
  - Circle Intersection Area
  - Speed Adaption Factor

Conclusion

- WindProof; a Validation Framework
  - Easily extendable
  - 50+ Scenario Library
  - Random Scenarios
- Using WindProof on WindFarm3D and PyWake
  - Power Curve Interpolation
  - Wake Decay Factor
  - Sector Width
  - Circle Intersection Area
  - Speed Adaption Factor
  - Elevation

Conclusion

- WindProof's Limitations

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming
  - Only empirical

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming
  - Only empirical
  - Scenario quality

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming
  - Only empirical
  - Scenario quality
- Future Work

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming
  - Only empirical
  - Scenario quality
- Future Work
  - Fix errors

Conclusion

- WindProof's Limitations
  - Requires knowledge of Wind Model
  - Requires programming
  - Only empirical
  - Scenario quality
- Future Work
  - Fix errors
  - Extend WindProof

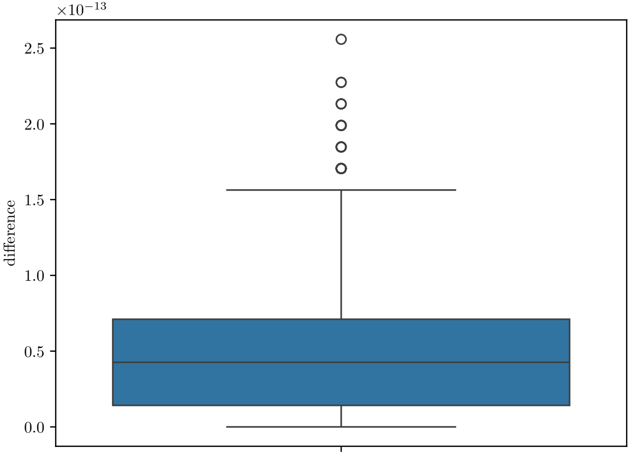
References

- Jensen, N. O. *A Note on Wind Generator Interaction*. Risø-M 2411. Roskilde, Denmark, 1983.
- Katic, I., Højstrup, J., and Jensen, N. O. "A simple model for cluster efficiency". In: *European wind energy association conference and exhibition*. A. Raguzzi. 1987, pp. 407–410.
- Maghnie, M. "Simulation and Layout Optimization of Offshore Wind Farms". MA thesis. LuFG Theory of Hybrid Systems at RWTH Aachen University, 2019.
- Miller, H. R. *Optimization: Foundations and Applications*. 2011.

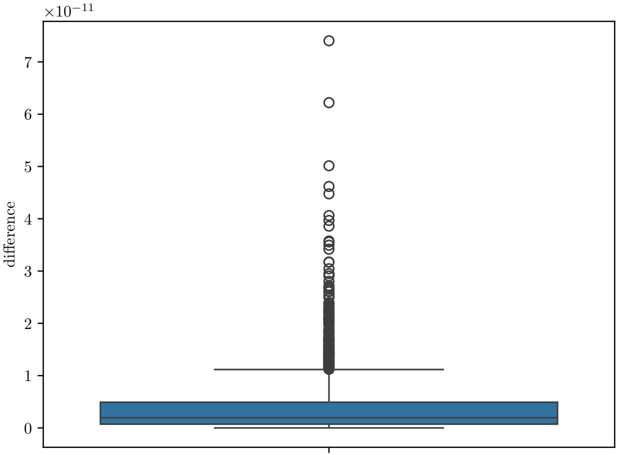
Back-Up Slides

Random Single Trubine Scenarios

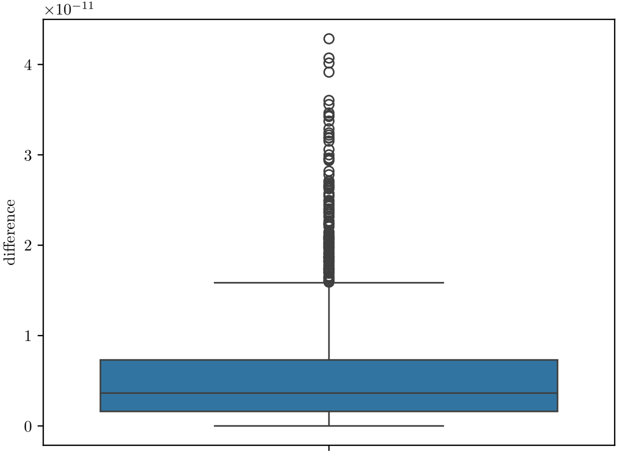
Random Single Trubine Scenarios



Random 2 Turbine Scenarios

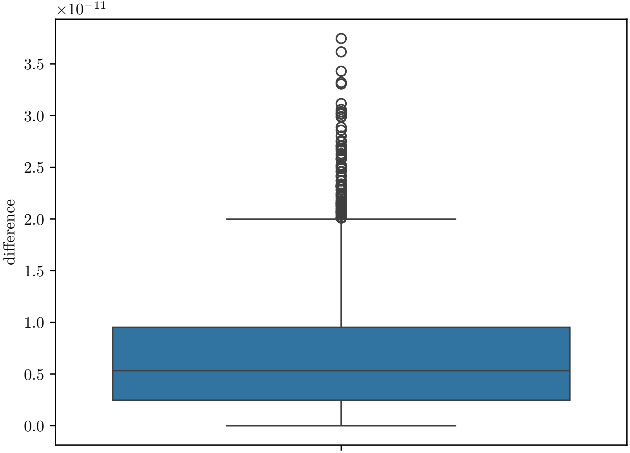


Random 3 Turbine Scenarios



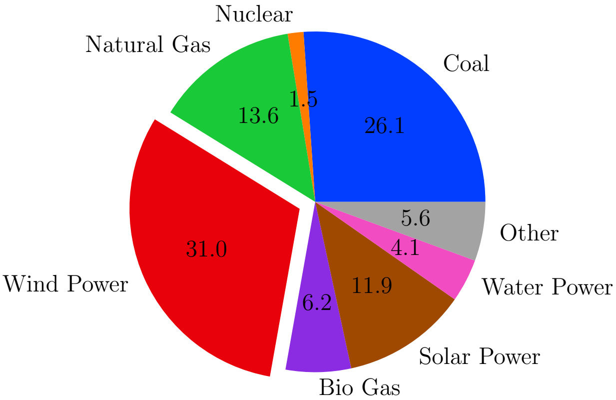
Random Flat Wind Parks

Random Flat Wind Parks

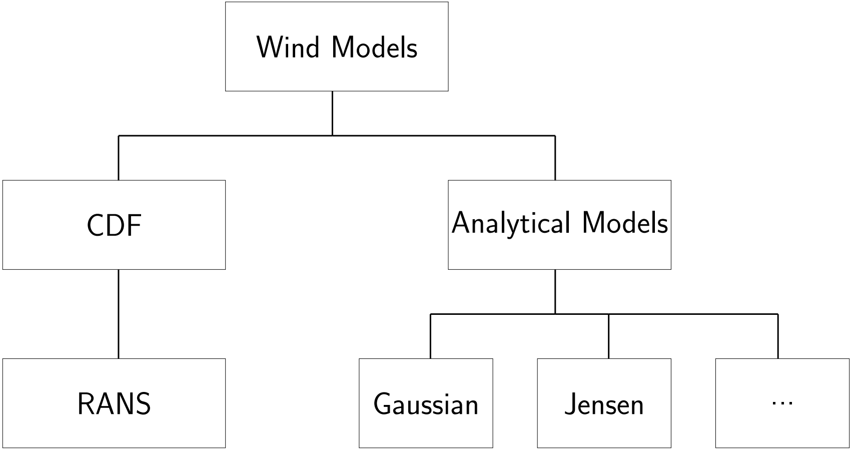


Energy Production in Germany

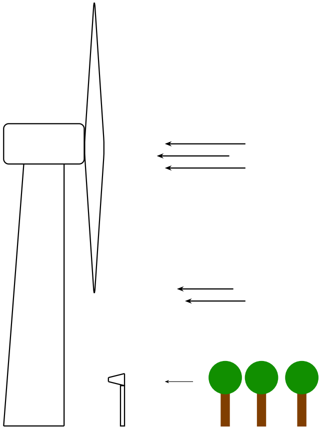
Energy Production in Germany



Model Types



Log Shear



Settings & Ranges

- Each setting has a value and a `Range`

Settings & Ranges

- Each setting has a value and a `Range`

Example settings and `Ranges`

Settings & Ranges

- Each setting has a value and a Range

Example settings and Ranges

Settings & Ranges

- Each setting has a value and a Range

Example settings and Ranges

- Wind Bin Size: $1 \in [0.1, 1]$

Settings & Ranges

- Each setting has a value and a `Range`

Example settings and `Ranges`

- Wind Bin Size: $1 \in [0.1, 1]$
- Shear Method: Log Shear $\in$ [Log Shear; Power Shear; No Shear]

Settings & Ranges

- Each setting has a value and a Range

Example settings and Ranges

- Wind Bin Size: $1 \in [0.1, 1]$
- Shear Method: Log Shear $\in$ [Log Shear; Power Shear; No Shear]

- Ranges restrict setting to sensible values

Settings & Ranges

- Each setting has a value and a Range

Example settings and Ranges

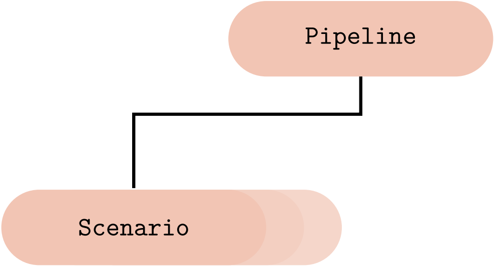
- Wind Bin Size: $1 \in [0.1, 1]$
- Shear Method: Log Shear $\in$ [Log Shear; Power Shear; No Shear]

- Ranges restrict setting to sensible values
- Allow iteration over possible values

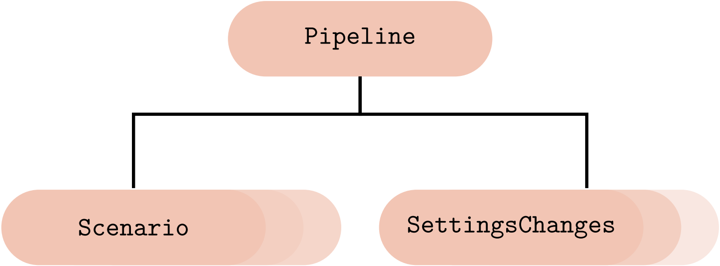
Pipeline & SettingsChanges

Pipeline

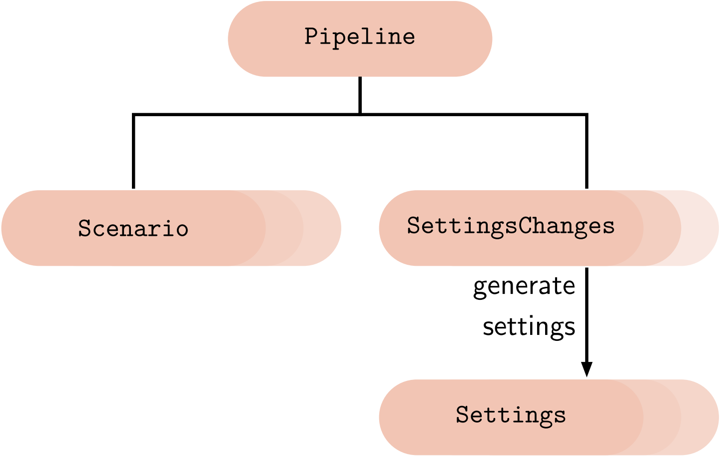
Pipeline & SettingsChanges



Pipeline & SettingsChanges



Pipeline & SettingsChanges



EvalData

- Class for saving and delivering data

EvalData

- Class for saving and delivering data
- Wrapper-class around pandas `DataFrame`

pandas

EvalData

- Class for saving and delivering data
- Wrapper-class around pandas `DataFrame`
- Familiarity

EvalData

- Class for saving and delivering data
- Wrapper-class around pandas `DataFrame`

pandas

- Familiarity
- User Interface

EvalData

- Class for saving and delivering data
- Wrapper-class around pandas `DataFrame`

pandas

- Familiarity
- User Interface
- Space Effciency

EvalData

- Class for saving and delivering data
- Wrapper-class around pandas `DataFrame`

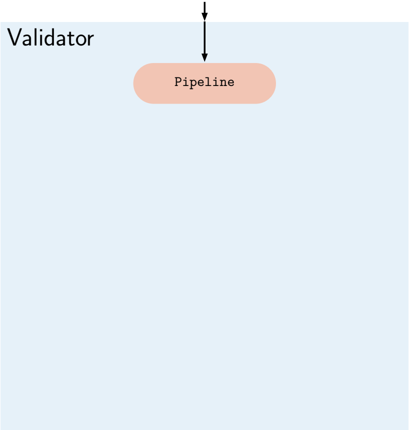
- Familiarity
- User Interface
- Space Effciency

- Ensures proper format and shortcuts interaction

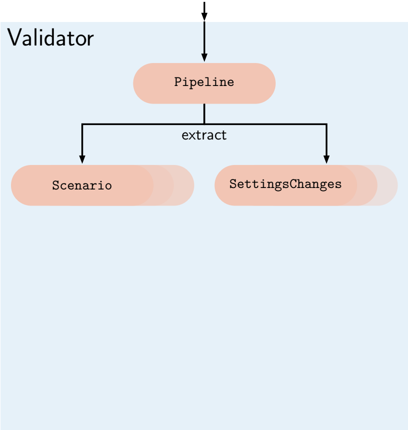
Validator

Validator

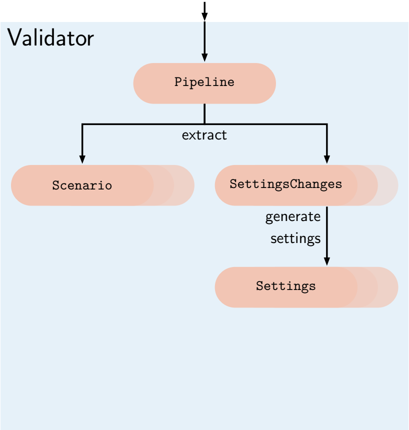
Validator



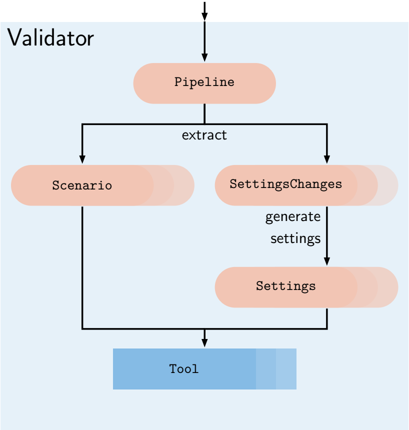
Validator



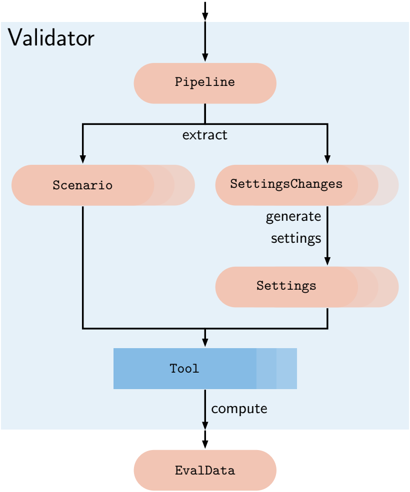
Validator


Validator
Pipeline
extract
Scenario
SettingsChanges
generate
settings
Settings

Validator


Validator
Pipeline
extract
Scenario
SettingsChanges
generate
settings
Settings
Tool

Validator



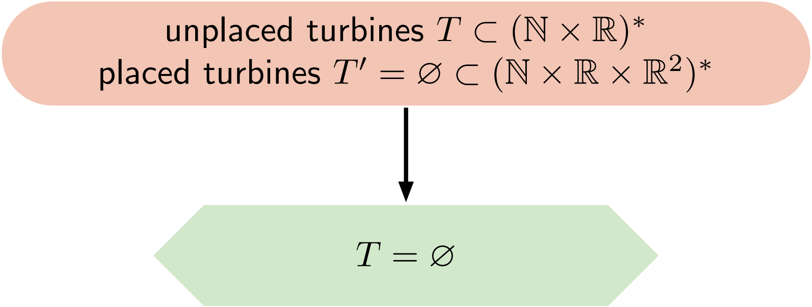
Validator
Pipeline
extract
Scenario
SettingsChanges
generate
settings
Settings
Tool
compute
EvalData

Random Factory

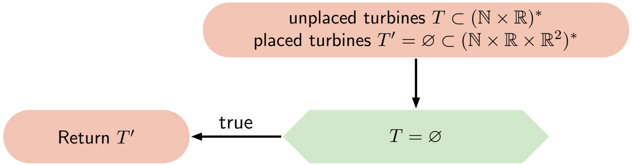
Random Factory

unplaced turbines $T \subset (\mathbb{N} \times \mathbb{R})^*$
placed turbines $T' = \varnothing \subset (\mathbb{N} \times \mathbb{R} \times \mathbb{R}^2)^*$

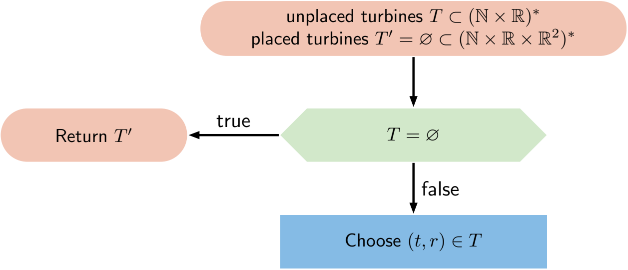
Random Factory



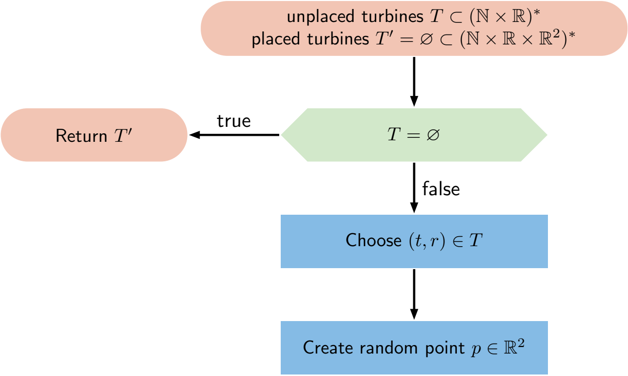
Random Factory



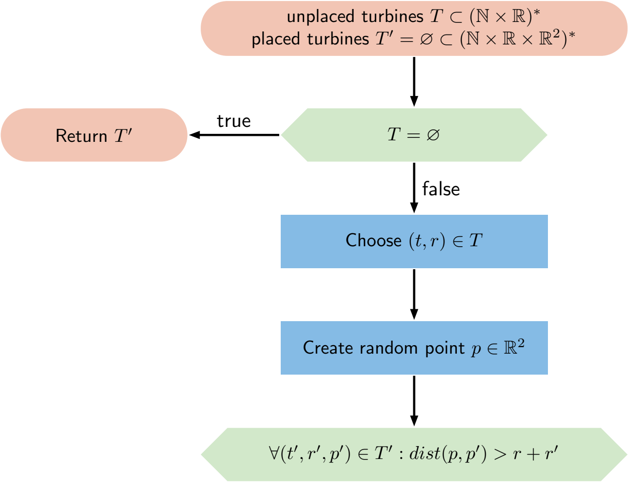
Random Factory



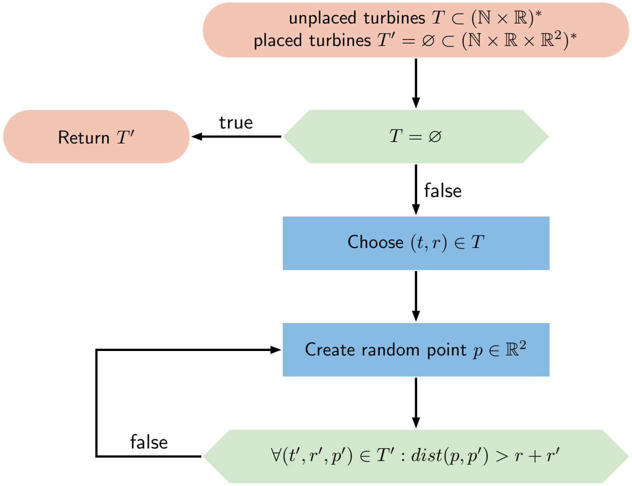
Random Factory



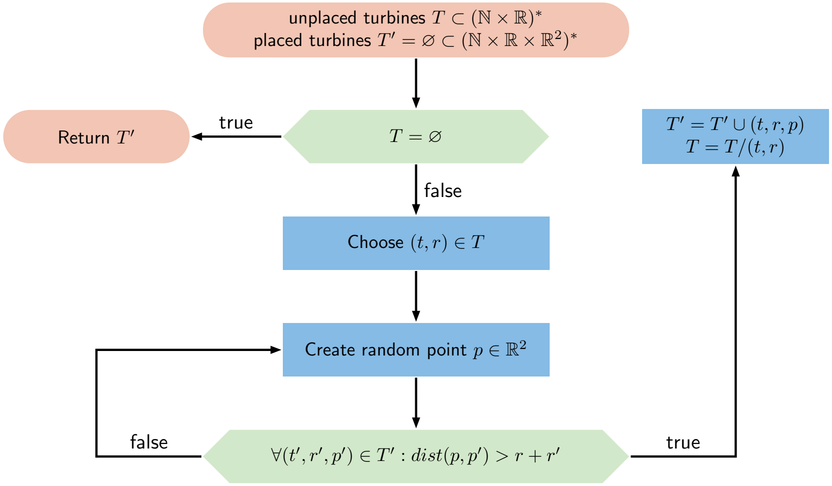
Random Factory



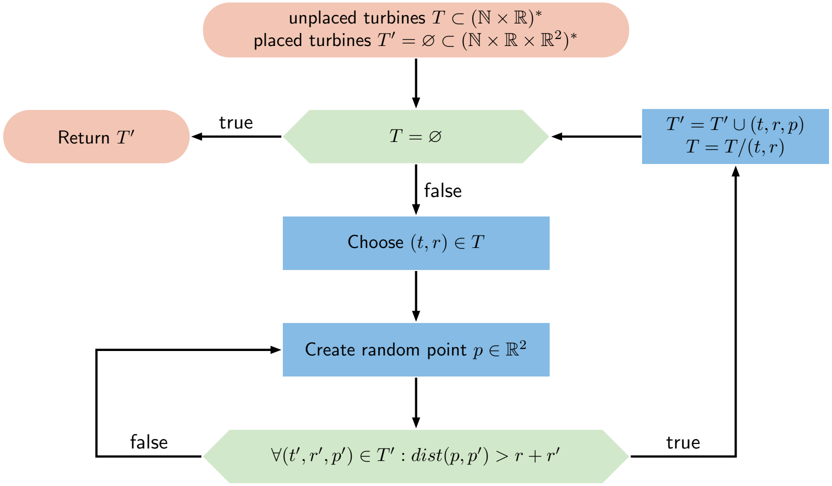
Random Factory



Random Factory



Random Factory



Eval

Scenario Name	Difference
Vestas Single Direction Group	
Low Constant Wind	$3.703\,703\,703\,703\,695\,3 \times 10^{0}$
Enercon Single Direction Group	
Low Constant Wind	$2.842\,170\,943\,040\,401 \times 10^{-14}$
Multiple Directions Group	
8 Sectors Multiple Constant Winds	$0.125\,805\,944\,330\,849\,68 \times 10^{0}$

Power Curve Interpretation

Possible Causes:

Power Curve Interpretation

Possible Causes:

- Cut-in Speed

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- Rotor Diameter

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- C_t/C_p Curve

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~

Power Curve Interpretation

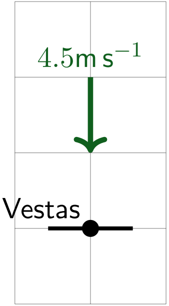
Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve



Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$(24[\mathsf{h/d}] \cdot 365[d]) \cdot \sum_{i=1}^{n} p_i \cdot \left(\sum_{v \in supp(X)} p(X = v) * T_p(v) \right)$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$8760[\mathrm{h}] \cdot \sum_{i=1}^{12} p_i \cdot \left(\sum_{v \in supp(X_i)} p(X_i = v) * V_p(v) \right)$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$8760[\mathrm{h}] \cdot \left(\sum_{v \in supp(X_9)} p(X_9 = v) * V_p(v) \right)$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$8760[\mathsf{h}] \cdot V_p(4.5)$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$8760[\text{h}] \cdot 208[\text{kW}]$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$1822080 [\text{kWh}]$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

$$1822.08[\text{MW h}]$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$1822.08 [\text{MW h}]$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$1892.16[\text{MW h}] = 8760[\text{h}] \cdot V_p^*(4.5)$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$1892.16[\mathrm{MW\,h}] = 8760[\mathrm{h}] \cdot V_p^*(4.5) \Rightarrow V_p^*(4.5) = 216[\mathrm{kW}]$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$V_p^*(4.5) = 216[\mathrm{kW}] = \frac{123[\mathrm{kW}] + 309[\mathrm{kW}]}{2}$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$V_p^*(4.5) = 216[\text{kW}] = \frac{123[\text{kW}] + 309[\text{kW}]}{2} = \frac{V_p(4) + V_p(5)}{2}$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve

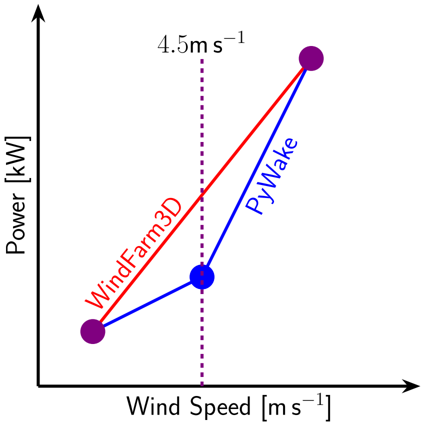
Tool Name	AEP [MW h]
WindFarm3D	1892.16
PyWake	1822.08

$$V_p^*(4.5) = 216[\text{kW}] = \frac{123[\text{kW}] + 309[\text{kW}]}{2} = \frac{V_p(4) + V_p(5)}{2}$$

Power Curve Interpretation

Possible Causes:

- ~~Cut-in Speed~~
- ~~Rotor Diameter~~
- ~~C_t/C_p Curve~~
- Power Curve



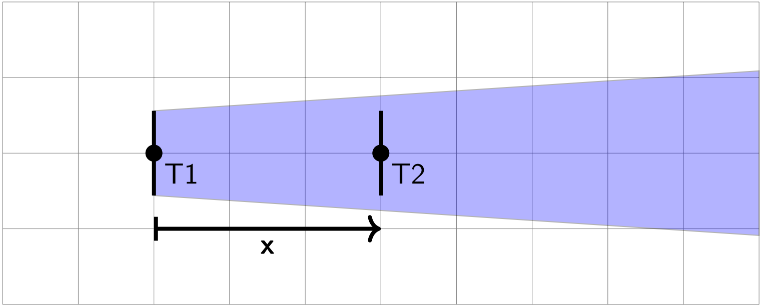
2 Turbine Test Cases

Scenario Name	Difference
Jensen Intensity Group	
No Interference	$1.421\,085\,471\,520\,200\,4 \times 10^{-14}$
Full Wake	$7.259\,577\,323\,929\,676 \times 10^{0}$
Multiple Turbine Types Group	
No Wind	$0.000\,000\,000\,000\,000\,00 \times 10^{0}$
No Interference	$0.000\,000\,000\,000\,000\,00 \times 10^{0}$
Basic Wake	$8.023\,289\,129\,267\,496 \times 10^{0}$
36 Sector Wake	$8.150\,410\,878\,579\,066 \times 10^{0}$
Wake 19 Degree	$7.190\,617\,965\,615\,274 \times 10^{0}$
No Wake	$0.000\,000\,000\,000\,000\,00 \times 10^{0}$
No Wake Interpolation	$1.421\,085\,471\,520\,200\,4 \times 10^{-14}$
Full Coverage	$8.046\,184\,311\,817\,555 \times 10^{0}$
Partial Coverage	$0.428\,264\,702\,225\,789\,05 \times 10^{0}$

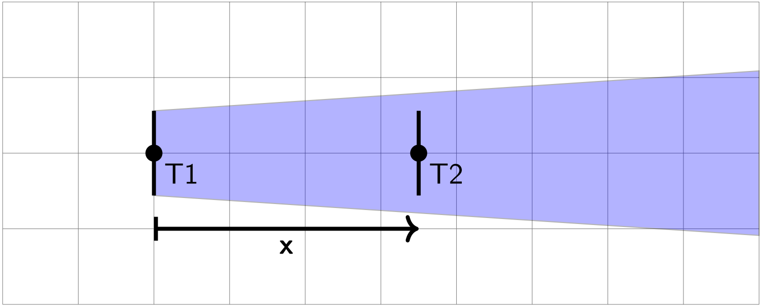
Closing on Wake Decay Factor

Scenario Name	Difference
Jensen Wake Intensity group	
Full Wake	$2.842\,170\,943\,040\,401 \times 10^{-14}$
Multiple Turbine Types Group	
Basic Wake	$2.842\,170\,943\,040\,401 \times 10^{-14}$
36 Sector Wake	$1.034\,155\,051\,941\,766\,6 \times 10^{0}$
Wake 19 Degree	$2.256\,683\,728\,774\,078\,2 \times 10^{-11}$
Full Coverage	$4.263\,256\,414\,560\,601 \times 10^{-14}$
Partial Coverage	$2.842\,170\,943\,040\,401 \times 10^{-14}$

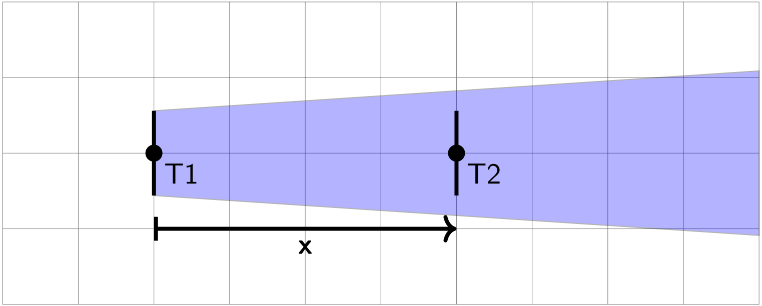
Increasing Distance Pipeline



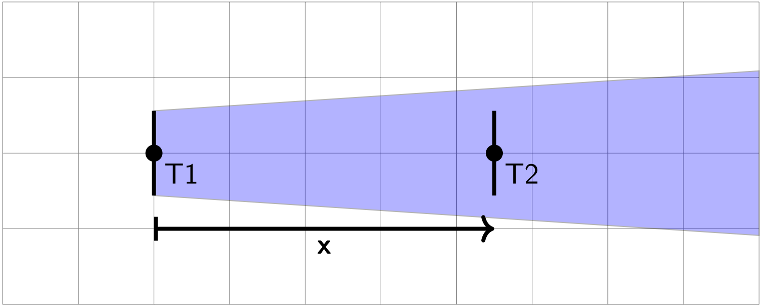
Increasing Distance Pipeline



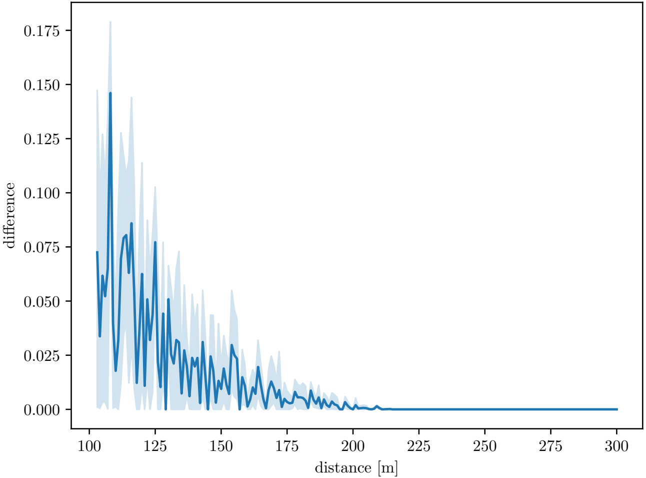
Increasing Distance Pipeline



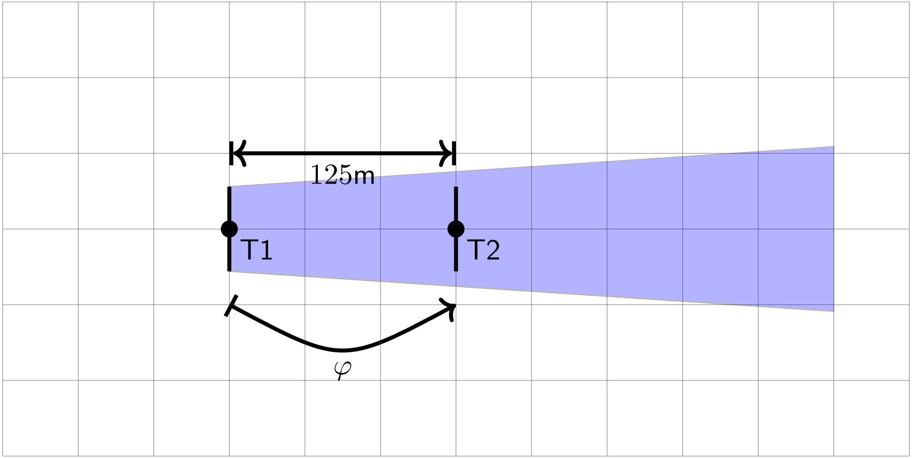
Increasing Distance Pipeline



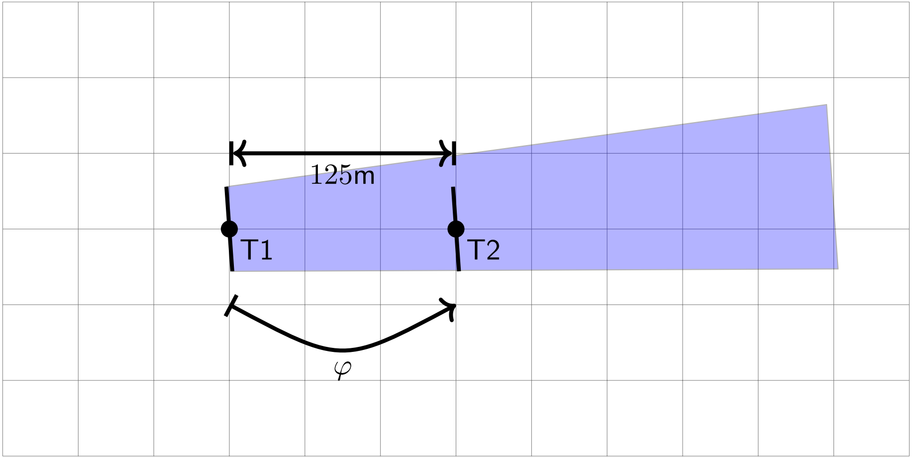
Increasing Distance Pipeline Results



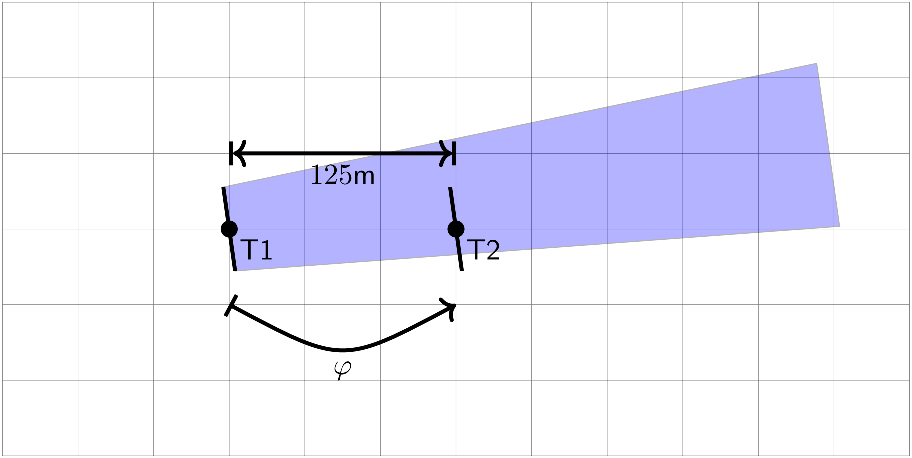
Increasing Angle Pipeline



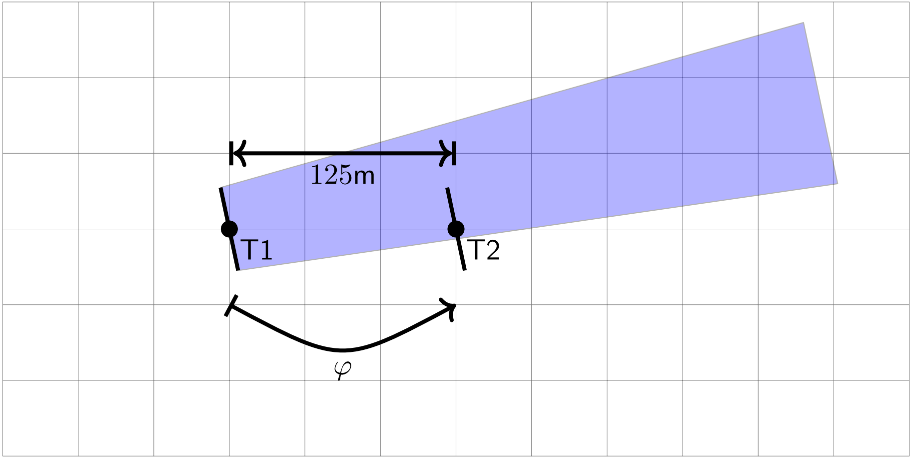
Increasing Angle Pipeline



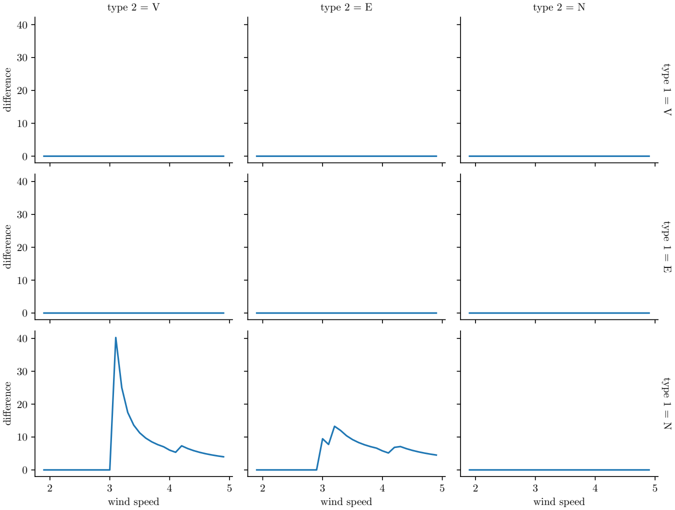
Increasing Angle Pipeline



Increasing Angle Pipeline



Turbine Combinations Pipeline



Circle Intersection Calculations

from \ to	Vestas	Enercon	Nordex
Vestas	(63.121; 56)	(63.121; 57.85)	(63.121; 45)
Enercon	(64.971; 56)	(64.971; 57.85)	(64.971; 45)
Nordex	(52.121; 56)	(52.121; 57.85)	(52.121; 45)

Circle Intersection Calculations

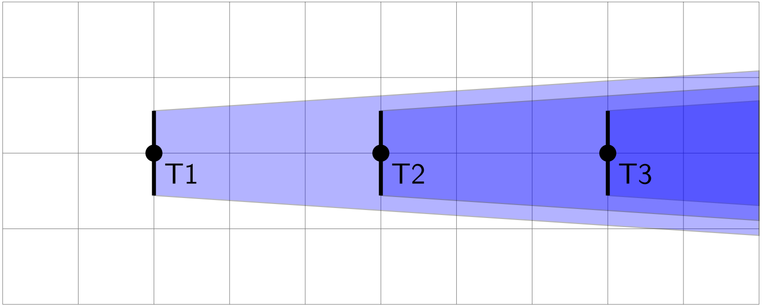
Tool Name	Wake Radius	Receiving Turbine Radius	Area $[m^2]$
WindFarm3D	52.121	57.85	3007.6114
PyWake	52.121	57.85	3286.4581
WindFarm3D	52.121	45	1779.9863
PyWake	52.121	45	1779.9863
WindFarm3D	52.121	53	2519.5157
PyWake	52.121	53	2563.3179

3 Turbine Cases

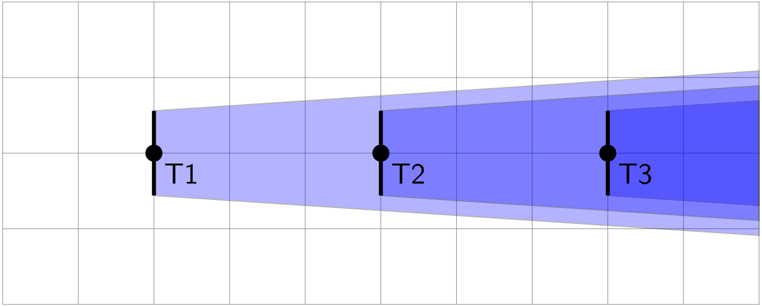
3 Turbine Cases

Scenario Name	Difference
Basic Wake Intersection	$2.842\,170\,943\,040\,401 \times 10^{-14}$
Complex Wake Intersection	$2.842\,170\,943\,040\,401 \times 10^{-14}$
Three in a Row Group	
No Interference	$0.000\,000\,000\,000\,000\,00 \times 10^{0}$
Turbine 2 Inoperable	$4.263\,256\,414\,560\,601 \times 10^{-14}$
Turbine 3 Inoperable	$1.421\,085\,471\,520\,200\,4 \times 10^{-14}$
All Turbines Operable	$7.180\,730\,119\,283\,453 \times 10^{0}$

All Turbines Operable

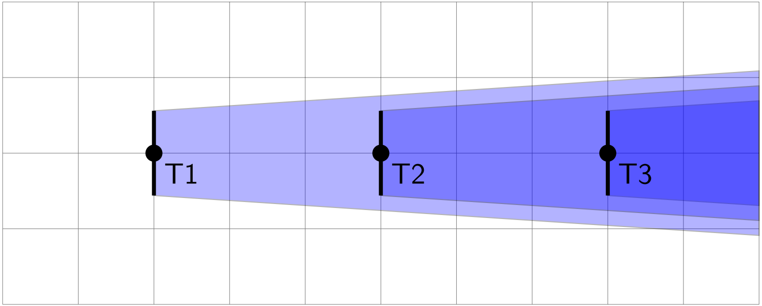


All Turbines Operable



$$\text{(WF)}\ 36270.36304[\text{MW h}]$$
$$\text{(PW)}\ 39076.32874[\text{MW h}]$$

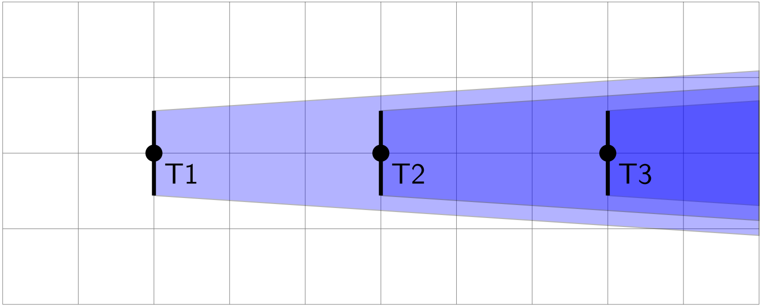
All Turbines Operable



(WF) 36270.36304[MW h] -32748.42776[MW h]

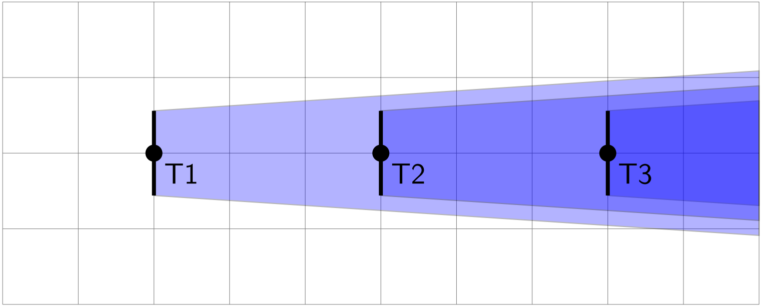
(PW) 39076.32874[MW h] -32748.42776[MW h]

All Turbines Operable



$$
\begin{aligned}
&\text{(WF)}\ 36270.36304[\mathsf{MW\,h}] \qquad -32748.42776[\mathsf{MW\,h}] = 3521.93528[\mathsf{MW\,h}]\\
&\text{(PW)}\ 39076.32874[\mathsf{MW\,h}] \qquad -32748.42776[\mathsf{MW\,h}] = 6327.90098[\mathsf{MW\,h}]
\end{aligned}
$$

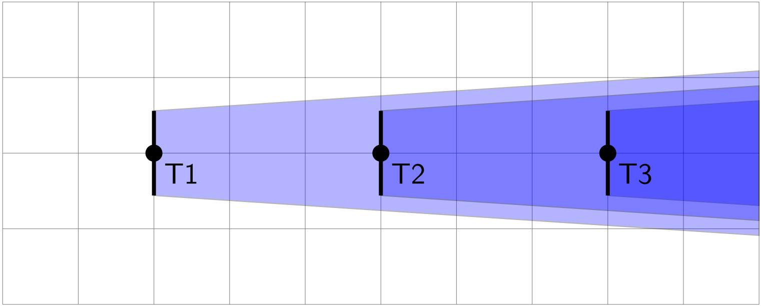
All Turbines Operable



$$(WF)\ 3521.93528[\mathrm{MW\,h}]$$

$$(PW)\ 6327.90098[\mathrm{MW\,h}]$$

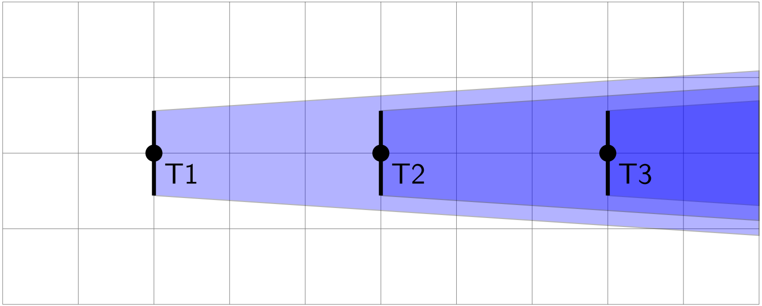
All Turbines Operable



(WF) $3521.93528[\mathsf{MW\,h}]/8760[\mathsf{h}]$

(PW) $6327.90098[\mathsf{MW\,h}]/8760[\mathsf{h}]$

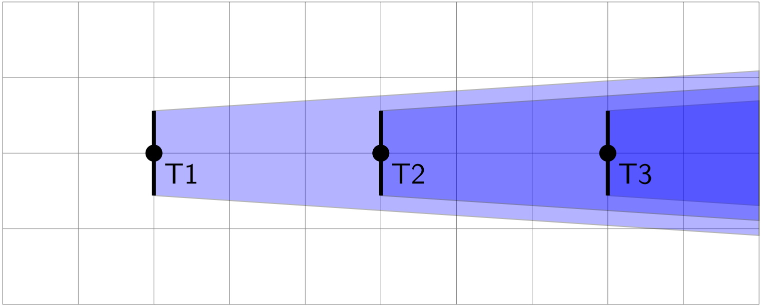
All Turbines Operable



$$\text{(WF) } 3521.93528[\text{MW h}]/8760[\text{h}] \cdot 1000$$

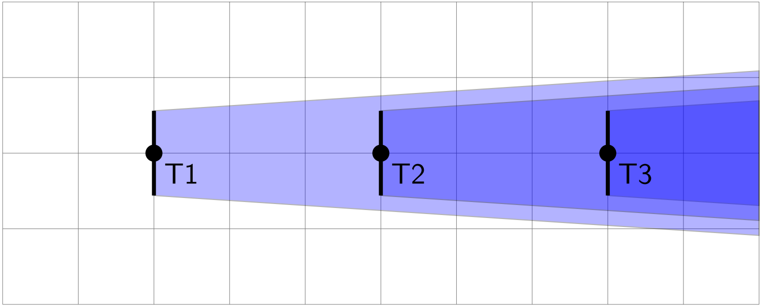
$$\text{(PW) } 6327.90098[\text{MW h}]/8760[\text{h}] \cdot 1000$$

All Turbines Operable



$$\text{(WF) } 3521.93528[\text{MW h}]/8760[\text{h}] \cdot 1000 = 402.04741[\text{kW}]$$
$$\text{(PW) } 6327.90098[\text{MW h}]/8760[\text{h}] \cdot 1000 = 722.36313[\text{kW}]$$

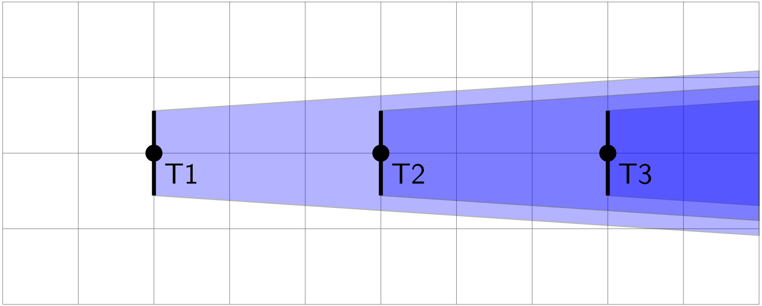
All Turbines Operable



(WF) $402.04741[\text{kW}]$

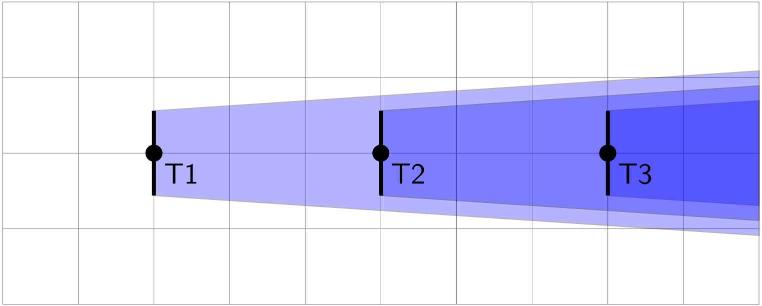
(PW) $722.36313[\text{kW}]$

All Turbines Operable



$$\text{(WF) } 402.04741[\text{kW}] = 309 + (567 - 309) \cdot (v - 5)$$
$$\text{(PW) } 722.36313[\text{kW}] = 567 + (927 - 567) \cdot (v - 6)$$

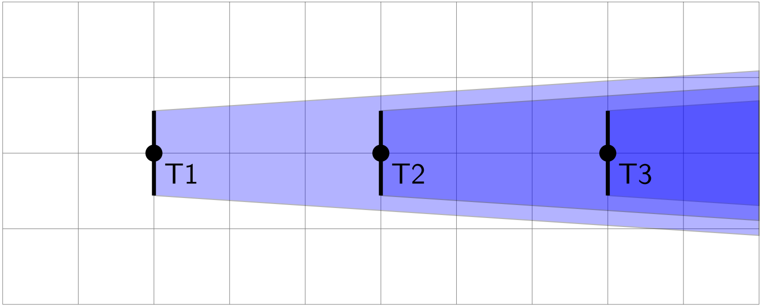
All Turbines Operable



$$\text{(WF)}\ 402.04741[\text{kW}] = 309 + (567 - 309) \cdot (v - 5) \Rightarrow v = 5.36065\,\text{m}\,\text{s}^{-1}$$

$$\text{(PW)}\ 722.36313[\text{kW}] = 567 + (927 - 567) \cdot (v - 6) \Rightarrow v = 6.43156\,\text{m}\,\text{s}^{-1}$$

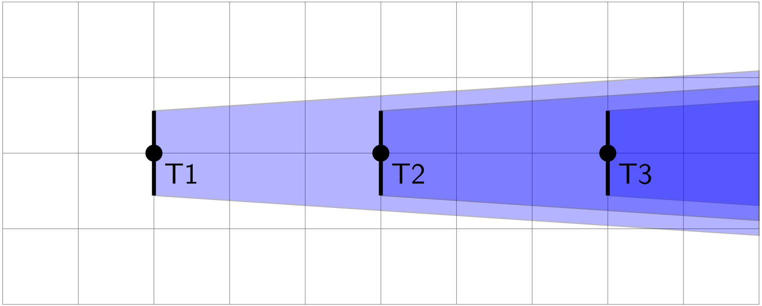
All Turbines Operable



(WF) $5.36065 \,\mathrm{m\,s^{-1}}$

(PW) $6.43156 \,\mathrm{m\,s^{-1}}$

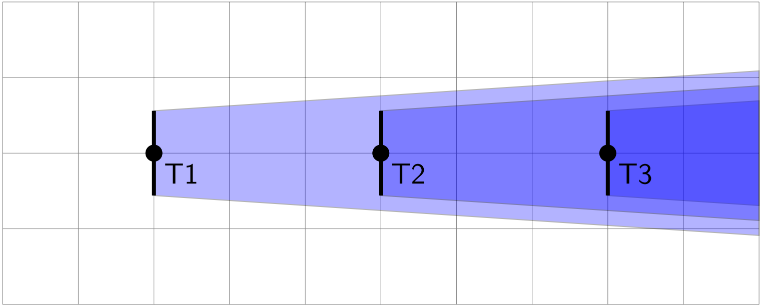
All Turbines Operable



$$\text{(WF)}\ 5.36065\,\mathrm{m\,s^{-1}} = 10\,\mathrm{m\,s^{-1}} \cdot (1-\delta)$$

$$\text{(PW)}\ 6.43156\,\mathrm{m\,s^{-1}} = 10\,\mathrm{m\,s^{-1}} \cdot (1-\delta)$$

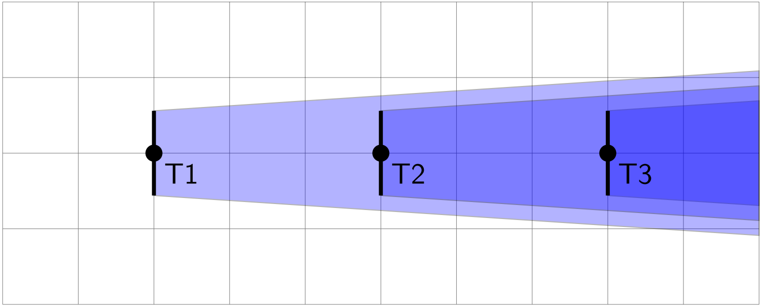
All Turbines Operable



$$\text{(WF) } 5.36065\,\mathrm{m\,s^{-1}} = 10\,\mathrm{m\,s^{-1}} \cdot (1-\delta) \Rightarrow \delta = 0.463935$$

$$\text{(PW) } 6.43156\,\mathrm{m\,s^{-1}} = 10\,\mathrm{m\,s^{-1}} \cdot (1-\delta) \Rightarrow \delta = 0.356844$$

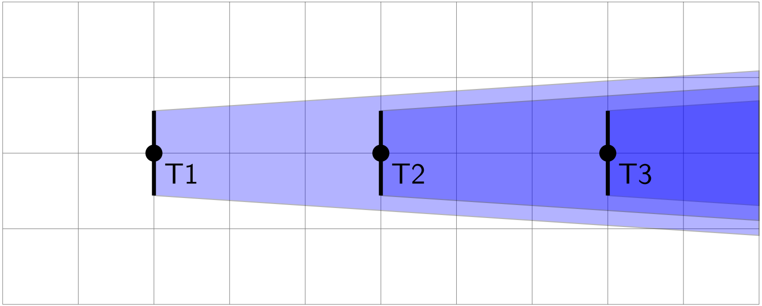
All Turbines Operable



(WF) 0.463935

(PW) 0.356844

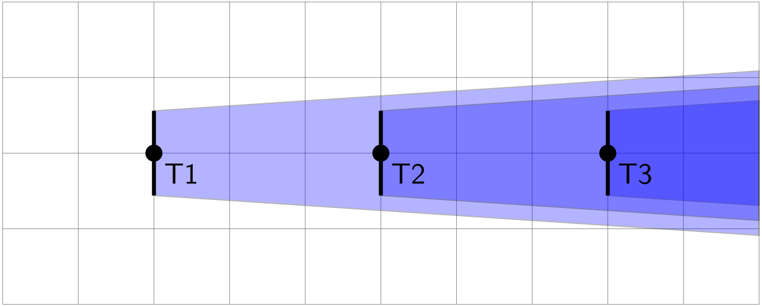
All Turbines Operable



$$\text{(WF) } 0.463935 = \sqrt{0.173072^2 + \delta^2}$$

$$\text{(PW) } 0.356844 = \sqrt{0.173072^2 + \delta^2}$$

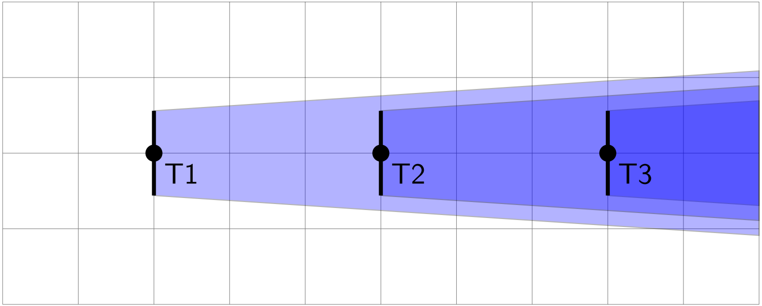
All Turbines Operable



$$\text{(WF) } 0.463935 = \sqrt{0.173072^2 + \delta^2} \Rightarrow \delta = 0.430444$$

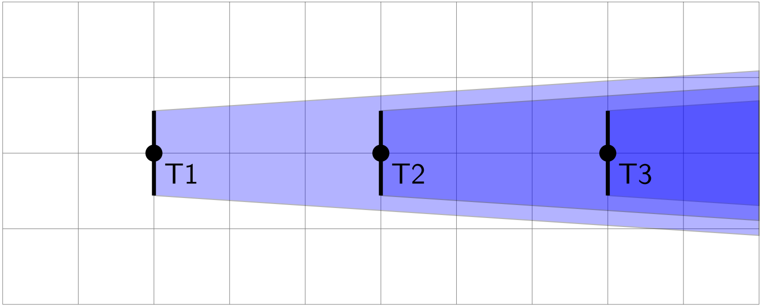
$$\text{(PW) } 0.356844 = \sqrt{0.173072^2 + \delta^2} \Rightarrow \delta = 0.312064$$

All Turbines Operable



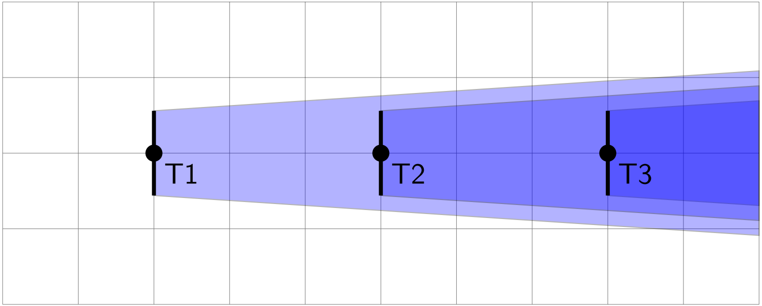
$$\frac{0.430444}{0.312064}$$

All Turbines Operable



$$\frac{0.430444}{0.312064} \approx 1.376872$$

All Turbines Operable



$$\frac{0.430444}{0.312064} \approx 1.376872 \approx \frac{10}{7.26284}$$