Modeling and Analysis of Hybrid Systems - SS 2013

Written Exam II

Wednesday, September 18, 2013

Forename and surname:	**Matriculation number:**
Sign here:	

- Do not open the exam until we give the start signal.
- Please place your student identity card on your desk for identification purposes.
- The duration of the exam is 120 minutes.
- Use a blue or black (permanent) pen only.
- Please write your name and matriculation number on each page of this exam.
- Please write clear and legible answers.
- Please use a separate sheet for each task. If you need more sheets, indicate this by a hand signal.
- Please clearly cross out parts you do *not* wish to be evaluated.
- If you have problems understanding a task, indicate this by a hand signal.
- You are not allowed to use auxiliary material except for a pen. In particular, switch off your electronic devices! Cheating disqualifies from the exam.

Task:	1.)	2.)	3.)	4.)	5.)	6.)	**Total**
Maximum score:	7	11	7	8	9	8	50
Reached score:							

Good luck!

Task 1. Hybrid System Modeling

(3 + 2 + 2 points)

Figure 1 depicts a *water tank*, where x denotes the height of the water in the tank. The water flows out of the tank through an aperture at the bottom. Additionally, the tank can be filled with water by turning a pump on. If the pump is off (and the tank is not empty), the water height change rate is $\frac{dx}{dt} = -0.5$. Otherwise, if the pump is on, the water height changes according to $\frac{dx}{dt} = 1$.

A controller tries to keep the water level in a safe range by turning the pump on when the water height reaches 3 and turning the pump off when the water height reaches 7.

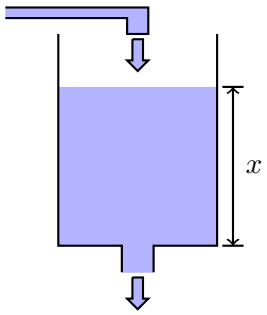
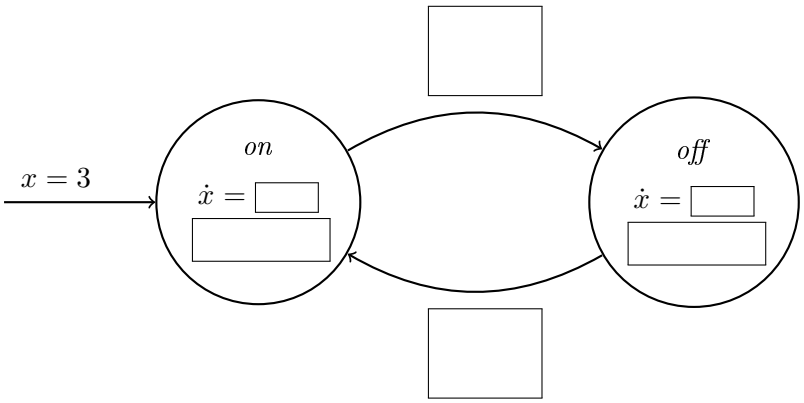


Figure 1: Water tank

(1) Please define the missing components of the following hybrid automaton to *model* the water tank system:



(2) Please give a definition of *Zeno paths*.

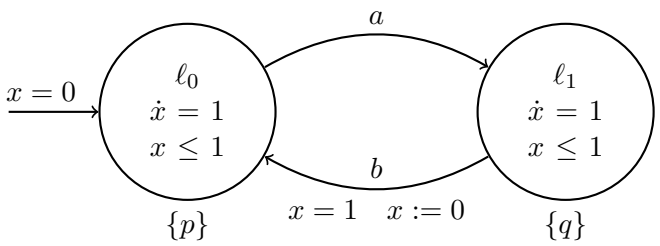
(3) Does the above hybrid automaton *has* a Zeno path? Justify your answer!

Task 2. Timed Automata

(3 + 6 + 2 points)

(1) Please describe the basic steps of *TCTL model checking* on timed automata.

(2) Consider the following timed automaton $\mathcal{T}$:



We want to check whether $\mathcal{T}$ satisfies the TCTL formula $AGAF^{\leq 1}\, p$. Please give the reachable fragment of the *region transition system* generated during model checking.

(3) The TCTL model checking algorithm can be applied to *Zeno-free* timed automata only. Why?

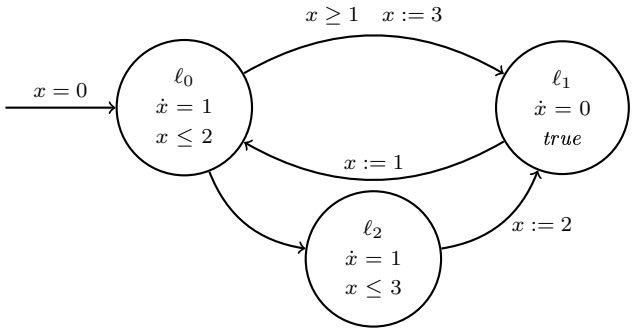
Task 3. Rectangular Automata

(2 + 2 + 3 points)

(1) Please explain the differences between *rectangular automata* and *stopwatch automata*.

(2) Is the reachability problem on *initialized rectangular automata* decidable? Describe the structure of the proof!

(3) Please *transform* the following initialized stopwatch automaton to a timed automaton. Here, we allow non-zero reset values in the resulting timed automaton.



Task 4. Linear Hybrid Automata (2 + 4 + 2 points)

(1) Please explain the differences between *linear hybrid automata* and *general hybrid automata*.

(2) For the state set defined by $S = \langle \ell, y \leq x \wedge 1 \leq x \rangle$, please compute (a) its *forward time closure* $\mathcal{T}_l^+(S)$ under the dynamics $\dot{x} = 2, \dot{y} = -1$ with invariant $x \leq 2$, and (b) its *postcondition* $\mathcal{D}^+_{(\ell,\ell')}(S)$ via a jump to ℓ' with guard $x \leq 2$ and reset $x := 0$.

(3) Is the reachability problem on linear hybrid automata *decidable*? Justify your answer!

Task 5. Reachability Analysis

(1 + 2 + 2 + 4 points)

(1) We mentioned different kinds of *geometric representations* for state sets. Please name at least two.

(2) We want to implement a fixedpoint-based reachability analysis algorithm using a given state set representation. Which *operations* are needed on the state set representation? Please mention at least four.

(3) What is a *flow pipe segmentation*?

(4) Please explain the basic principle of *minimization*.

Task 6. Convex Polytopes

(2 + 2 + 2 + 2 points)

(1) Please define what it means that a set is *convex*.

(2) Please define the *convex hull* of a set. How can the convex hull be computed for a finite set $V = \{v_1, v_2, \ldots, v_n\}$?

(3) How can we check whether a point x_0 belongs to a $\mathcal{V}$*-polytope* P?

(4) How can we check whether a point x_0 belongs to an $\mathcal{H}$*-polytope* P?