Model-Based Optimization of the Building Envelope for Energy Efficiency

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Aachen, 15.10.2025

hybr2d Theory of Hybrid Systems Informatik 2 | RWTH AACHEN UNIVERSITY

Motivation for Algorithmic Optimization

Example

- Family in an old building with high energy demand.

  $\Rightarrow$ Lower energy demand by 20%.
- Construction measures: new windows, wall insulation, and more
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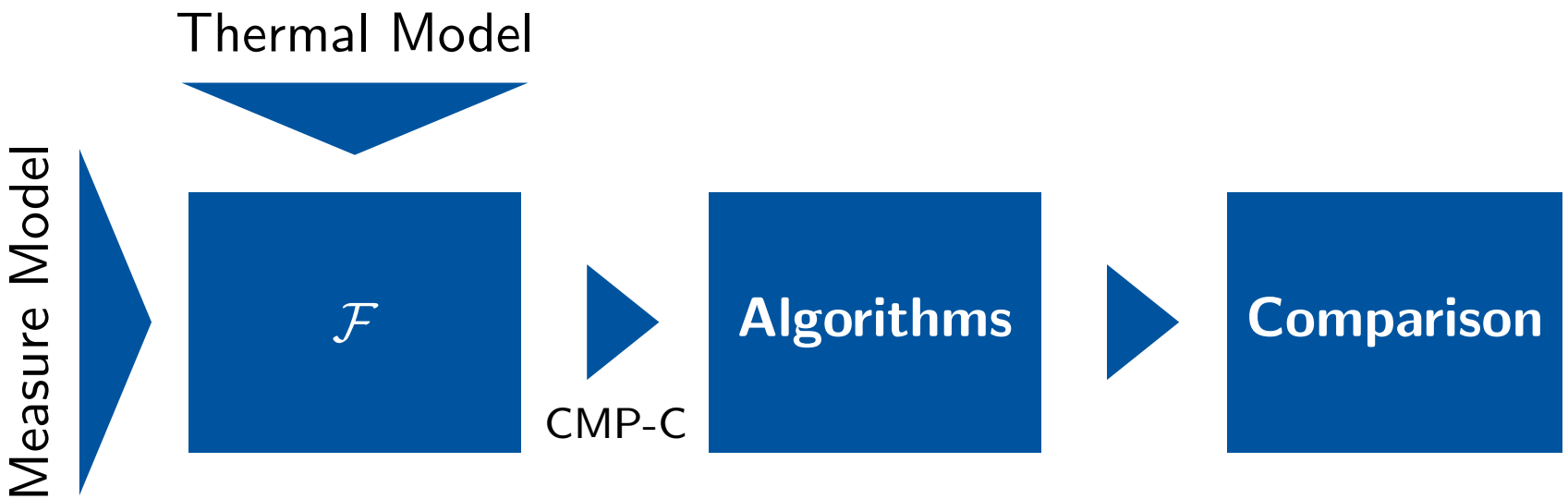
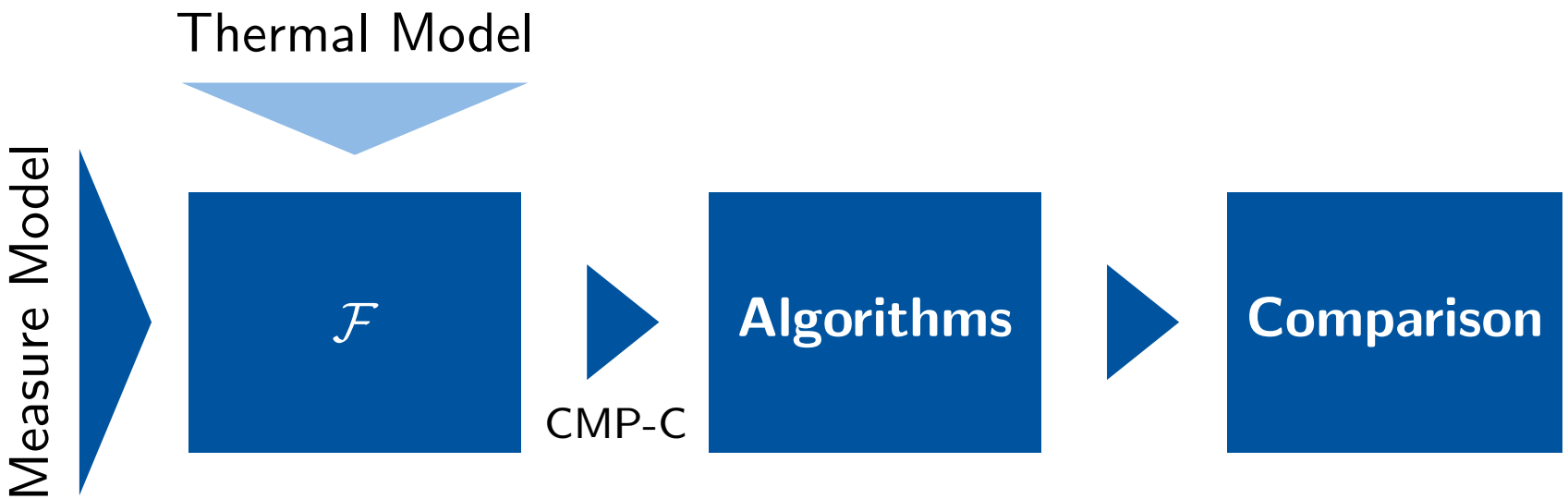
Outline


Thermal Model
Measure Model
$\mathcal{F}$
CMP-C
Algorithms
Comparison

Thermal Model



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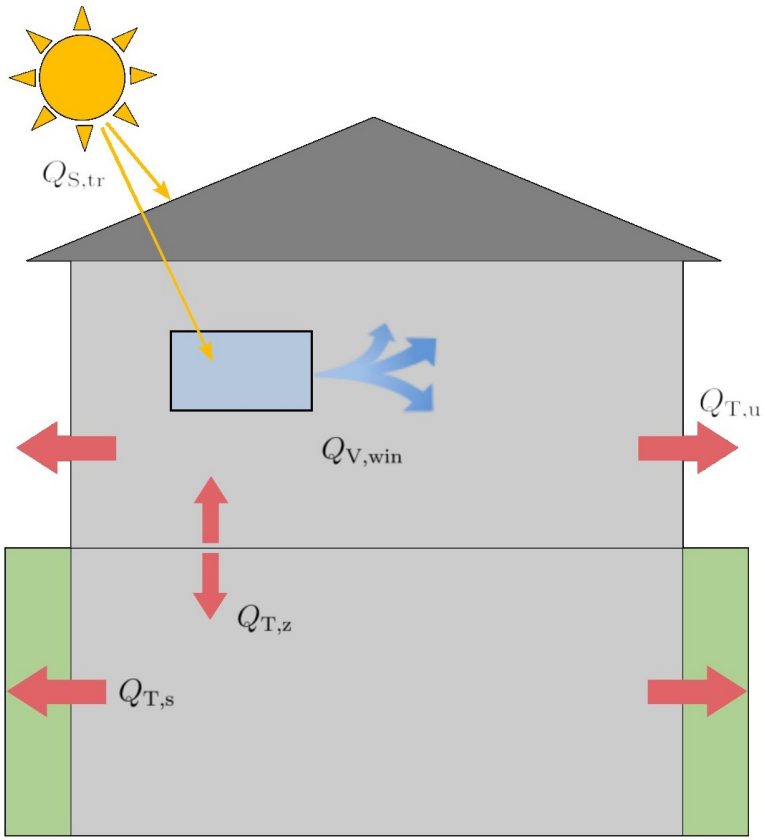
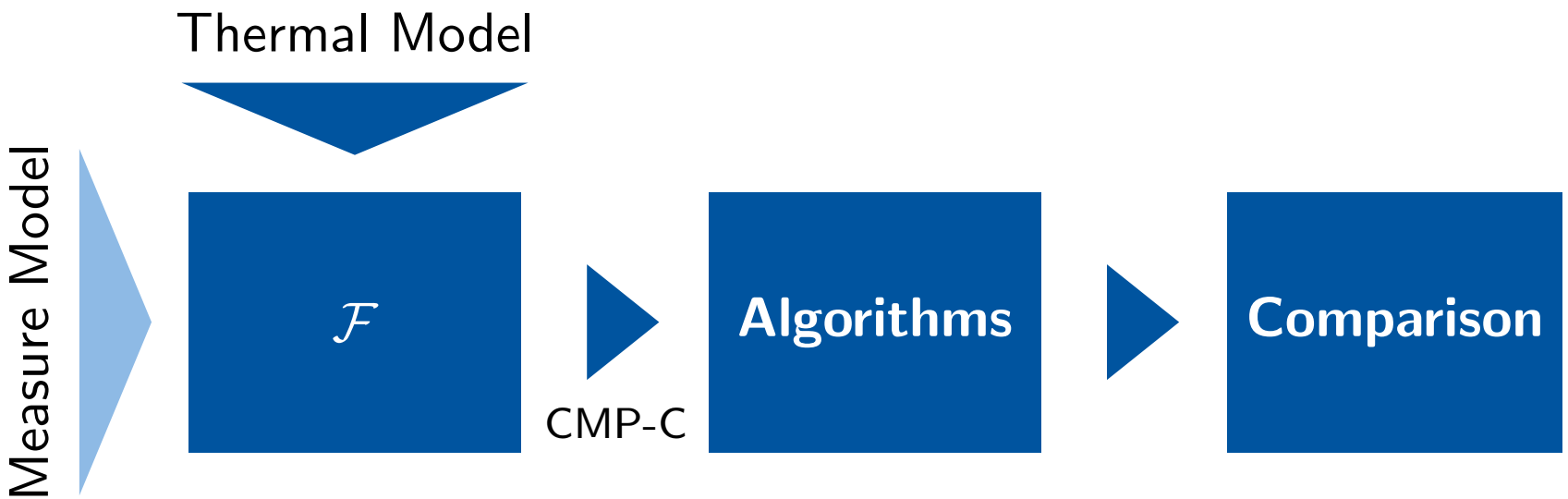


- **Energy flows in buildings:**
  - *Heat losses* via the envelope (transmission) and ventilation (Q_T, Q_V)
  - *Heat gains* from solar radiation and internal sources (Q_S)
- **Primary Energy Demand:**
  - $Q_p = Q_{p,in} - Q_{p,out}$
  - Accounts for generation and transport losses
- **Model input:**
  - 10 specific *test cases*

Measure Model



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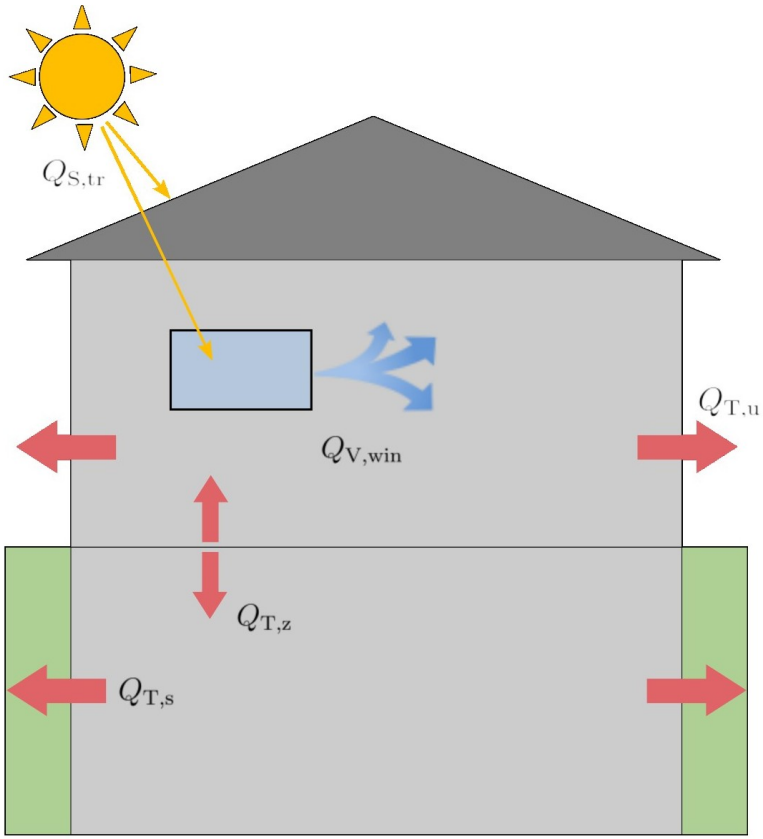

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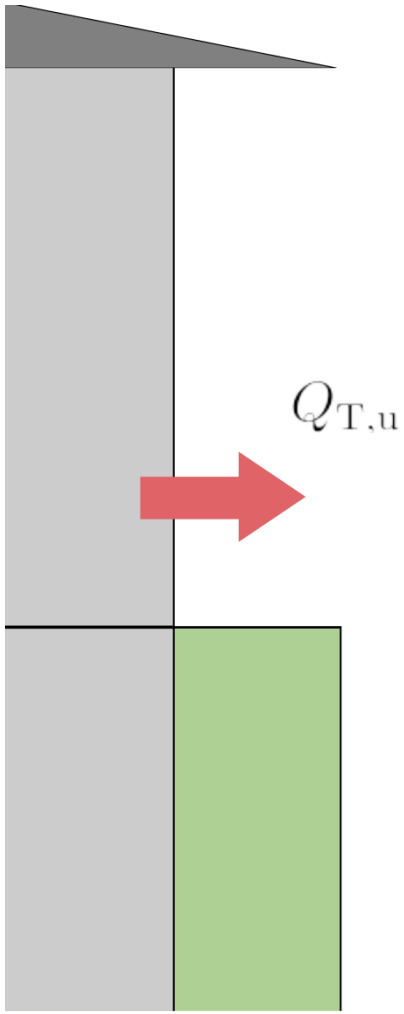

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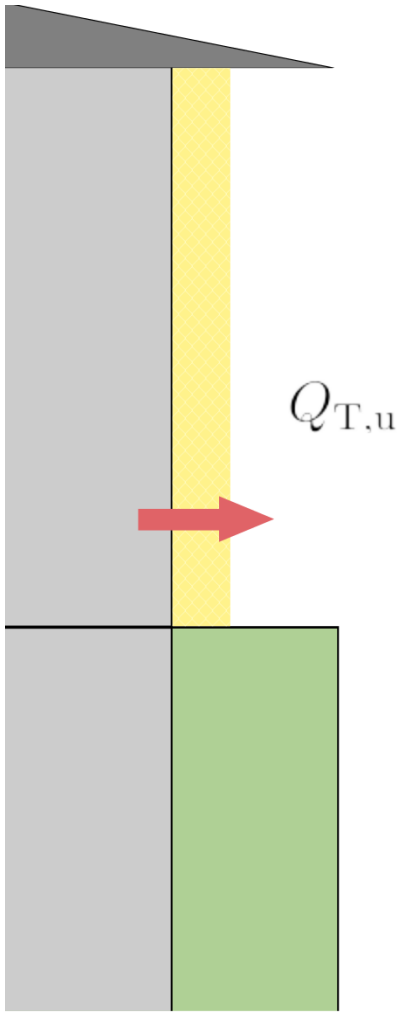

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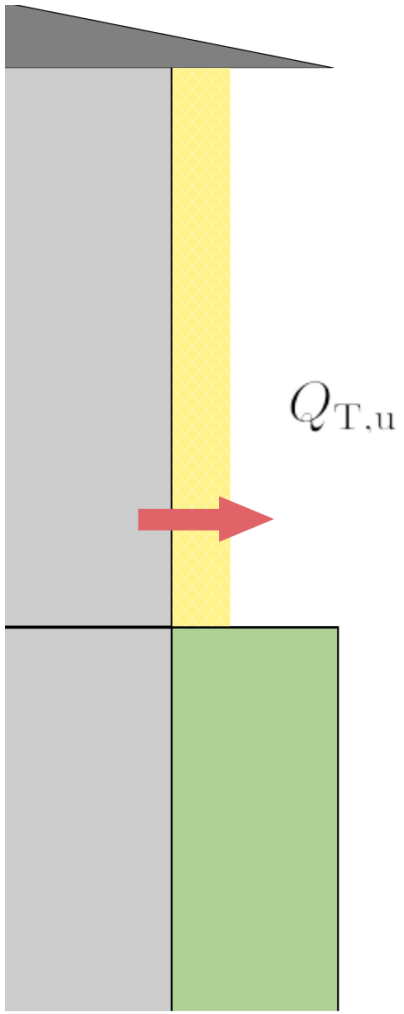

- **Model Properties:**
  - Cost: material & procedure dependent
  - Efficiency improvement: component-specific
  - Conflicts: exclude simultaneous measures

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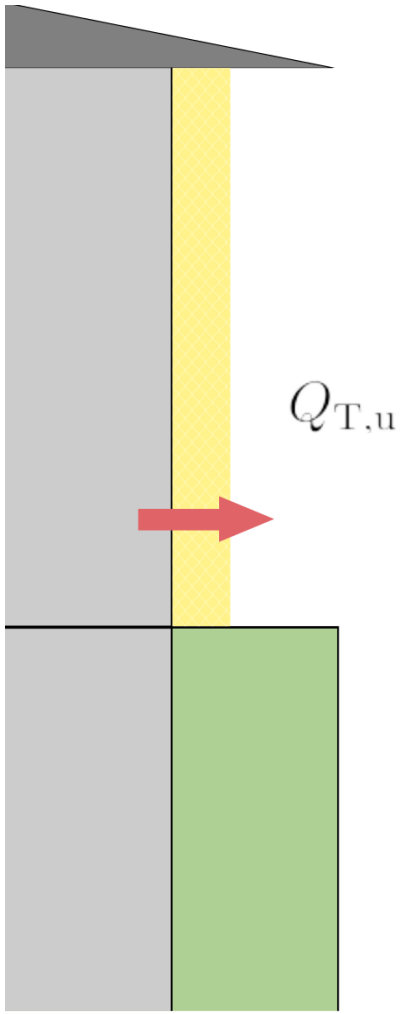
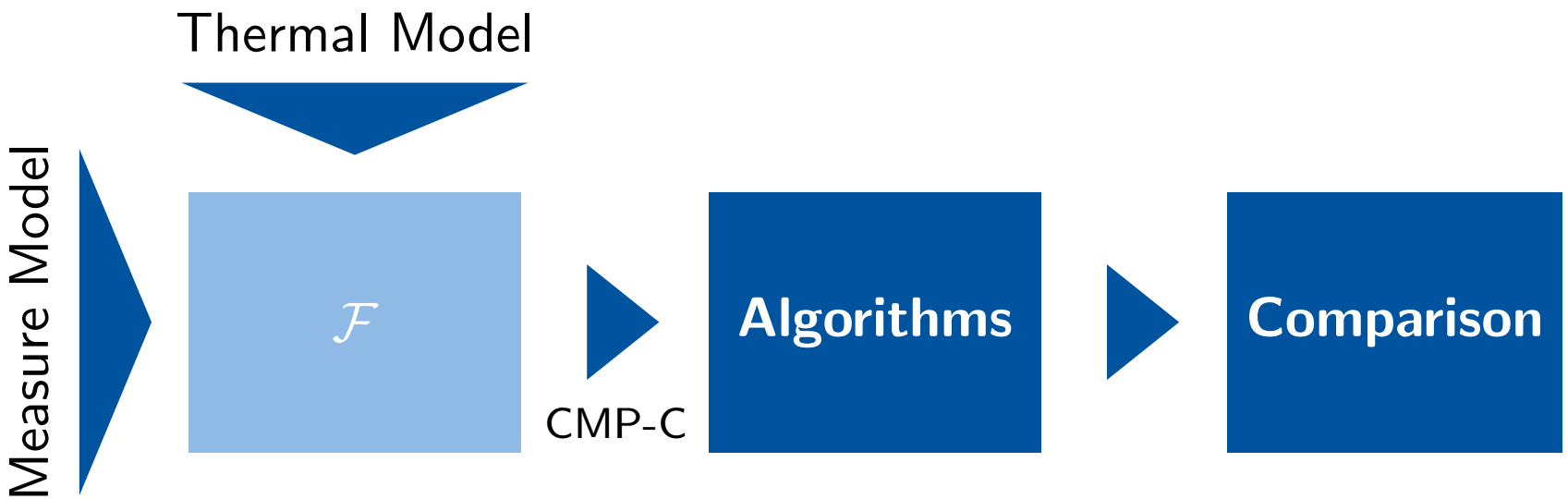
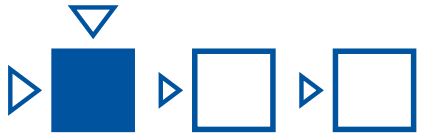


- **Model Properties:**
  - Cost: material & procedure dependent
  - Efficiency improvement: component-specific
  - Conflicts: exclude simultaneous measures
- **Measure Catalogue $\mathcal{M}$:**
  - Components: walls, roofs, windows, doors, base plates
  - Variants per component: procedures, materials, thicknesses

Bridging Thermal and Measure Model



Definition and Role of $\mathcal{F}$



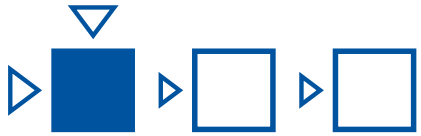
Definition

$\mathcal{F} : \mathcal{B} \times 2^{\mathcal{M}} \to \mathbb{R}$

For a building configuration $B \in \mathcal{B}$ and a set of measures $M \subseteq \mathcal{M}$, the function returns the resulting **primary energy demand** of the renovated building.

- $\mathcal{B}$ – all possible thermal model configurations (buildings).
- $\mathcal{M}$ – catalogue of all available construction measures.

Examination of Additivity



Definition of Additivity

$$f(u_1 + u_2) = f(u_1) + f(u_2)$$

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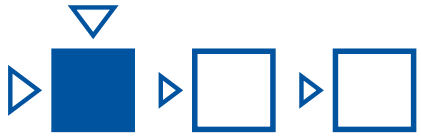

Reformulation of Additivity

$$\triangle\mathcal{F}(B, X \cup Y) = \triangle\mathcal{F}(B, X) + \triangle\mathcal{F}(B, Y)$$

The joint benefit of measure sets equals the sum of their separate benefits.

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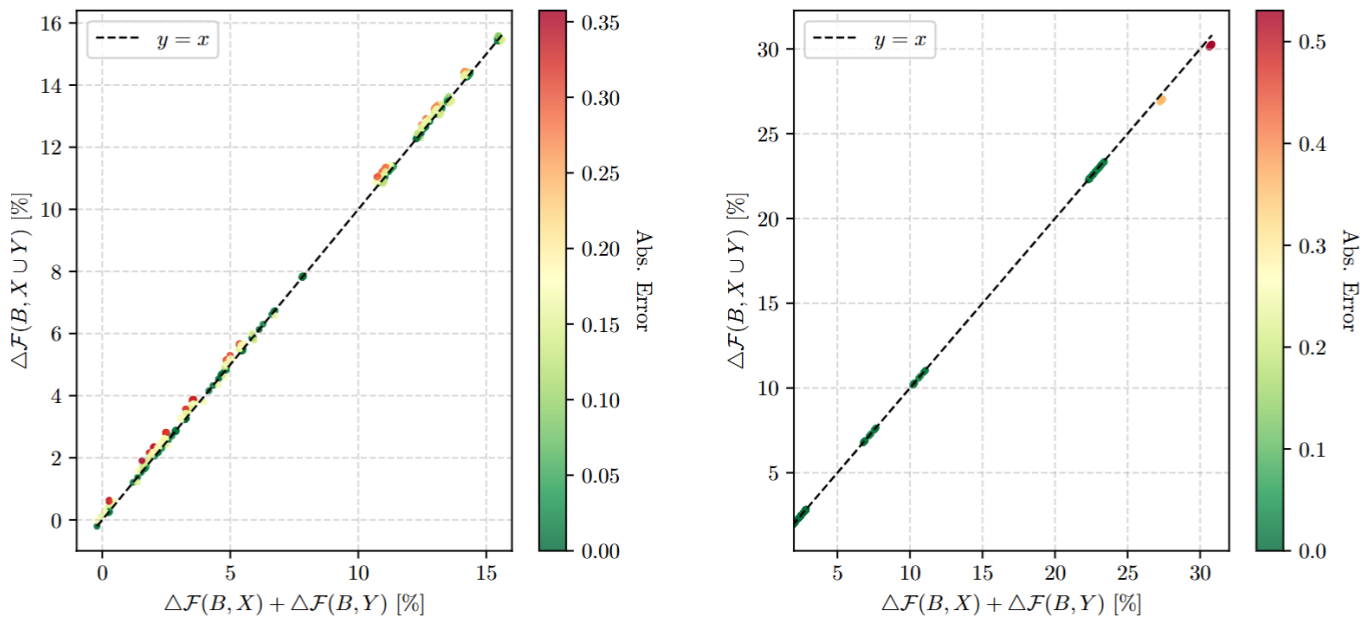

Figure: Comparison of combined vs. summed benefits for SFB 11 and AB 7.

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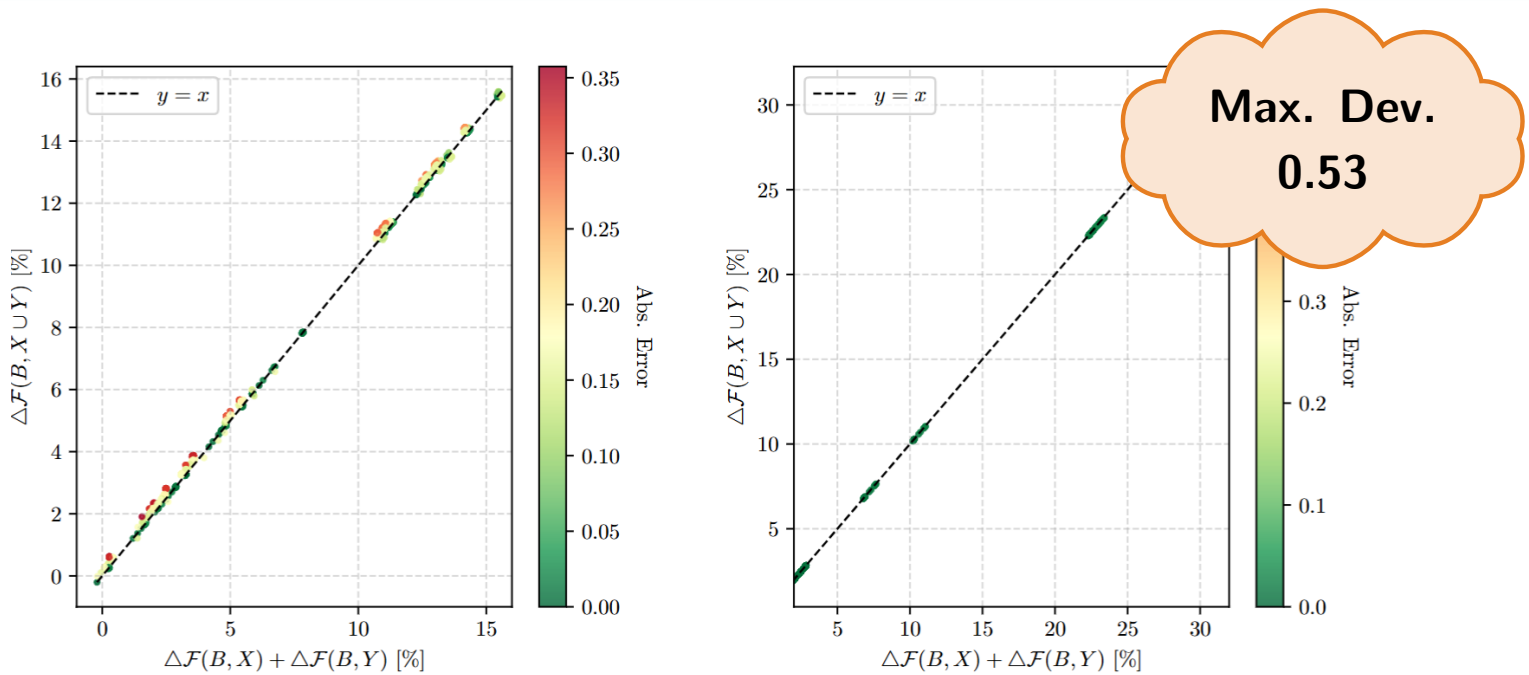

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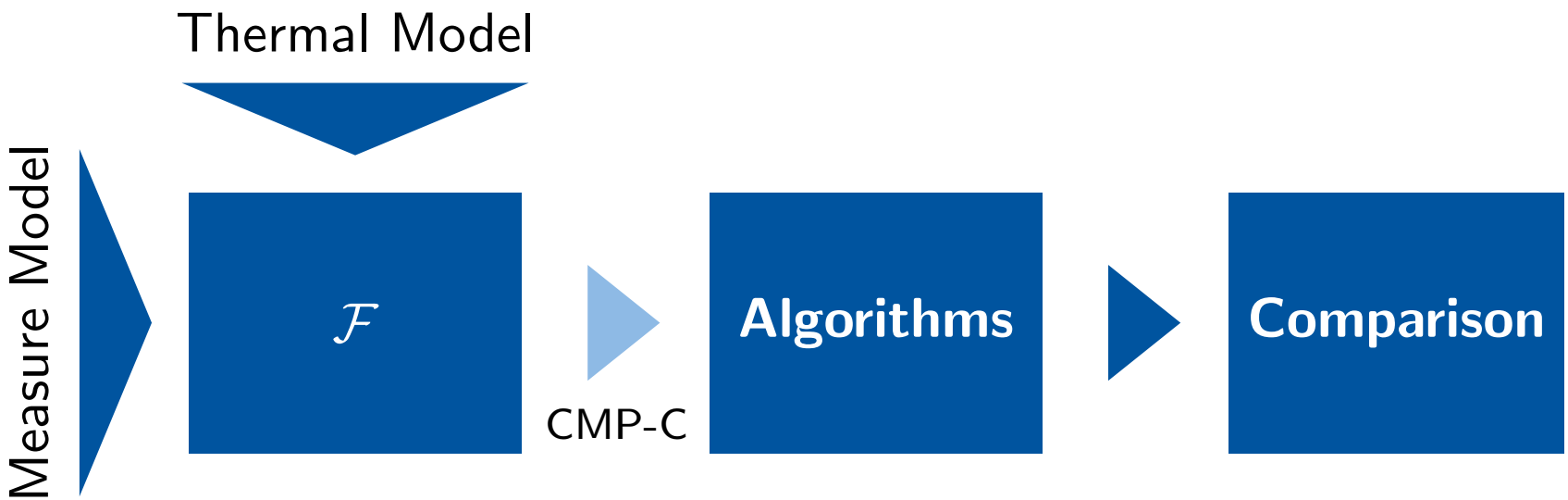
CMP-C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Cost Minimization Problem with Conflicts

Which simultaneously applicable measures should the family choose to reach the goal while keeping costs low?

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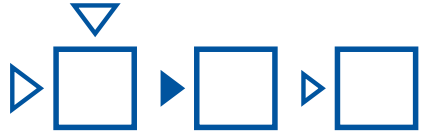

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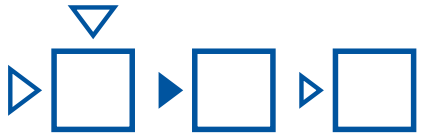

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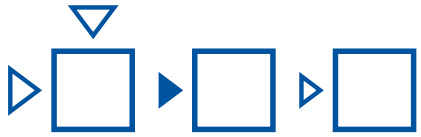
(CMP-C)

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{M}$$

Variables: x_j binary decision (select j or not)

Cost Minimization Problem with Conflicts



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$$\text{(CMP-C)}\ \min \quad \sum_{j \in \mathcal{M}} c_j x_j$$

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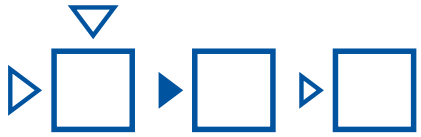
Objective: Minimize total cost

Cost Minimization Problem with Conflicts



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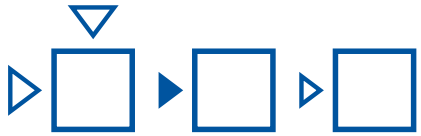
Constraint: Achieve required target value b

Cost Minimization Problem with Conflicts



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\end{aligned}
$$

Constraint: Avoid conflicting measures

Cost Minimization Problem with Conflicts

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CMP-C is NP-hard

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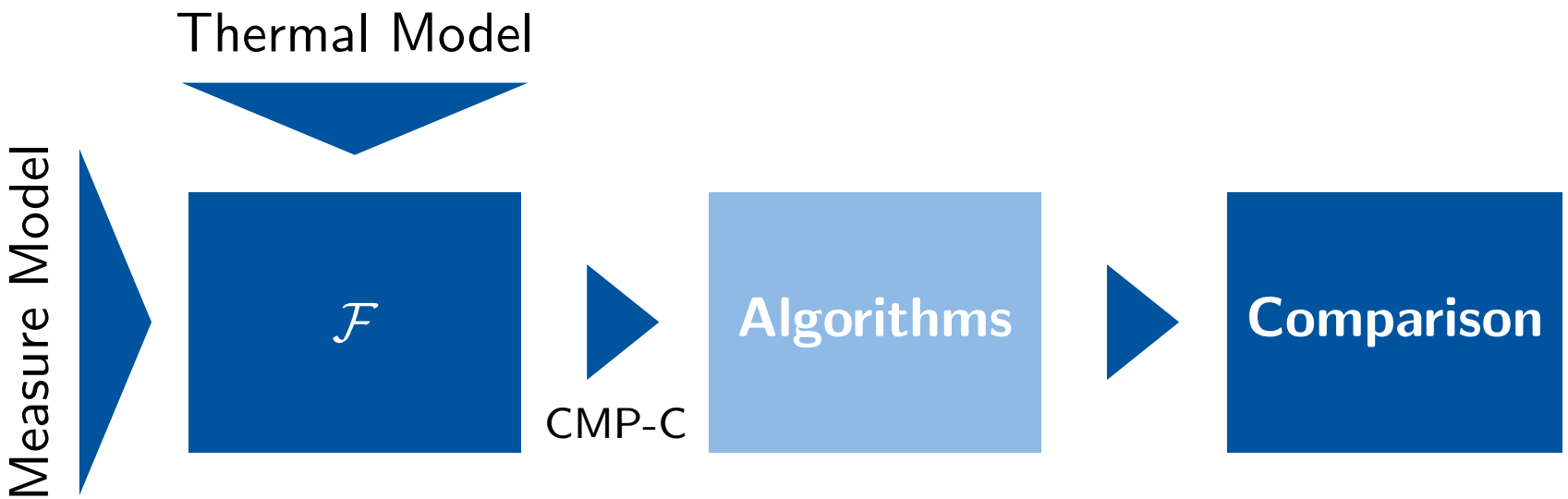
Algortihms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Branch-and-Bound

Core Components:

- **Search Strategy:** Depth-first search, left branch first $\rightarrow$ faster valid solutions
- **Branching Strategy:** Binary branching (apply / skip measure)
- **Pruning Rules:**
  - Conflict-based (feasibility): remove conflicting combinations
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Embedding Process:

- Additive relaxation of CMP-C
- Validate solution in the original CMP-C context

B&B with Validation-Driven Branching

Problem of Standard B&B:

- Assumes full additivity of $\mathcal{F}$
- In reality: $\mathcal{F}$ only partially additive
- Validation only after traversal $\rightarrow$ invalid results possible

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Modification: Validation-Driven Branching

- Use real model $\mathcal{F}$ selectively during branching
- Not only sum benefits — evaluate $\mathcal{F}$ for critical branches
- A branch is considered critical if:
  - Pair deviation $\geq 1\%$
  - Combination size > 5
- Each provisional solution is validated with $\mathcal{F}$

B&B with Iterative Evaluation

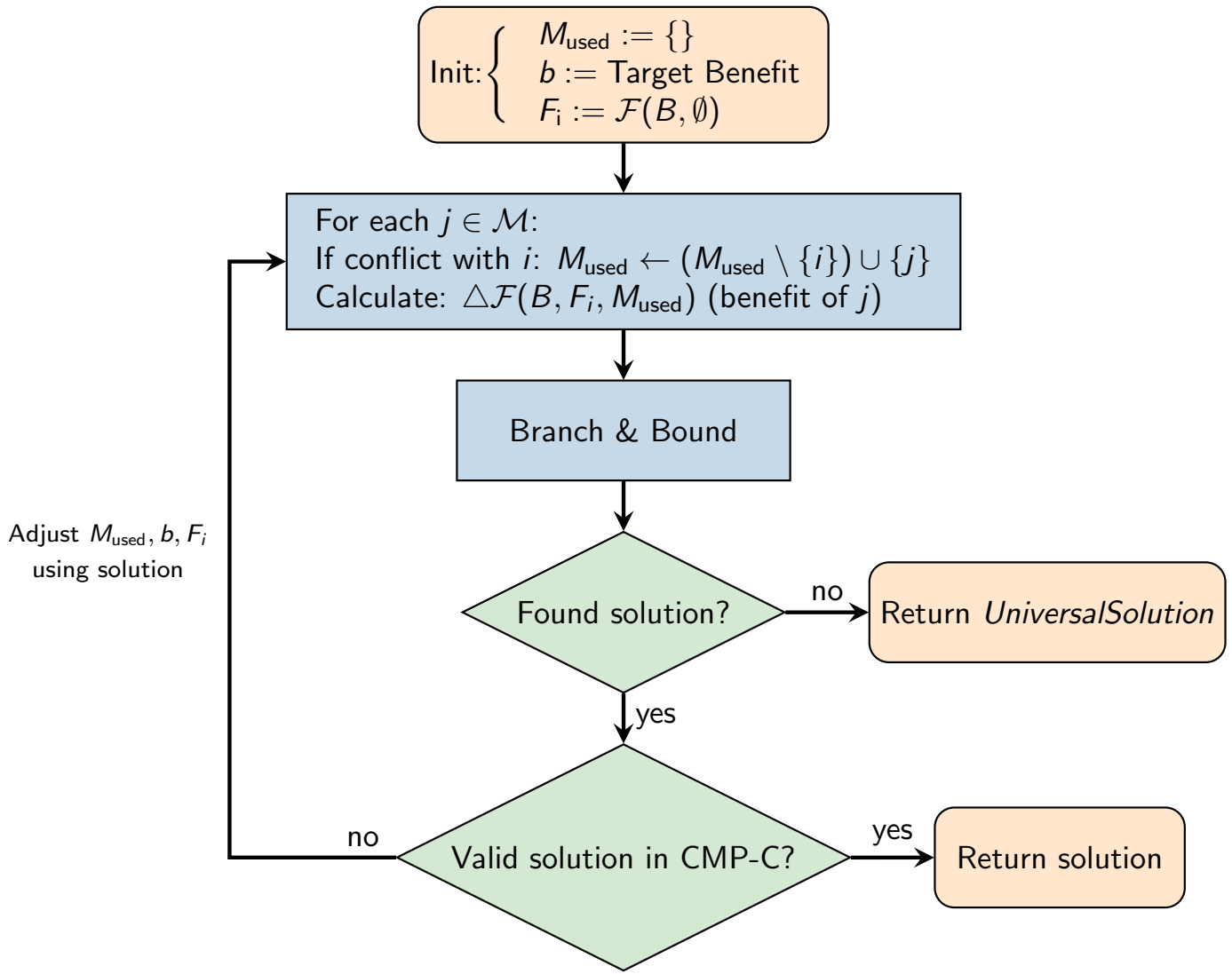
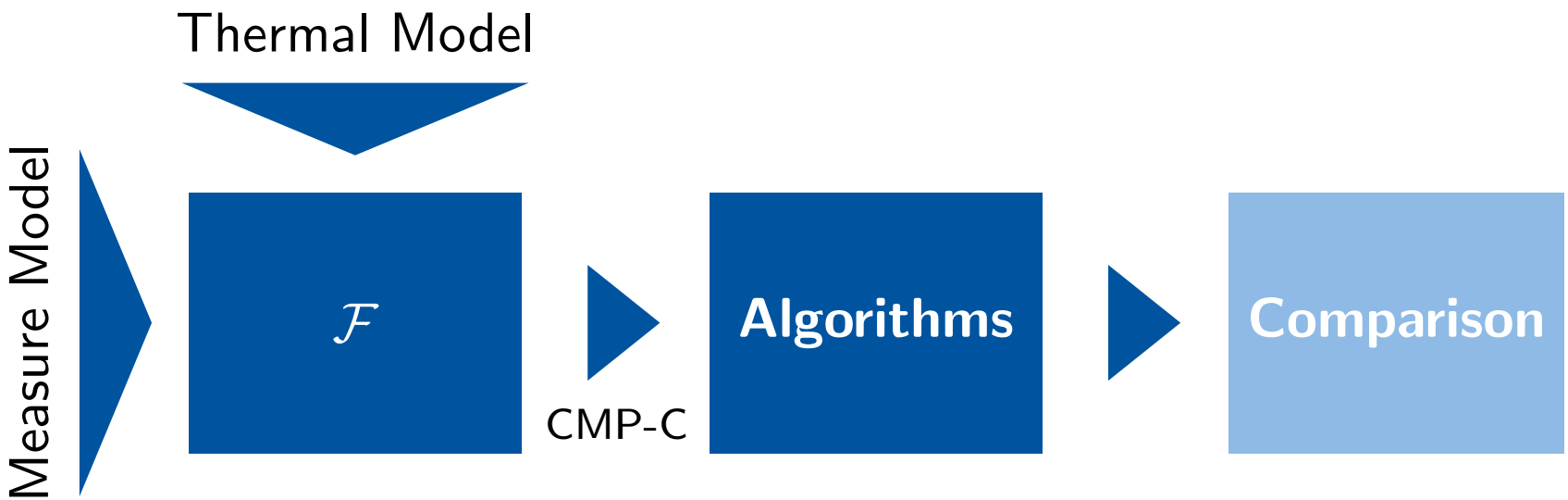


Figure: Iterative Evaluation B&B flowchart.

Comparison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Establishing a Comparison Basis

CMP-C Instances
defined by $(B_i, b_j, \mathcal{M})$

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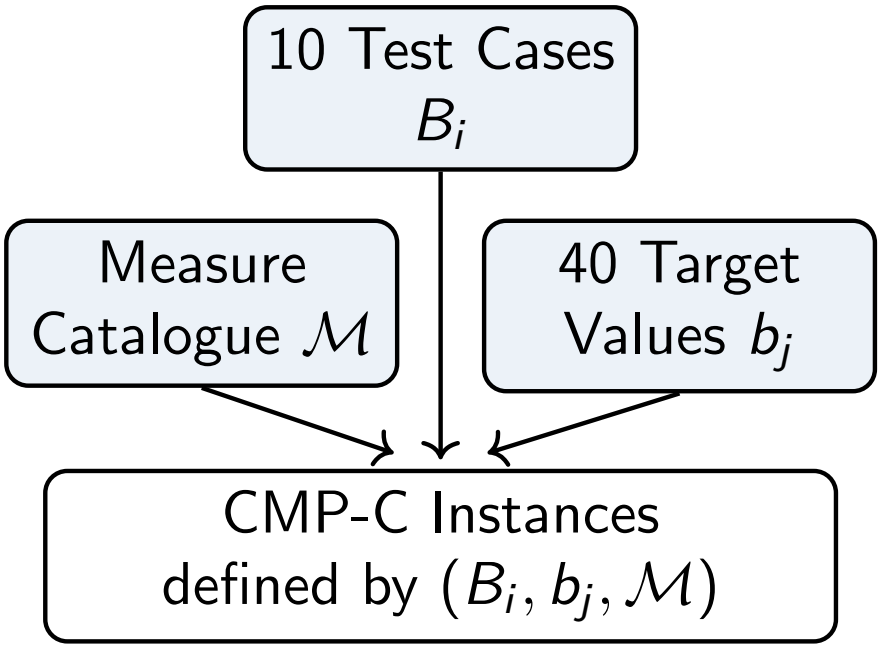

- 10 test cases
- $\mathcal{M}$ with 31 measures
- b_j sampled from optimizing interval

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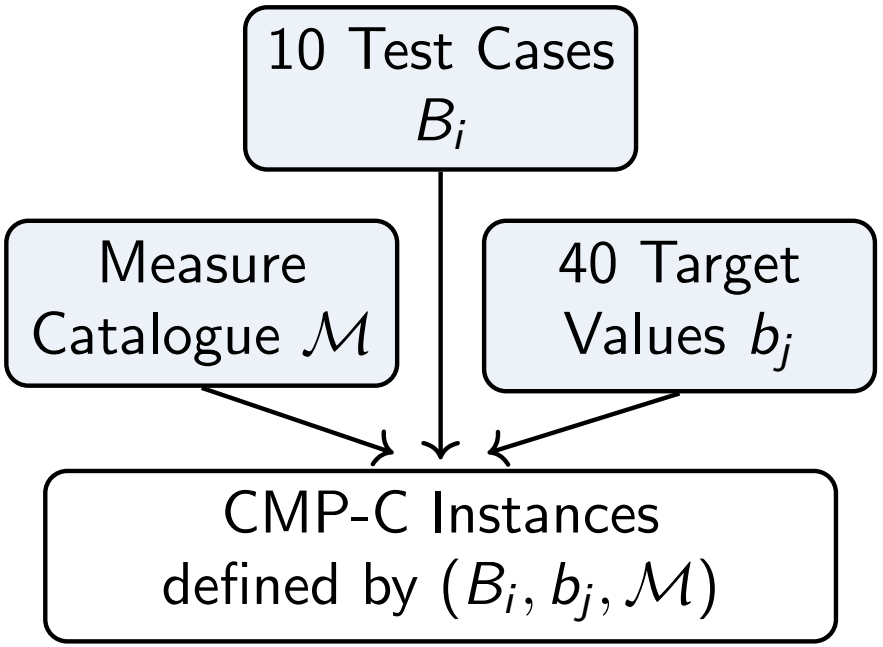

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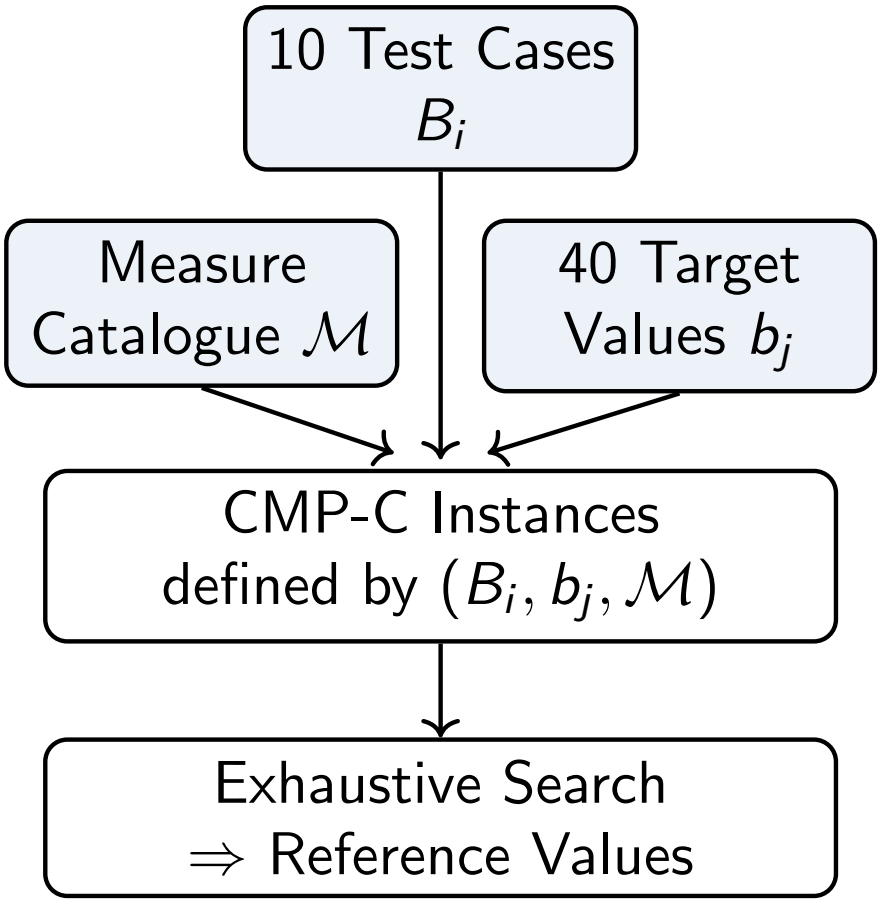

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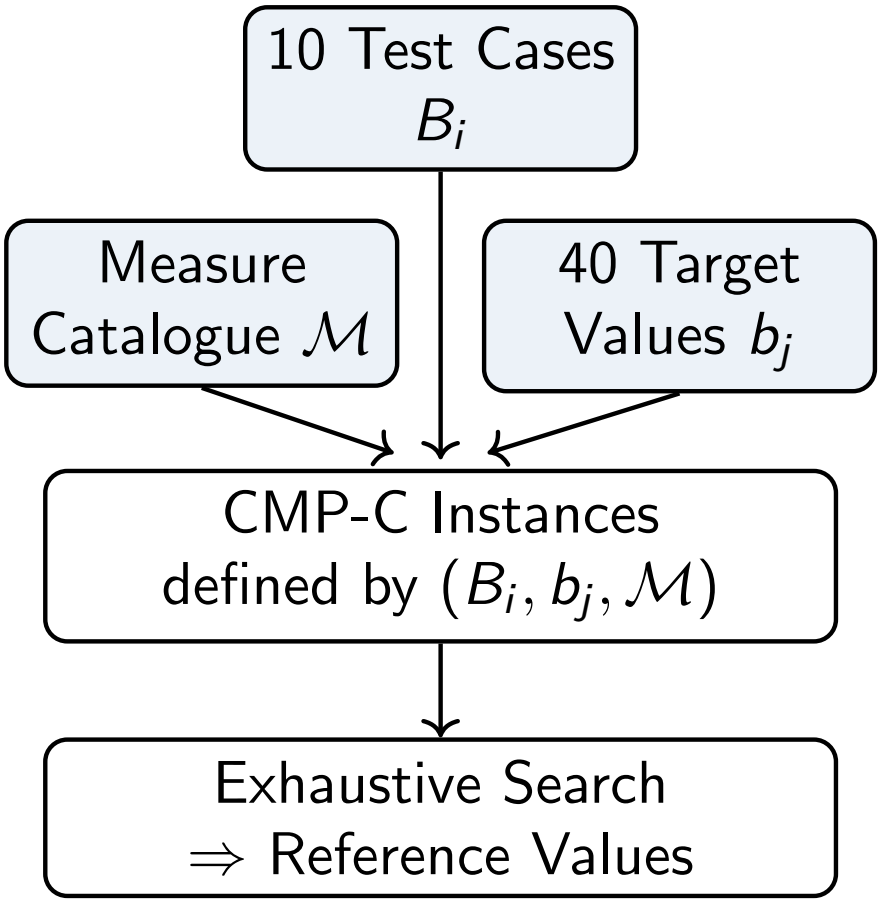

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$\Rightarrow$ Benchmark set with known optima for evaluating and comparing algorithm performance

Number of Model Evaluations

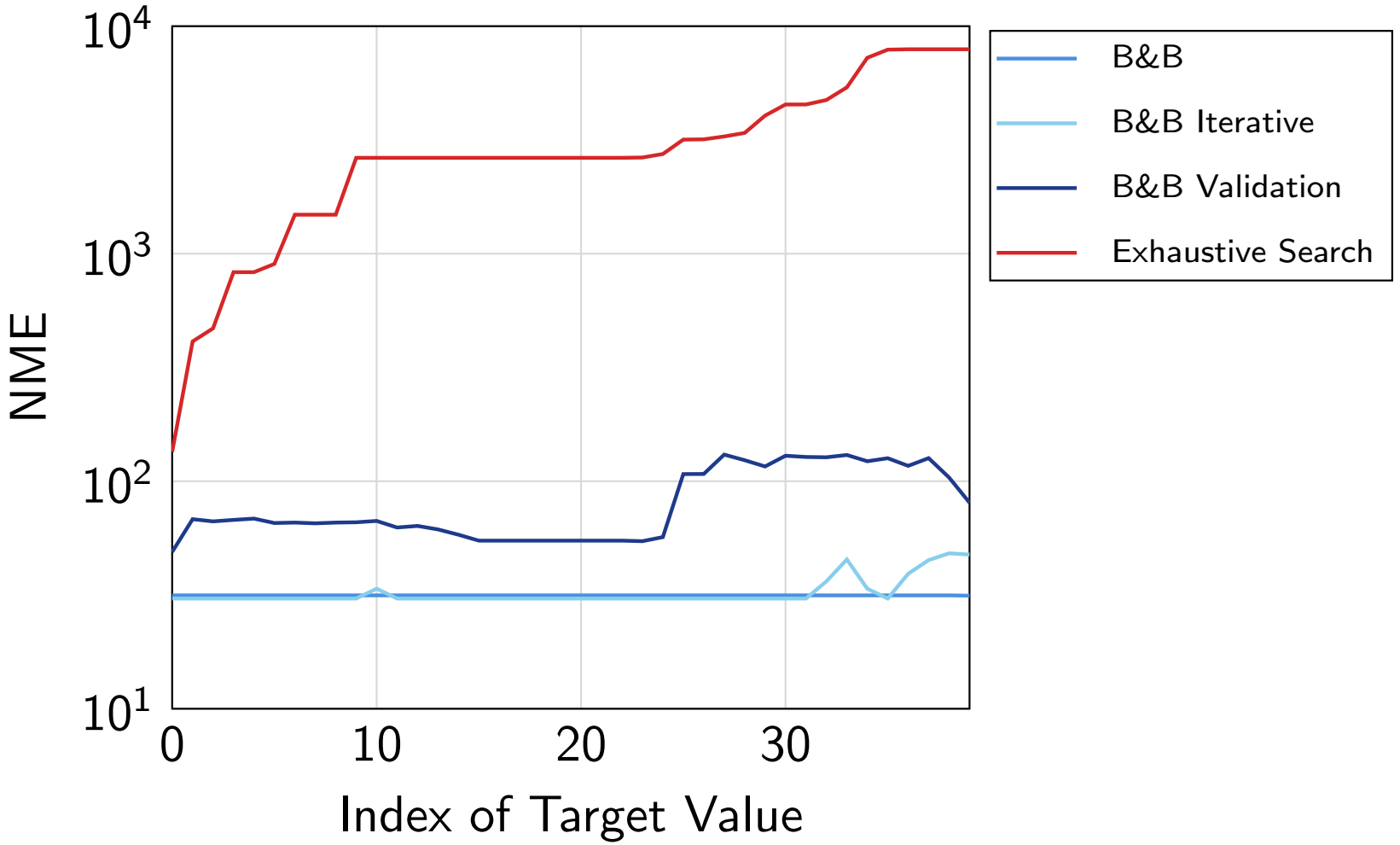

Figure: Mean of number of model evaluations for each index for each algorithm on each test case on a logarithmic scale.

Solution Quality

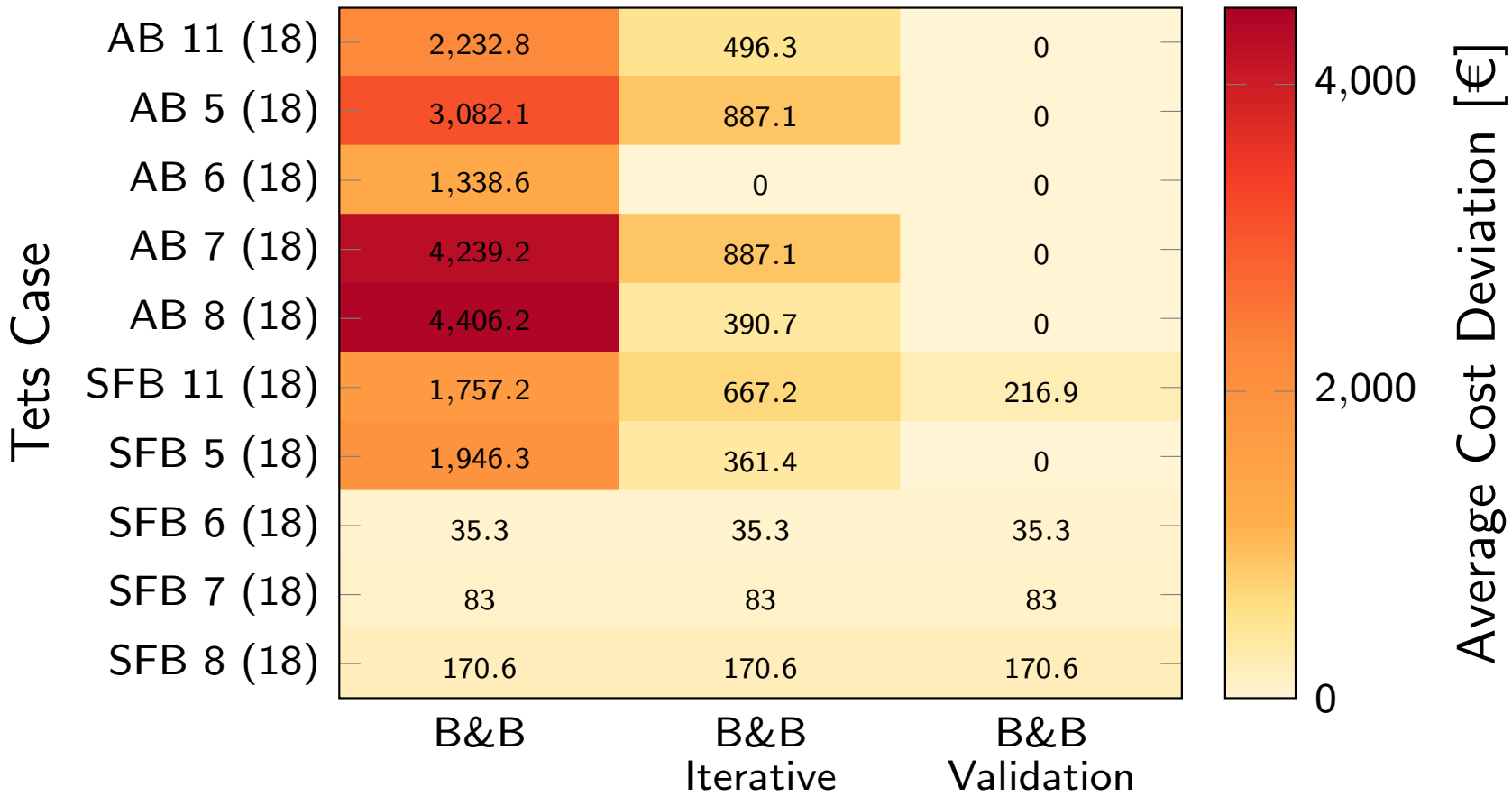

Figure: The average cost deviation from the reference optima, generated with exhaustive search, per test case.

Runtime-Quality Trade-off

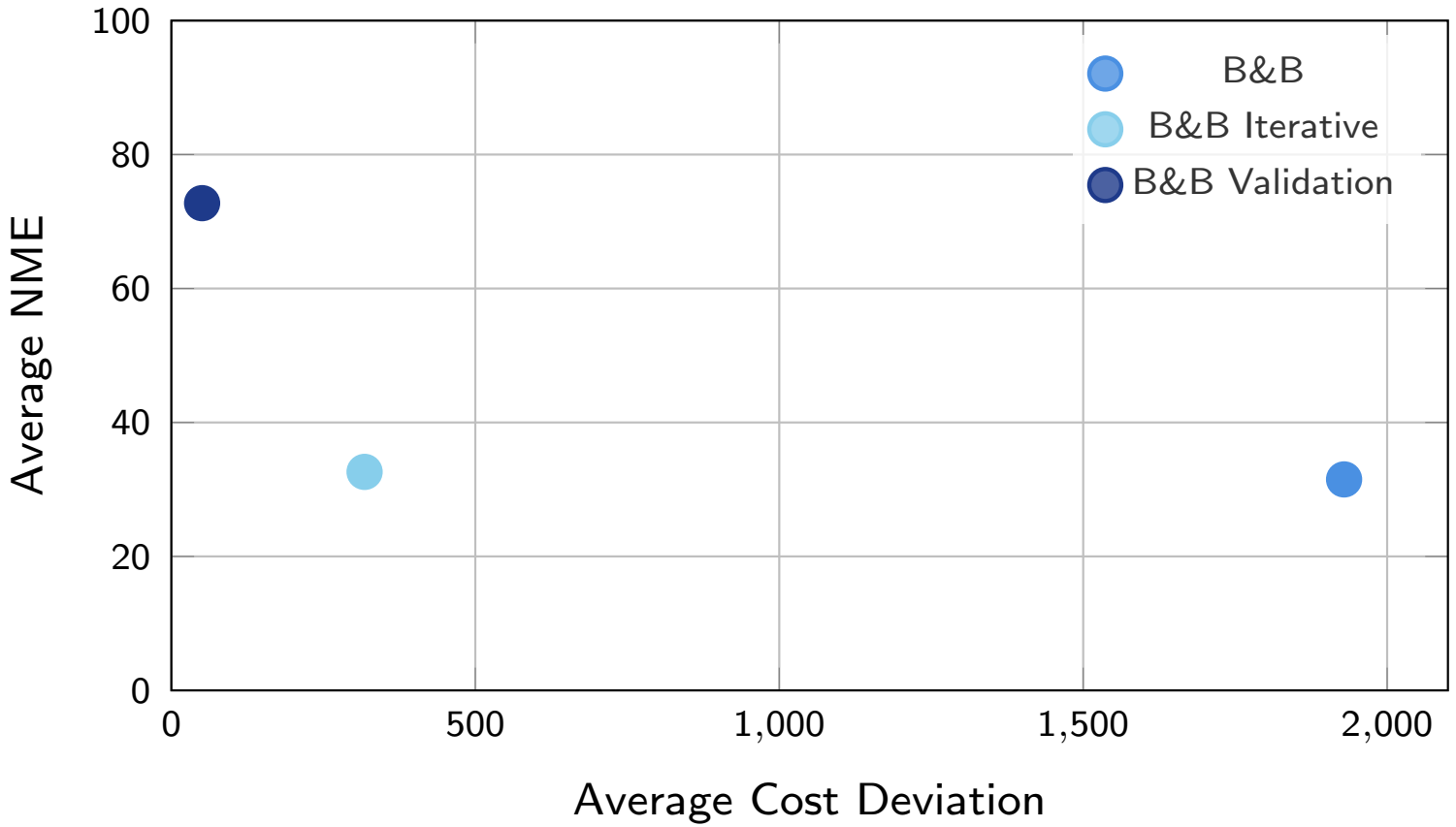

Figure: Runtime–quality trade-off for 400 CMP-C instances, showing average model evaluations (NME) versus average cost deviation.

Conclusion

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- The presented framework enables the optimization of the building envelope through construction measures
- Additive relaxation allows for efficient optimization
- B&B with validation-driven branching provides more **accurate** solutions at the cost of runtime
- B&B with iterative evaluation yields **faster** results with quality loss

Backup Slides

Analysis of the Distribution of Non-Additivity

Test Case	Pairs (Total)	Exact Add. Pairs	MAE	Share Pos. [%]	Share Neg. [%]
SFB 11 (2018)	2414	0	0.14	93.54	6.46
SFB 8 (2018)	2414	0	0.08	55.84	44.16
SFB 7 (2018)	2414	0	0.06	41.30	58.70
AB 5 (2018)	911	0	-0.04	0	100
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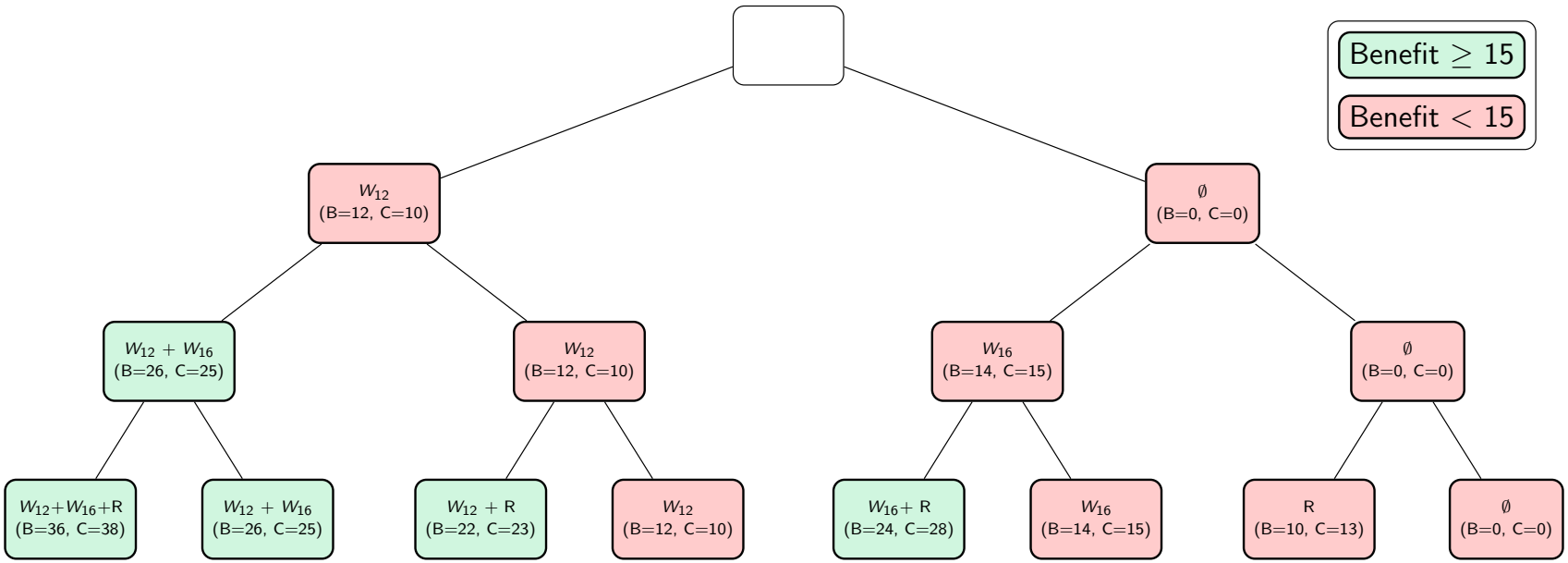
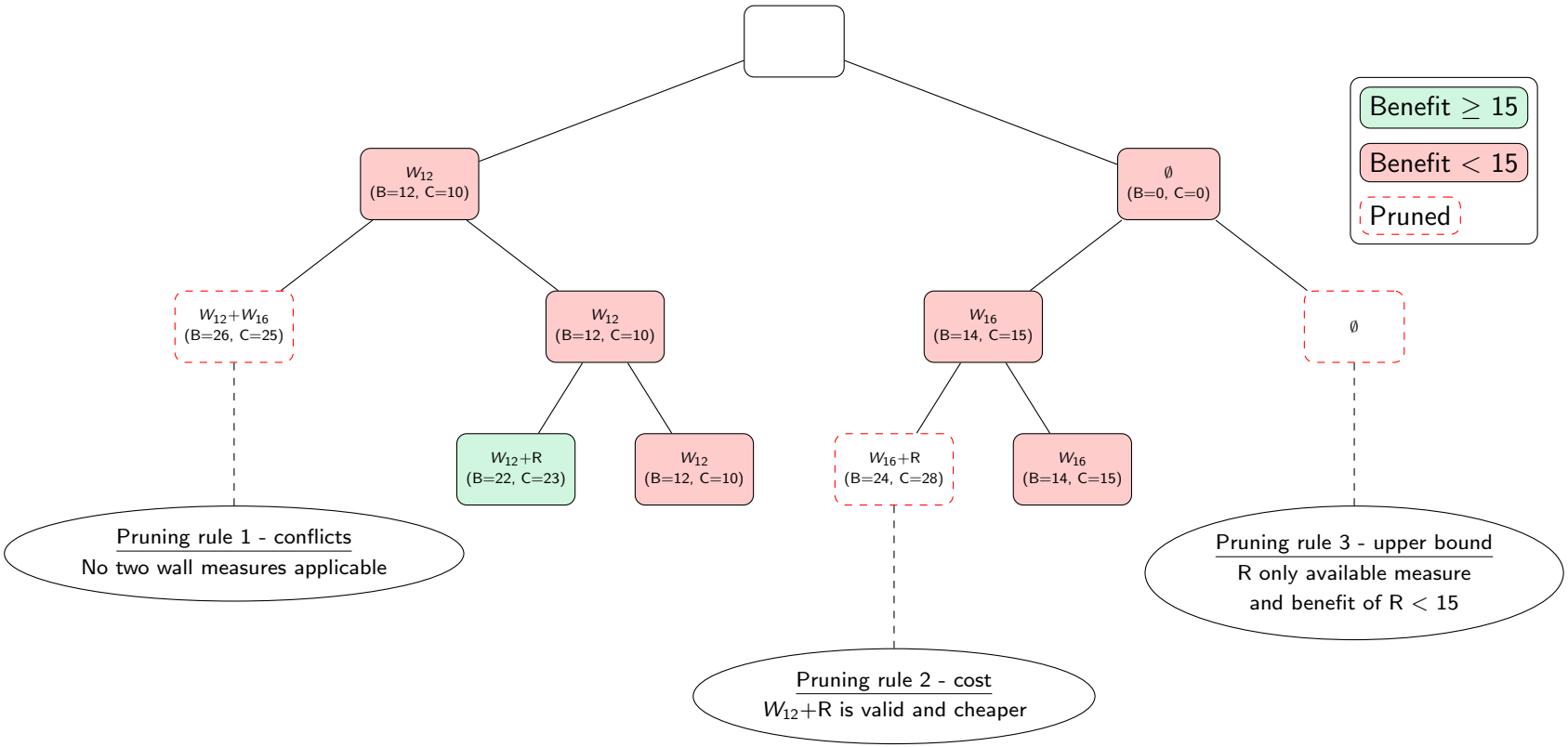
SFB 5 (2018)	2414	0	-0.03	5.43	94.57
AB 8 (2018)	911	0	-0.02	1.43	98.57
AB 11 (2018)	911	0	-0.02	8.78	91.22
AB 6 (2018)	911	0	0	44.24	55.76
SFB 6 (2018)	2414	0	0	24.73	75.27

Complete Search Tree Generated by B&B



Pruned Search Tree Generated by B&B



Additive Relaxation of CMP-C

$$\text{(CMP-C)} \qquad \min \sum_{j \in \mathcal{M}} c_j x_j$$

Black-box function

$$\text{s.t.} \quad \mathcal{F}\big(B, \{ j \in \mathcal{M} \mid x_j = 1 \}\big) \leq b, \quad B \in \mathcal{B}$$

$$\sum_{j \in S_i} x_j \leq 1, \quad \forall i \in L$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{M}$$

Additive Relaxation of CMP-C

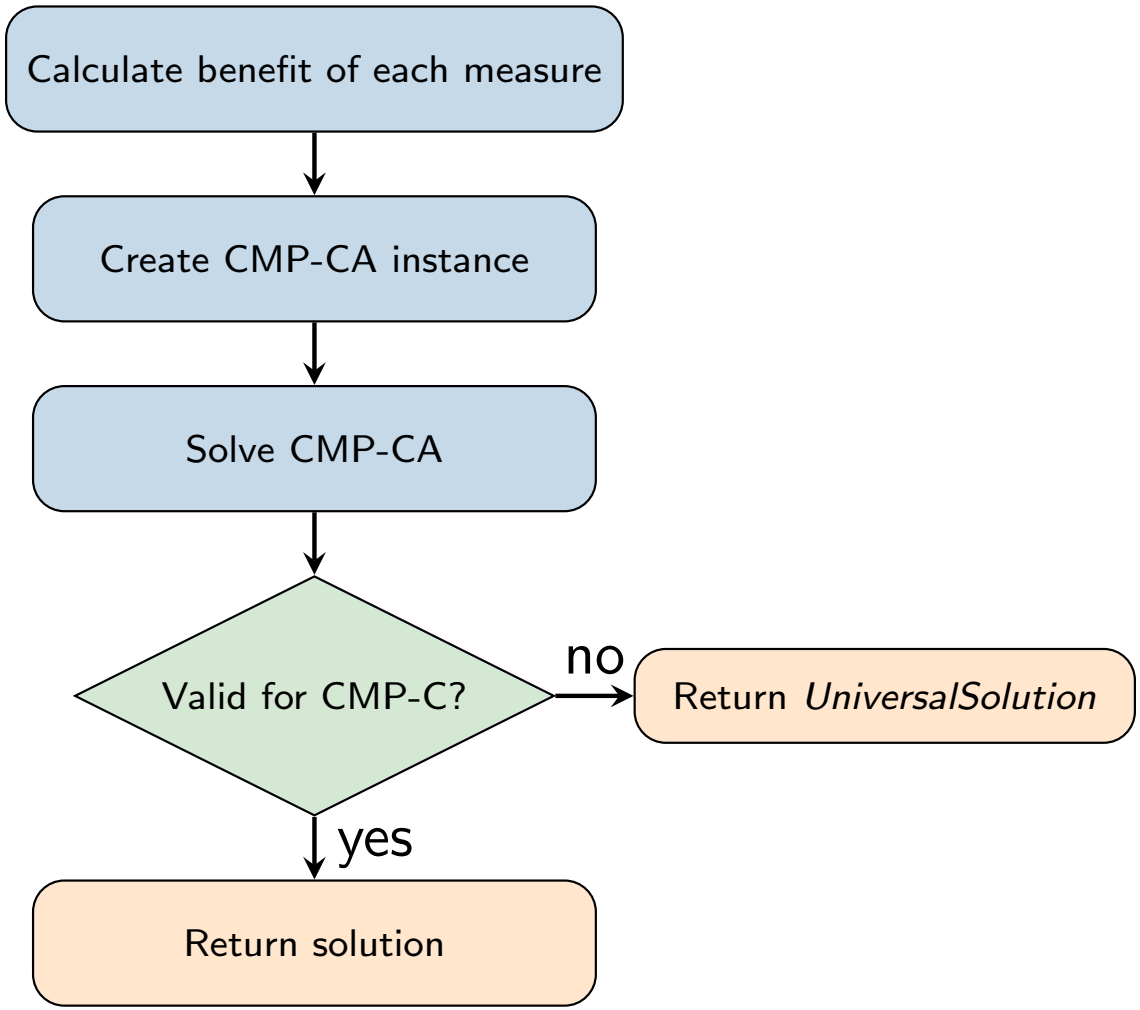
$$\text{(CMP-CA)} \quad \min \sum_{j \in \mathcal{M}} c_j x_j$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{M}} b_j x_j \geq b$$

$$\sum_{j \in S_i} x_j \leq 1, \quad \forall i \in L$$

$$x_j \in \{0, 1\}, \quad \forall j \in \mathcal{M}$$

Embedding Process of Additive Relaxation



Genetic Algorithm

- Included as a non-deterministic benchmark for completeness
- Suitable for black-box optimization — no explicit model formulation of $\mathcal{F}$ required

Fitness function:

- Penalizes invalid or conflicting combinations
- Discards solutions below target demand
- Minimizes total cost of valid configurations

Evolution process: evaluate $\rightarrow$ select $\rightarrow$ recombine $\rightarrow$ mutate $\rightarrow$ repeat (50 generations)

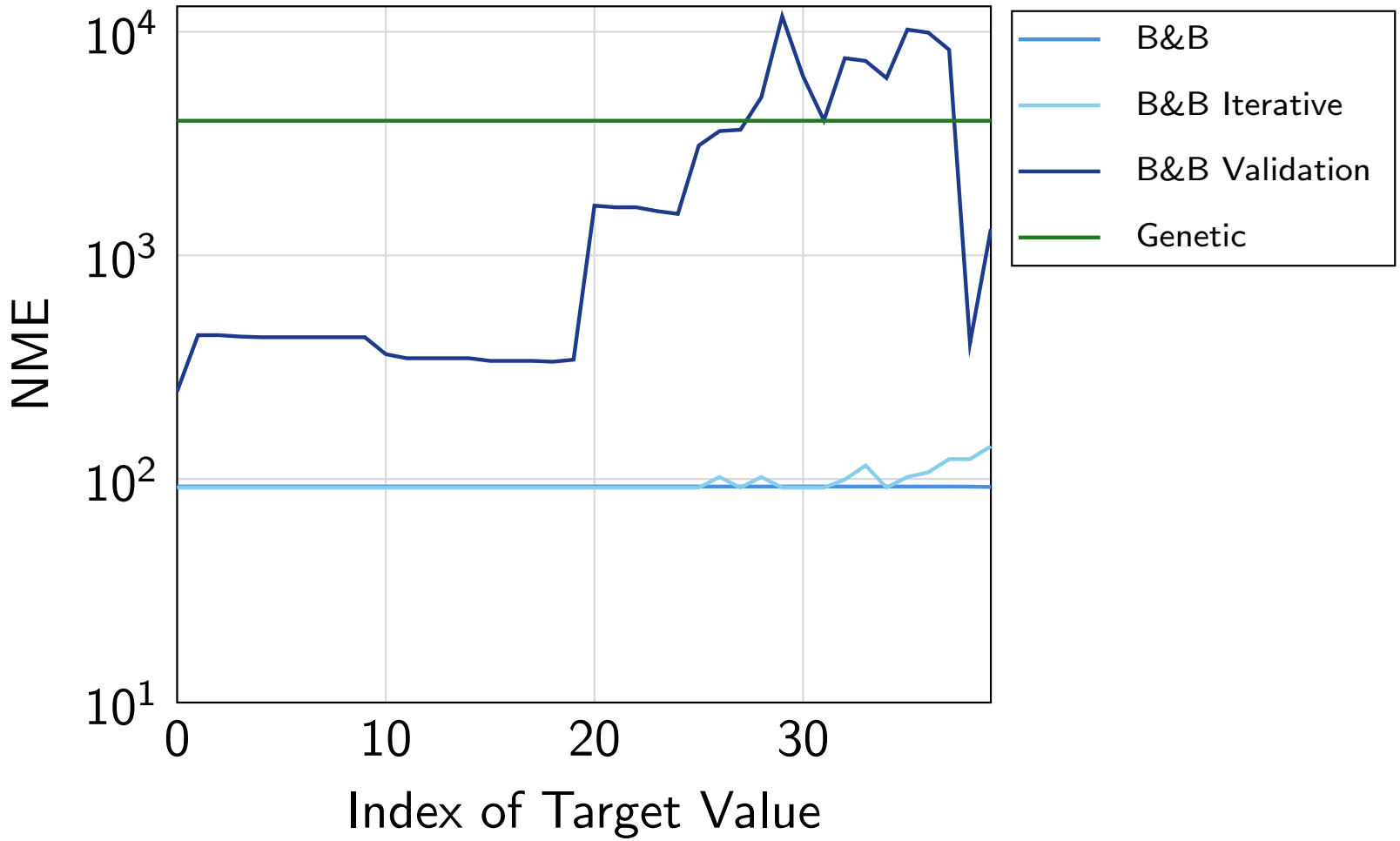
Measure Catalogue M_{small}

Component	Type	Materials	Thickness [cm]	Measures
Base Plate	Ins. Below	EPS, PUR	8, 10, 12	6
	Ins. Below With Cladding	GB	8, 10, 12, 14	4
Wall	ETICS	PUR, XPS	12, 14, 16, 18	8
Door	–	FG, HFG, HSI, HW, SH, SI, WO	–	7
Roof Window	–	2G, 3G, PHG	–	3
Window	–	2G, 3G, PHG3	–	3
			Total:	**31**

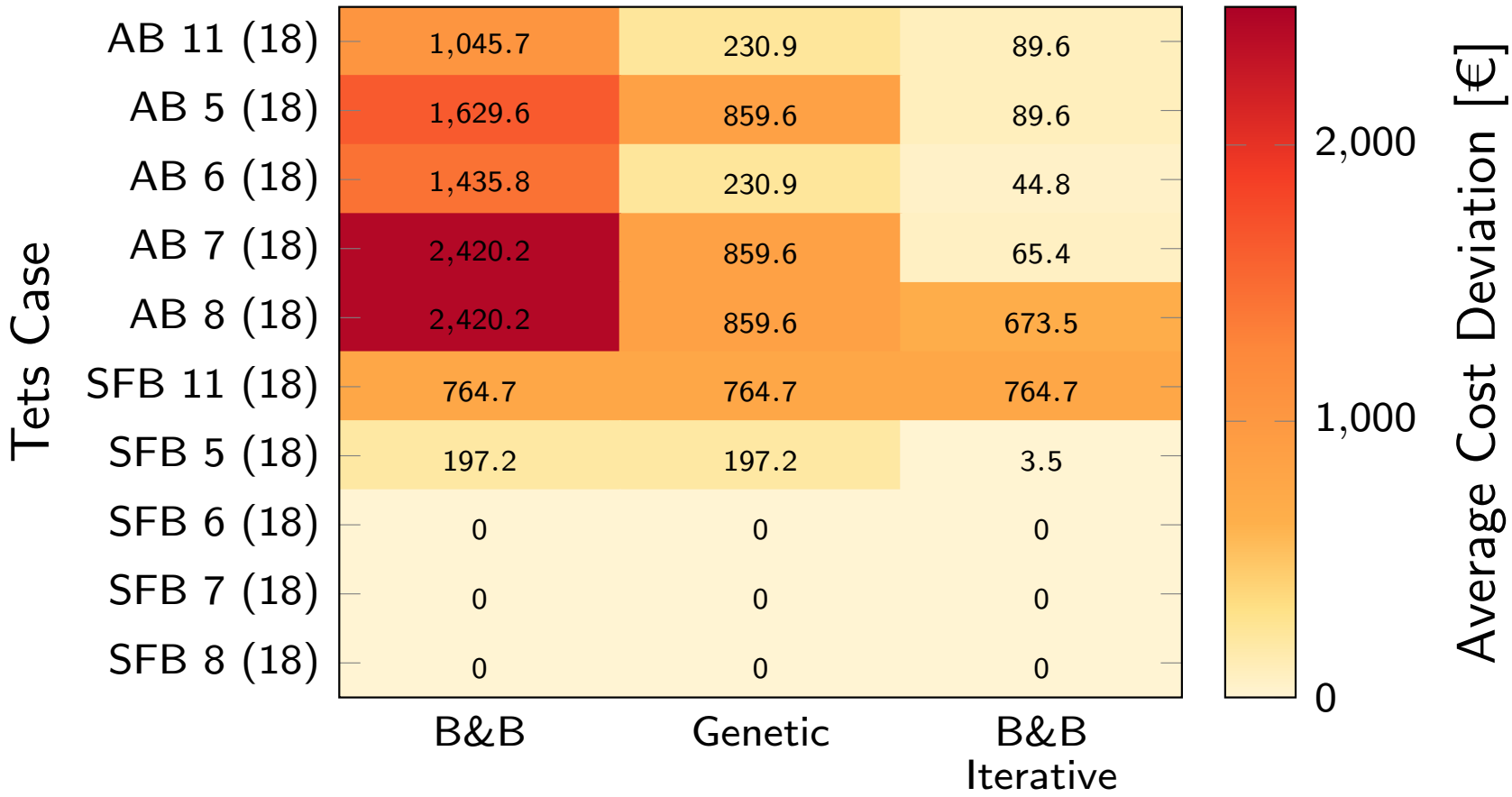
Measure Catalogue M_{all}

Component	Type	Materials	Thickness [cm]	No. of Measures
Base Plate	Ins. Below	EPS, MW, PUR, WF, XPS	8, 10, 12	15
	Ins. Below With Cladding	GB, WFC	8, 10, 12, 14	8
	Ins. Above	BMW, BWF, CEL, FGG	5, 6, 9	12
Roof	Above Rafters	EPS, PUR, WF	13, 17, 20, 23	12
	Between Rafters	GW, MW, WF	13, 17, 20, 23	12
Wall	Core Insulation	AER, EPSG, MW, PER	6, 8, 9	12
	ETICS	EPS, MW, PUR, WF, XPS	12, 14, 16, 18	20
Door	–	FG, SH, SI, WO, HFG, HSI, HW	–	7
Roof Window	–	2G, 3G, PHG	–	3
Window	–	2G, 3G, PHG	–	3
			Total:	**104**

NME under $\mathcal{M}_{\text{all}}$



Solution Quality under $\mathcal{M}_{\text{all}}$



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