Extending the RealySt tool to support linear hybrid automata

Final Talk

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LuFG Theory of Hybrid Systems

SS 2026

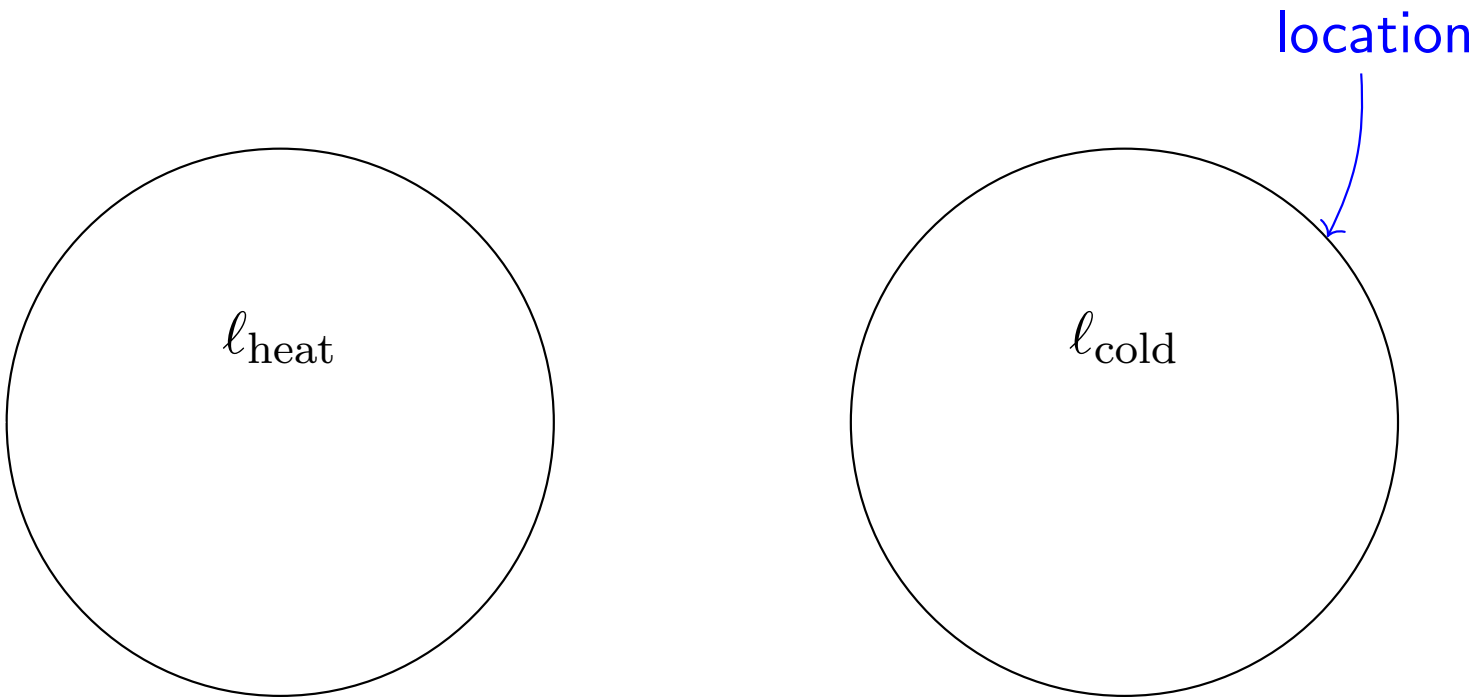
Motivation

- Hybrid automata describe physical phenomena
- Stochastic hybrid automata add probabilistic event
- Several tools which can analyze automata
- JANI was introduced since interoperability was limited
- REALYST only supported rectangular hybrid automata
- Linear hybrid automata are more expressive

Outline

1. Hybrid Automata
2. Reachability Analysis
3. Tools
4. JANI
5. Implementation
6. Results

Example Hybrid Automaton



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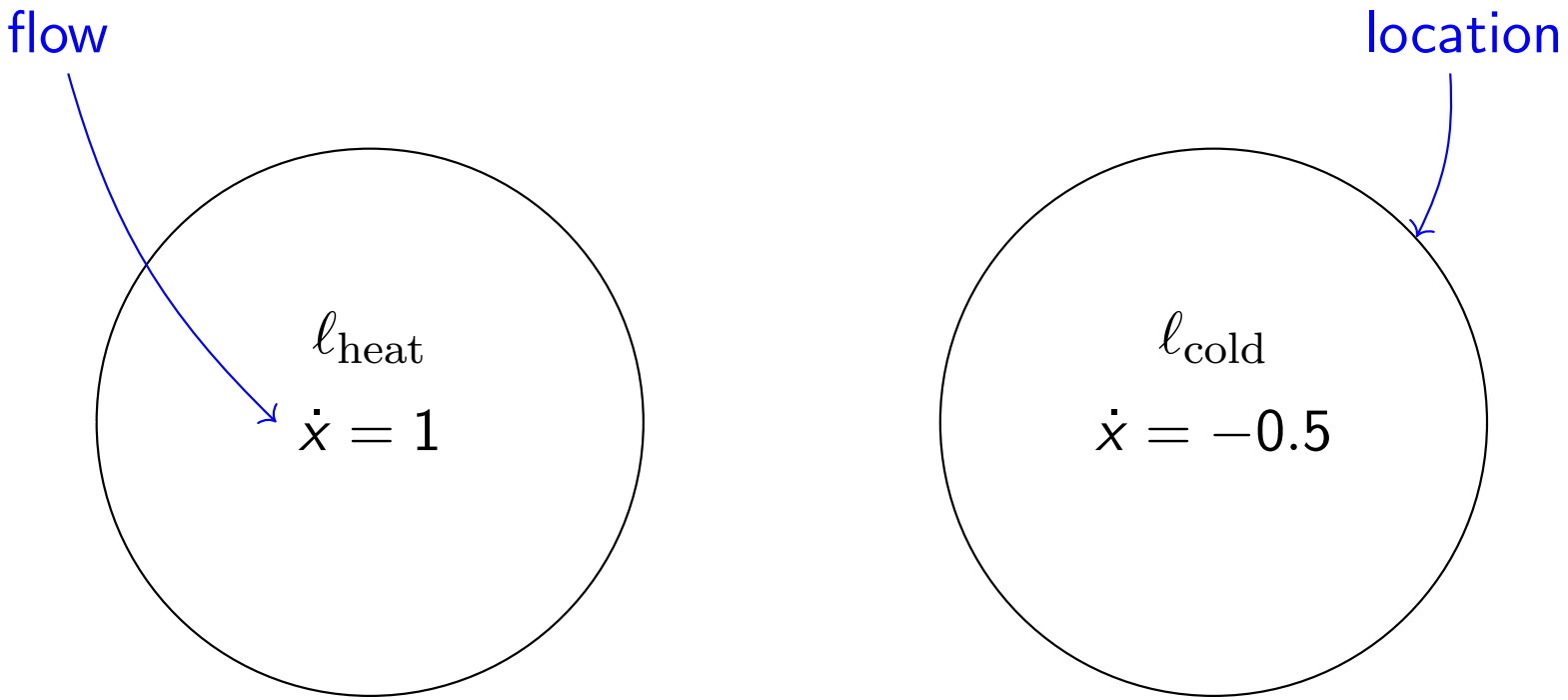

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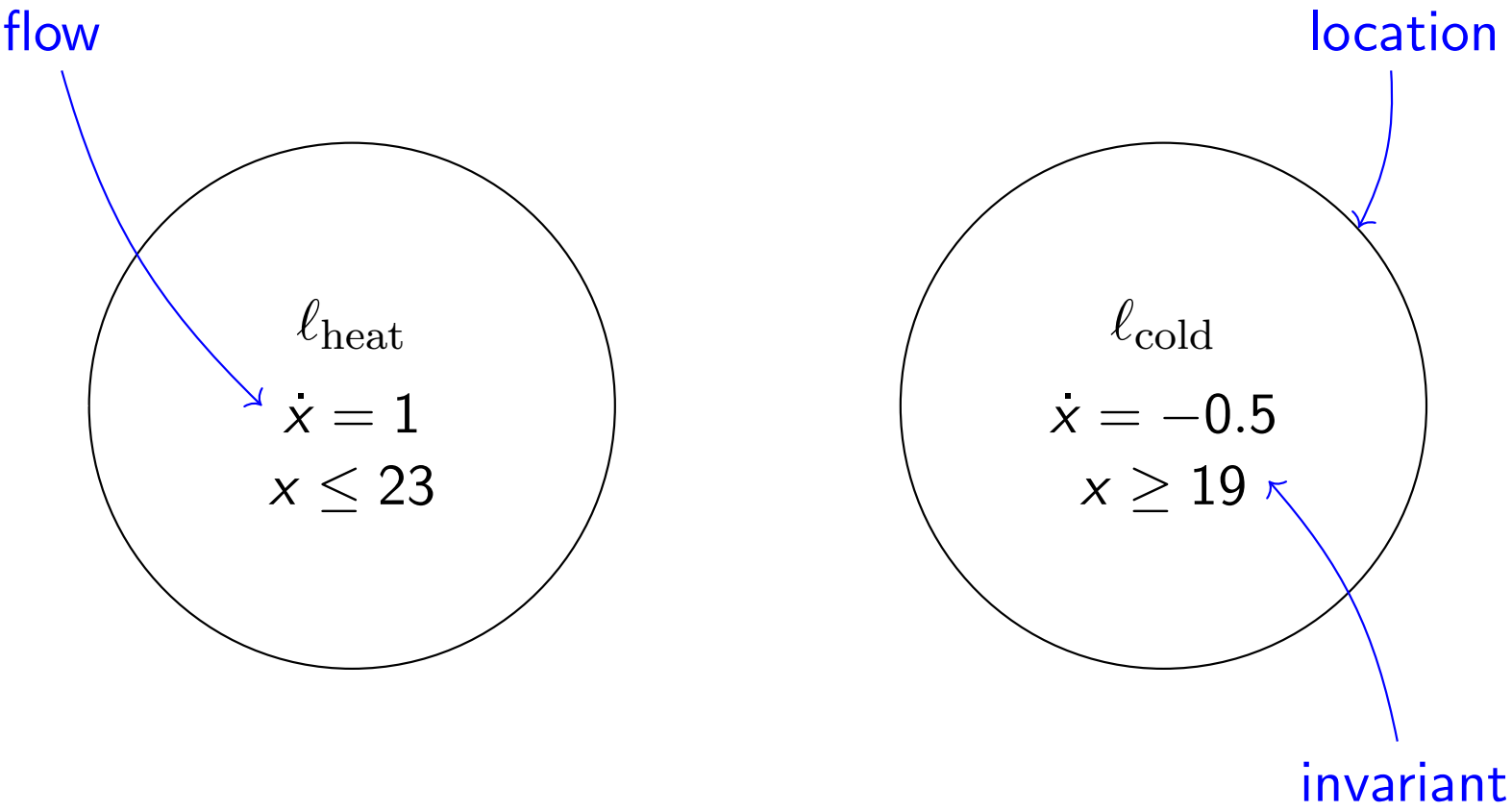

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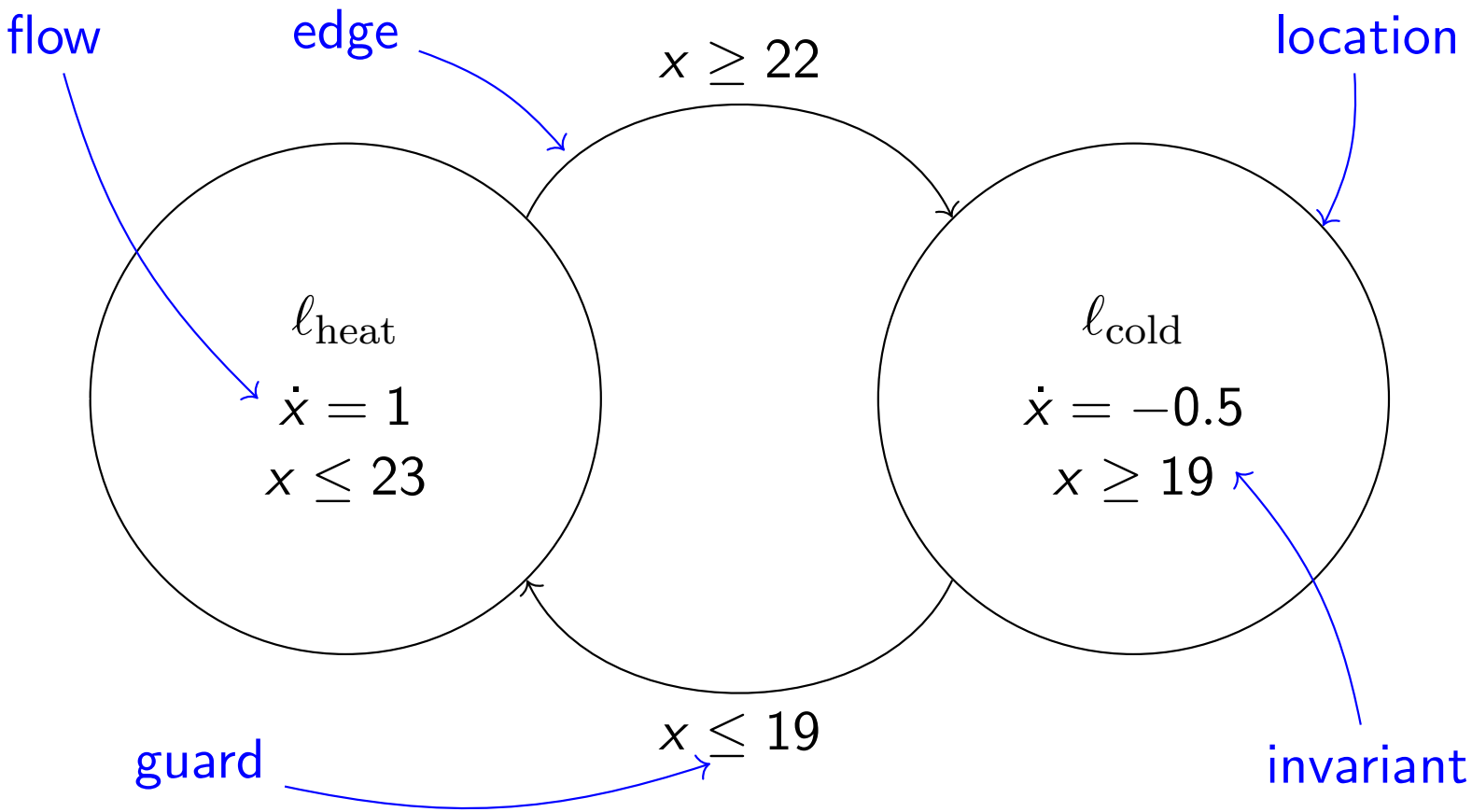

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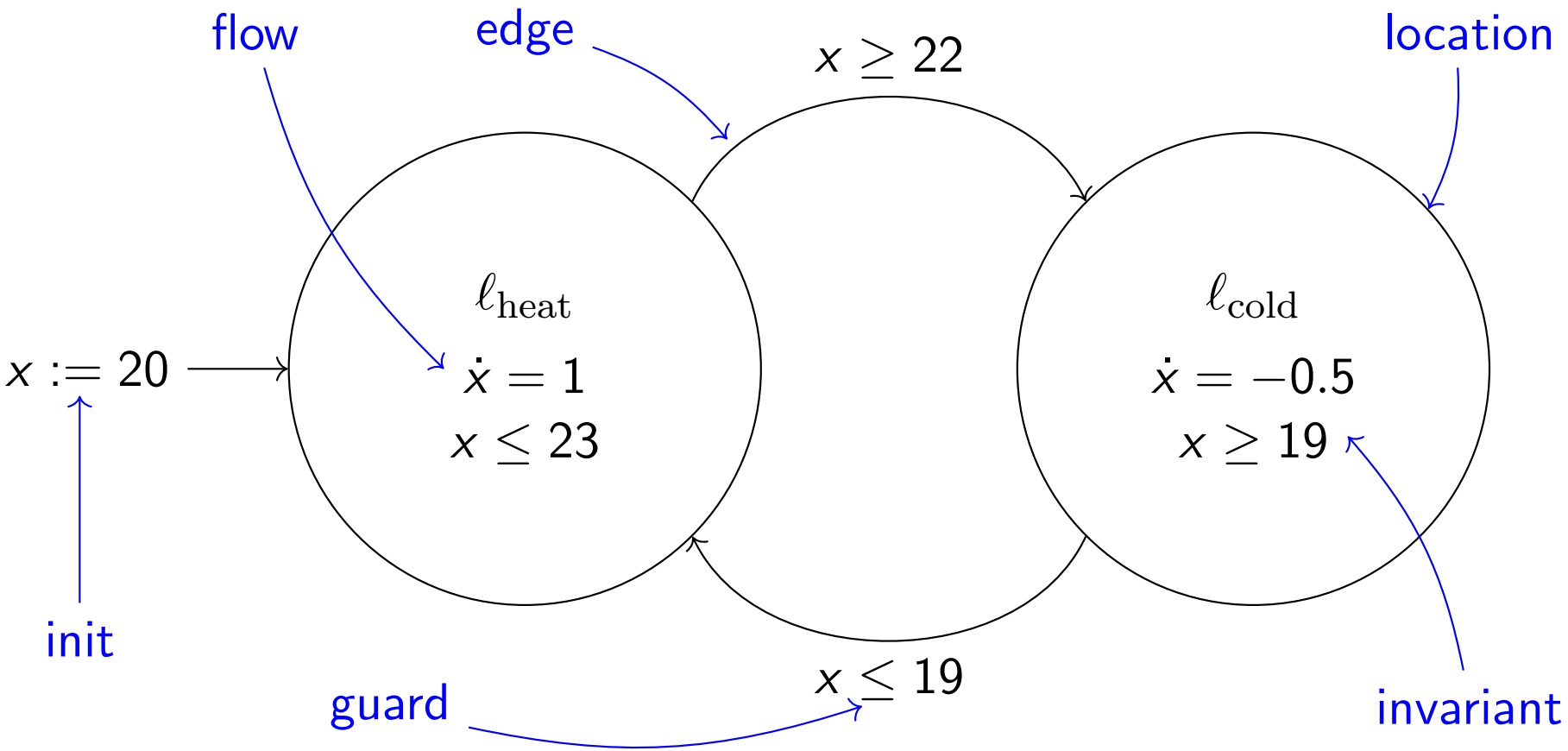

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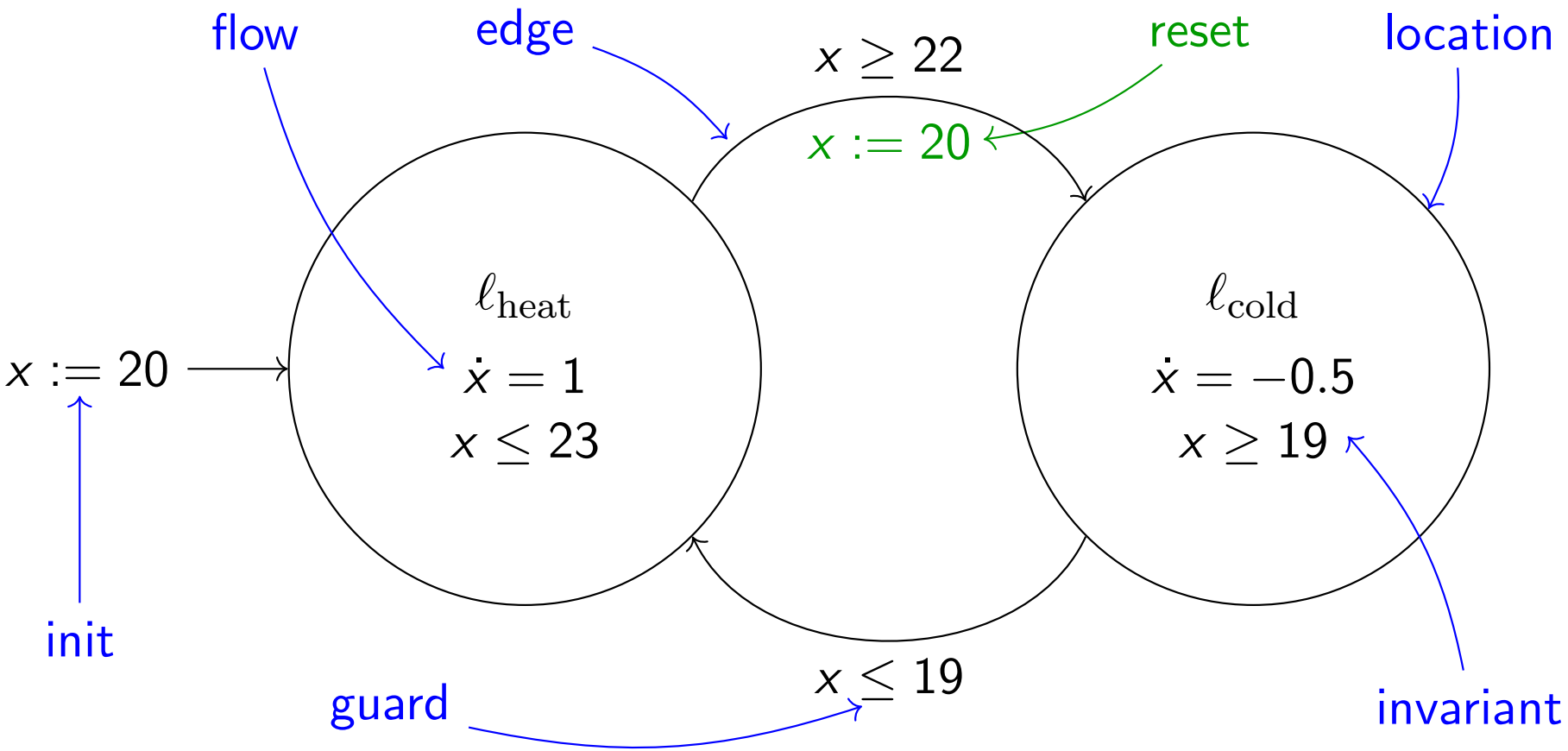

Subclasses of Hybrid Automata

Rectangular Hybrid Automata:

- Flows, invariants, guards and resets are defined by rectangular sets
- $[a, b]$ represents the rectangular set for a variable x with $a \leq x \leq b$

Linear Hybrid Automata:

- Invariants, guards and resets are defined either by a linear inequality or rectangular set
- Type I: flows are defined as rectangular sets
- Type II: flows are defined as linear ordinary differential equations $\dot{x} = A * x + b$ where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^n$

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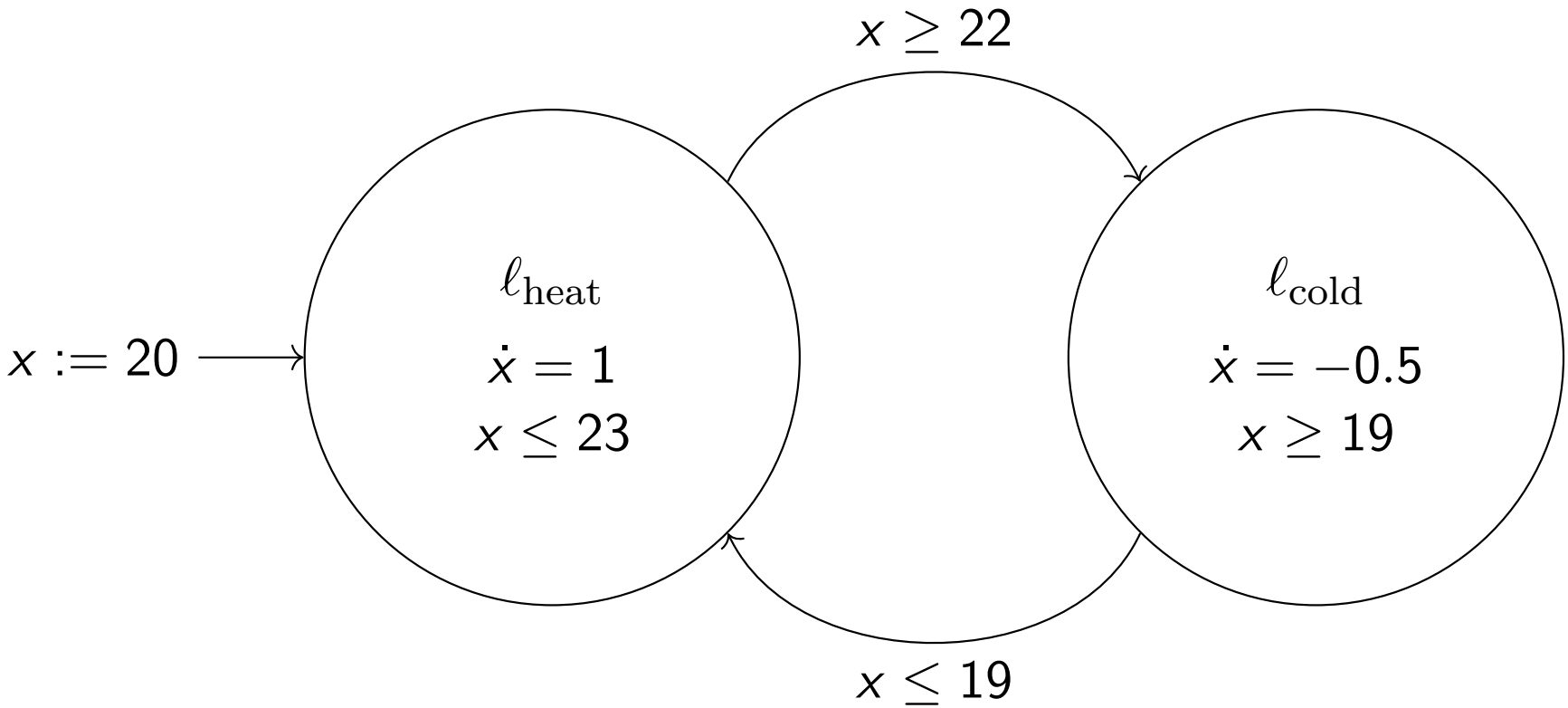
Example Linear Hybrid Automaton II



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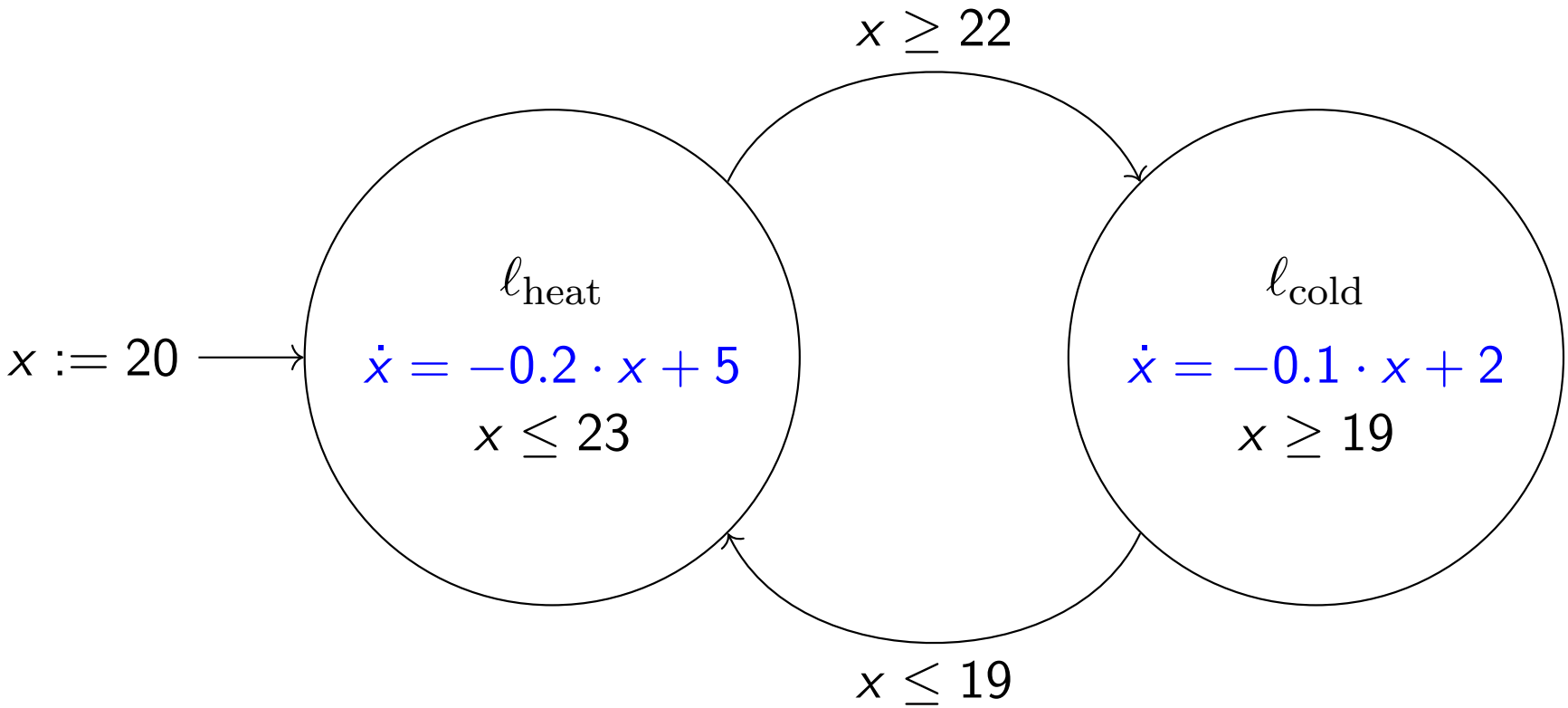
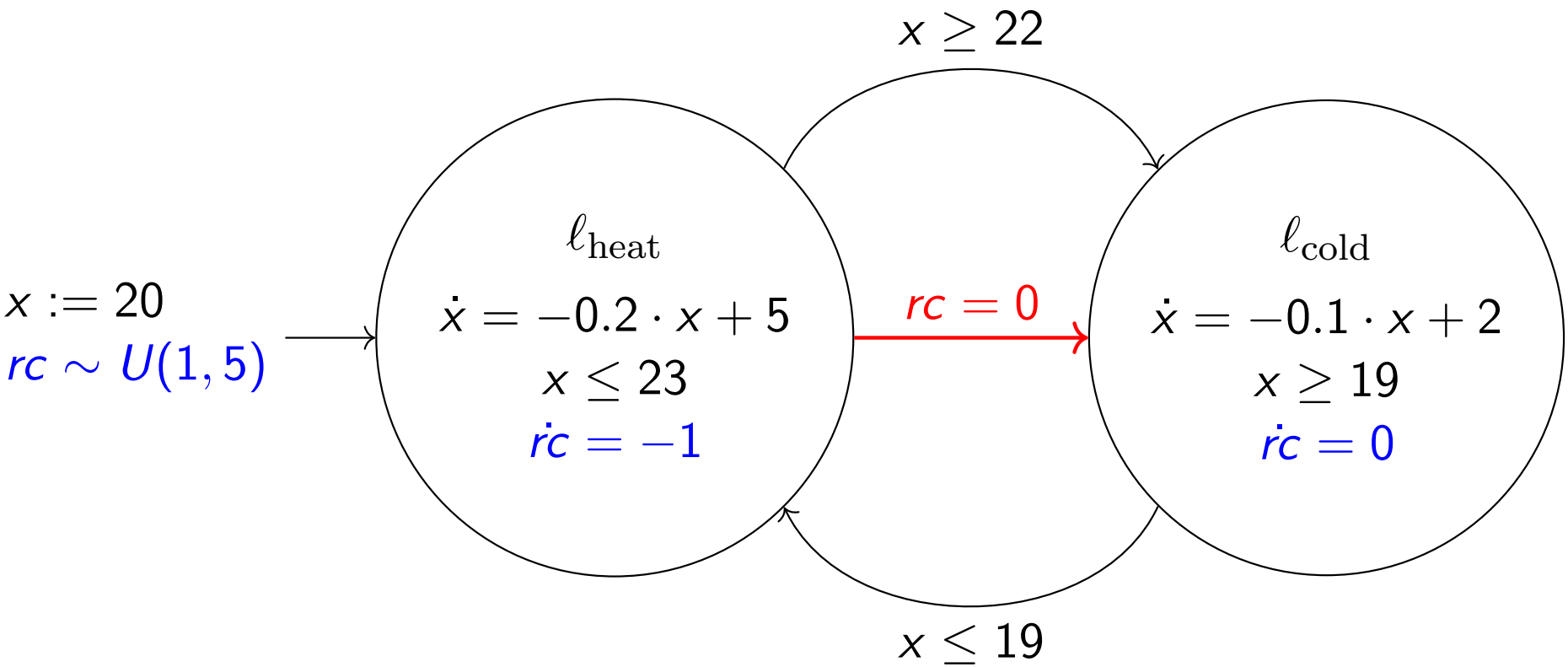


Linear Hybrid Automata II with Random Clocks

- Random clocks model nondeterministic choices
- Stochastic transitions fire when clocks expire
- Enables modeling uncertainty in systems

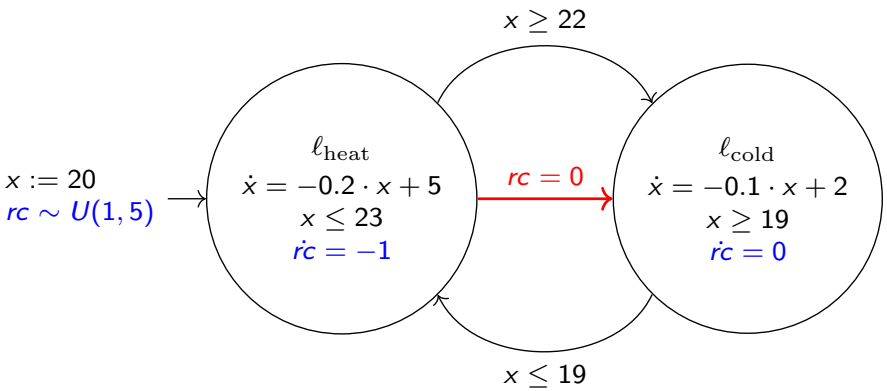
Example Linear Hybrid Automaton II with Random Clocks



Reachability Analysis

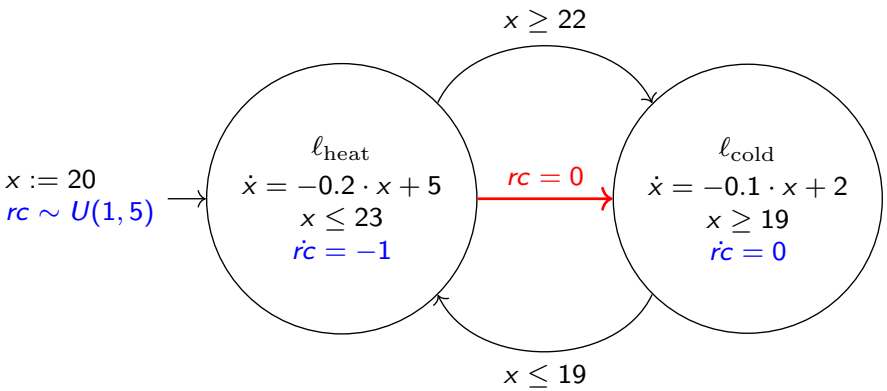
- Calculation how reachable a state from the initial location is
- Performed by iteratively computing the set of reachable states
- Problems: infeasible computation, Zeno execution
- Solutions: time-bound, limited jump depth

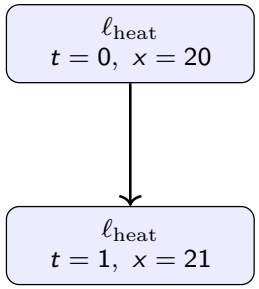
Reachability Tree for the Heating Example



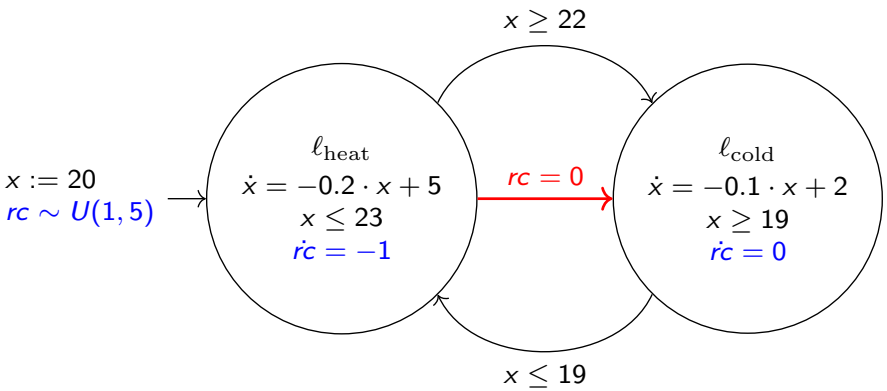
ℓ_{heat}
$t = 0,\ x = 20$

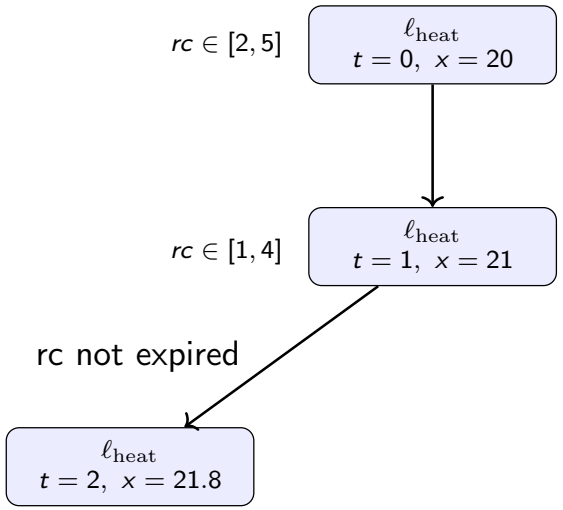
Reachability Tree for the Heating Example





Reachability Tree for the Heating Example





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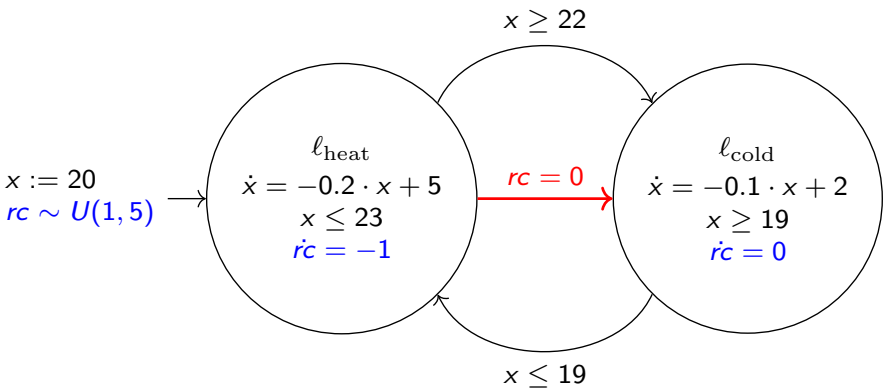

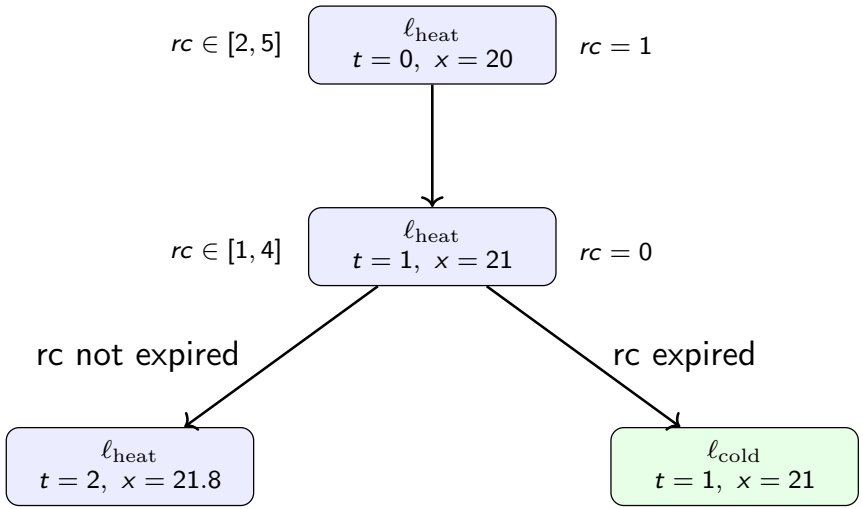

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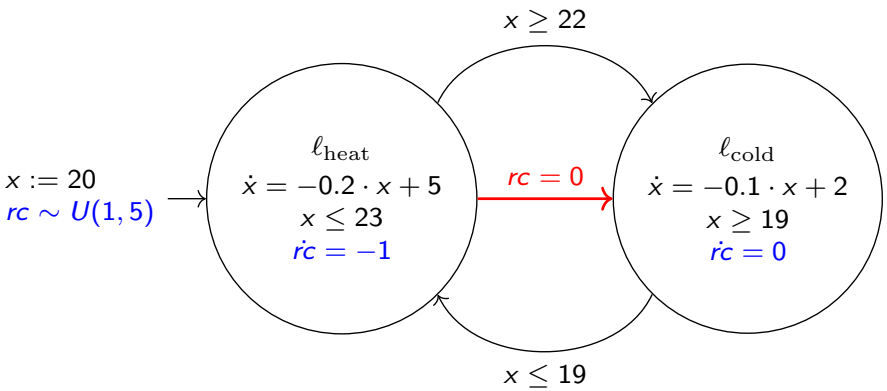
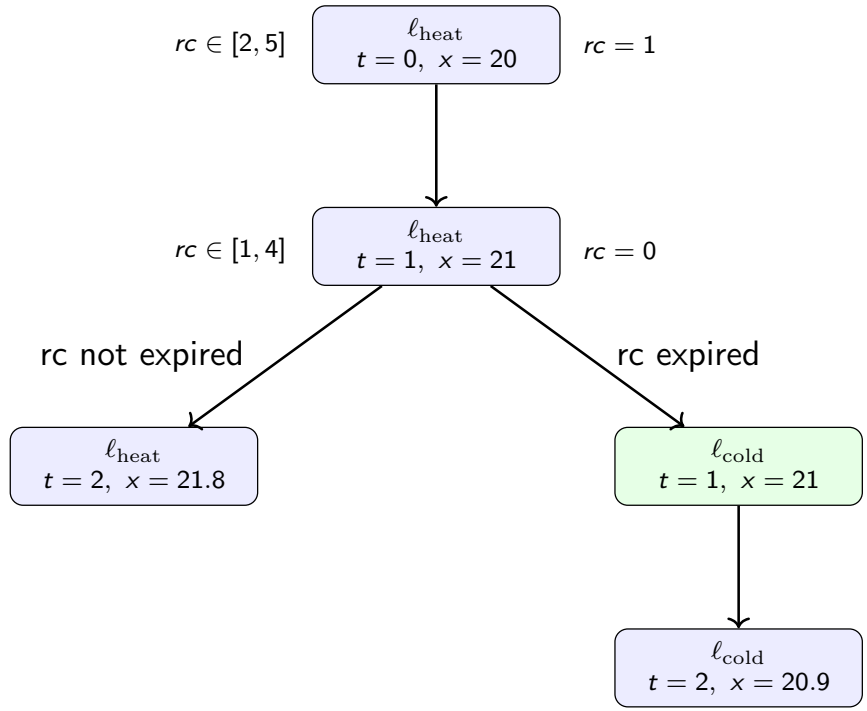




HyPro

- Provides implementations of state set representations for the reachability analysis
- Provides different geometric abstractions for polytopes
- Offers operations required for reachability analysis of linear hybrid automata

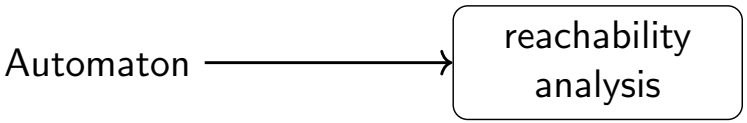
RealySt

- Open-source tool written in C++ for probabilistic reachability analysis
- Computes maximum reachability probabilities for singular automata with random clocks
- Uses HyPro for geometric operations and flowpipe construction

RealySt: Flow

Automaton

RealySt: Flow



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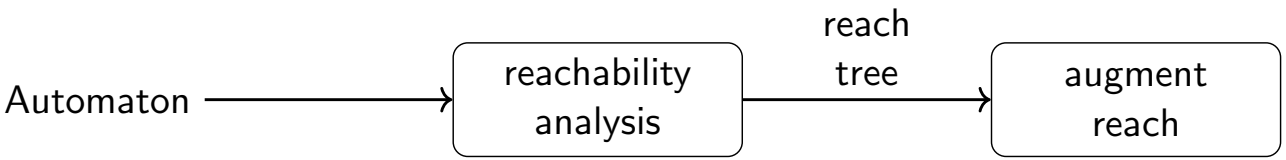

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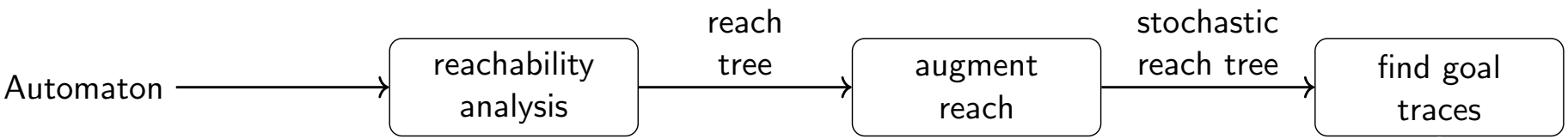

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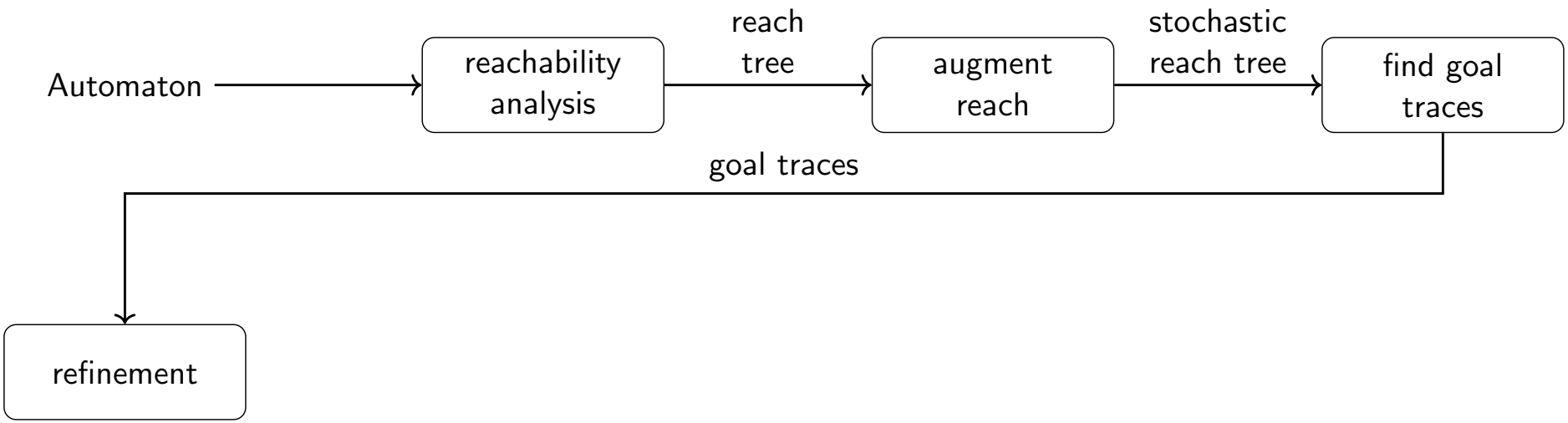

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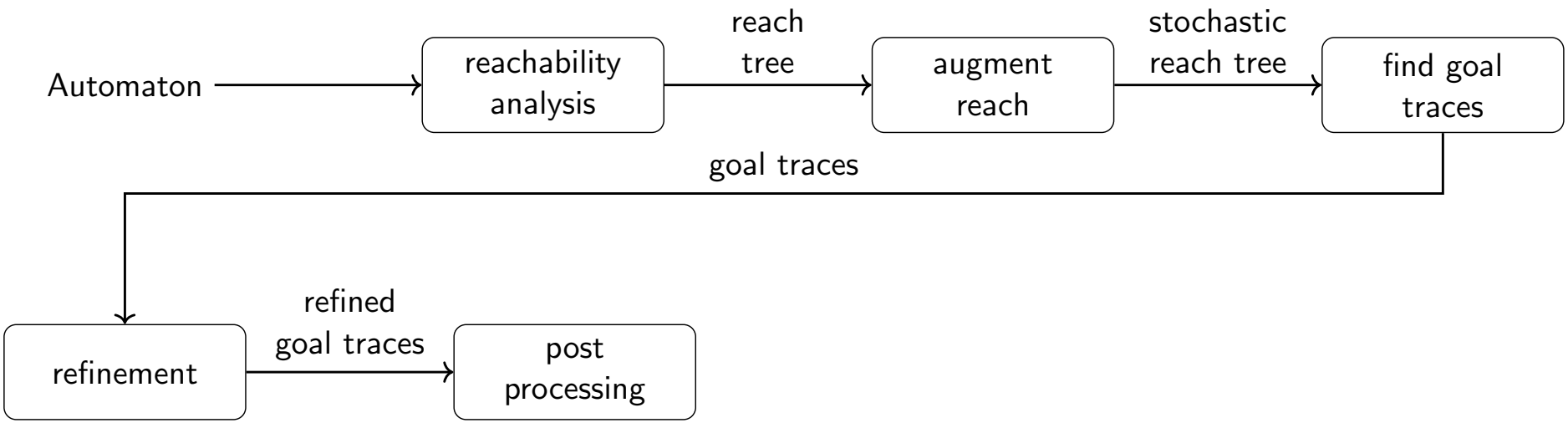

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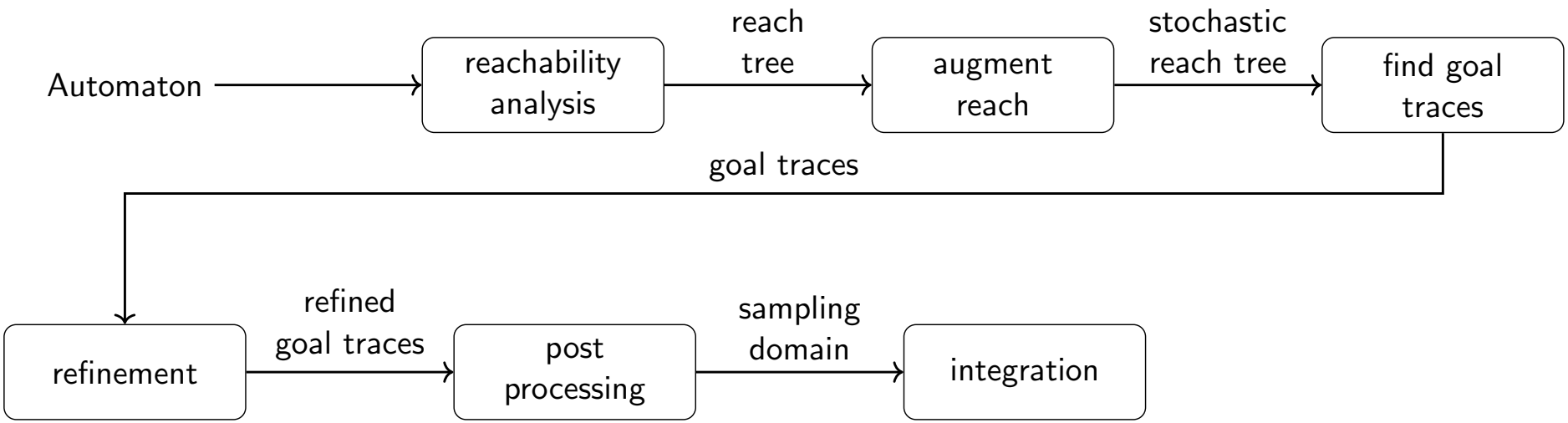

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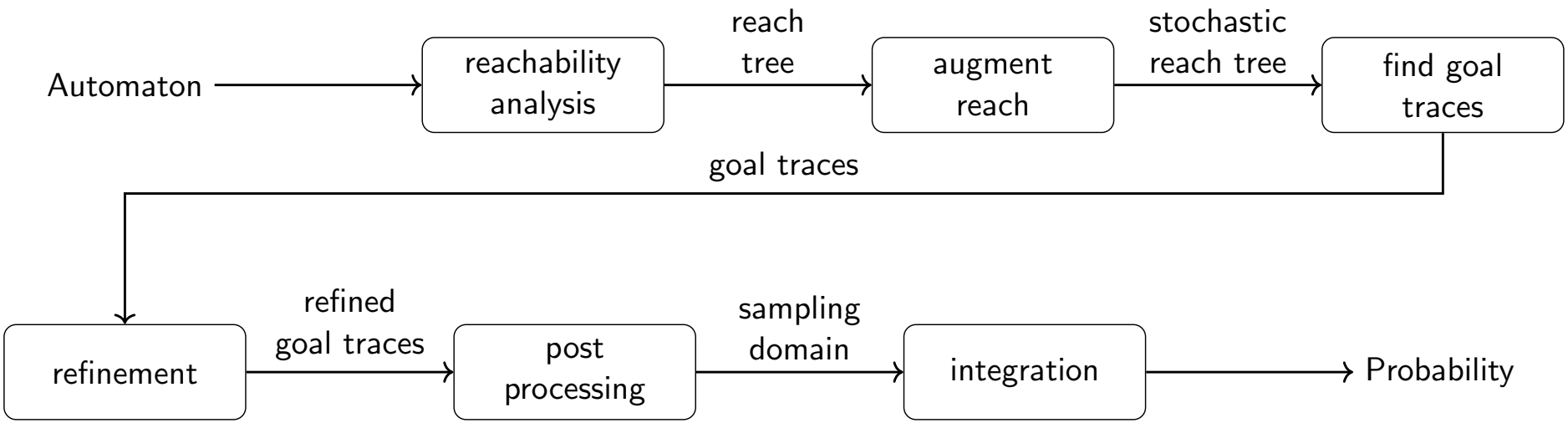

RealySt: Command Call

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- `-d [jumpdepth]`
- `-b [BENCHMARK]`
- `-m [CASE]`
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- `-l [info/trace/debug]`
- `--plotDimensions [index1] [index2]`
- `--LTIAnalysis`
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JANI

- Standardized exchange format
- JSON data format to encode models and messages
- Enables interoperability between tools
- Design also allows for future extensions

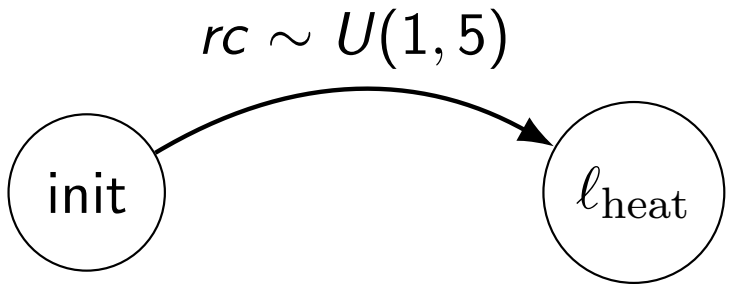
JANI: Model Header and Variables

```
{
 "jani-version": 1,
 "name": "heating_rc_example",
 "type": "sha",

 "variables": [
 { "name": "x", "type": "continuous",
 "initial-value": 20.0 },
 { "name": "rc", "type": "continuous" }
 ]
}
```

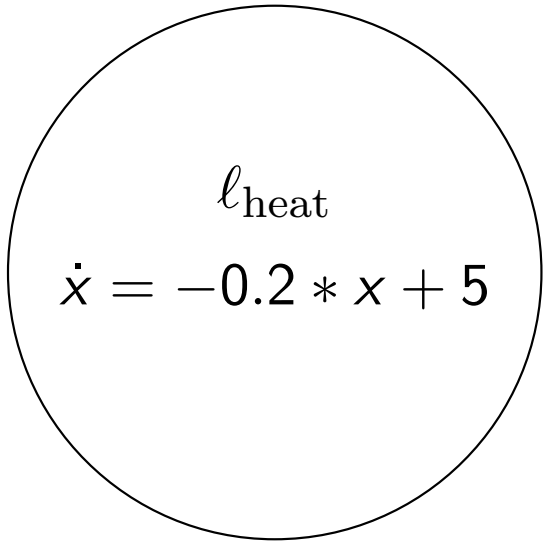
JANI: Stochastic Initialization

```
"location": "init_stochastic_variables",
"destinations": [
 {
  "assignments": [
   {
    "ref": "rc",
    "value": {
     "args": [1.0, 5.0],
     "distribution": "Uniform"
    }
   }
  ],
  "location": "l_heat"
 }
]
```



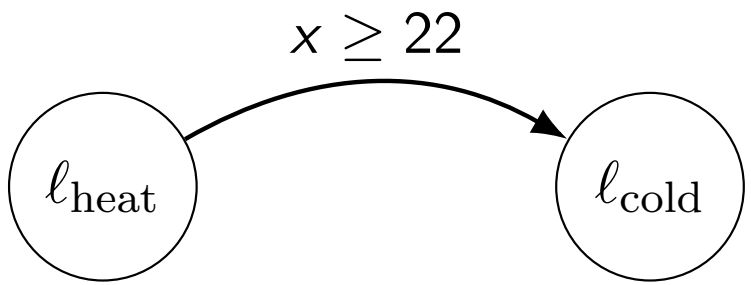
JANI: Location with Linear Flow

```
"name": "l_heat",
"time-progress": {
 "exp": {
  "left": { "op": "der", "var": "x" },
  "op": "=",
  "right": {
   "left": {
    "left": -0.2,
    "op": "*",
    "right": "x"
   },
   "op": "+",
   "right": 5.0
  }
 }
}
```



JANI: Guards for Transitions

```
"location": "l_heat",
"guard": {
 "exp": {
  "left": 22,
  "op": "≤",
  "right": "x"
 }
},
"destinations": [
 { "location": "l_cold" }
]
```



JANI: Example Specification

```
"properties": [
 {
  "expression": {
   "fun": "max",
   "op": "filter",
   "states": { "op": "initial" },
   "values": {
    "exp": {
     "left": true,
     "op": "U",
     "right": {
      "left": "x",
      "op": "≤",
      "right": 20.0
     },
     "time-bounds": {
      "upper": 20
     }
    },
   "op": "Pmax"
   }
  },
  "name": "specification"
 }
]
```

- Defines a reachability property
- Goal condition: $x \leq 20$
- Only in 20 time-steps
- RealySt checks whether this goal can be reached in any location

State JANI Parser in RealySt

- Supported rectangular hybrid automata before
- Linear hybrid automata were not supported
- Specifications were missing while creating a JANI-file
- Resets were not handled

Implementation – Structure of a Linear Expression

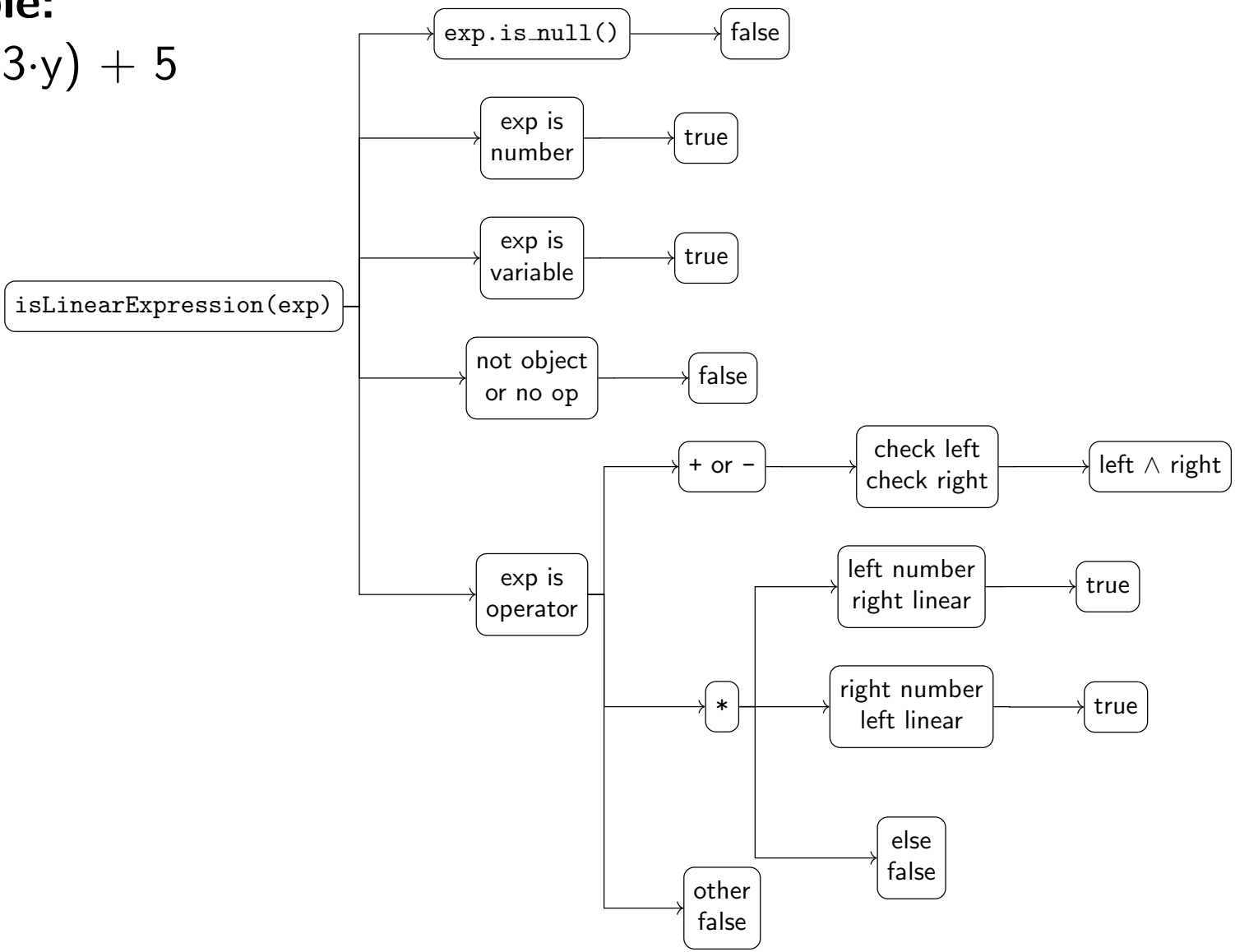
```
{
 "op": "+",
 "left": {
  "op": "+",
  "left": {
   "op": "*",
   "left": 2,
   "right": "x"
  },
  "right": {
   "op": "*",
   "left": 3,
   "right": "y"
  }
 },
 "right": 5
}
```

- $(2 \cdot x + 3 \cdot y) + 5$
- JANI stores expressions as trees
- Inner nodes are operators
- Leaves are variables or constants

Implementation – Detecting Linear Expression

Example:

$(2 \cdot x + 3 \cdot y) + 5$



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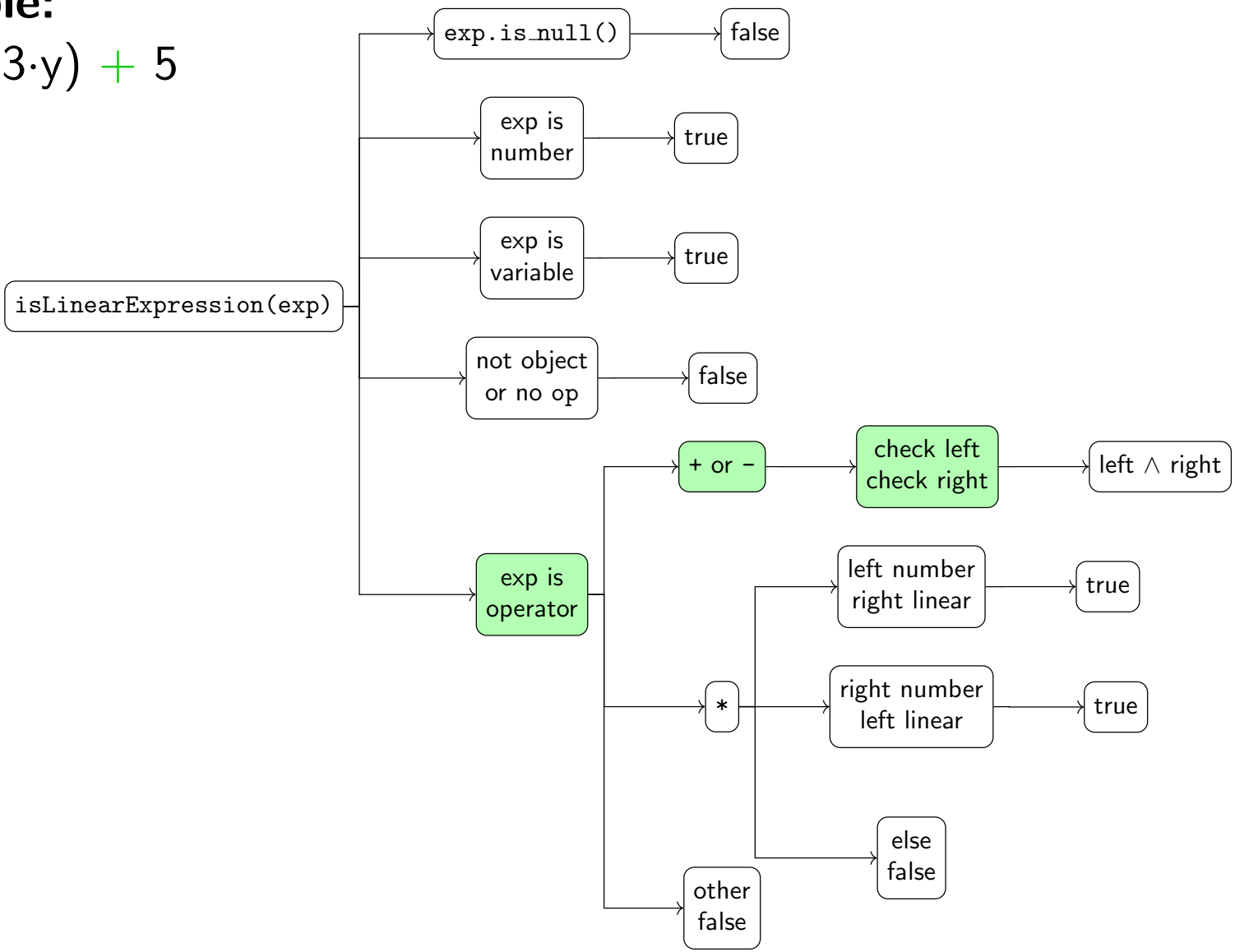

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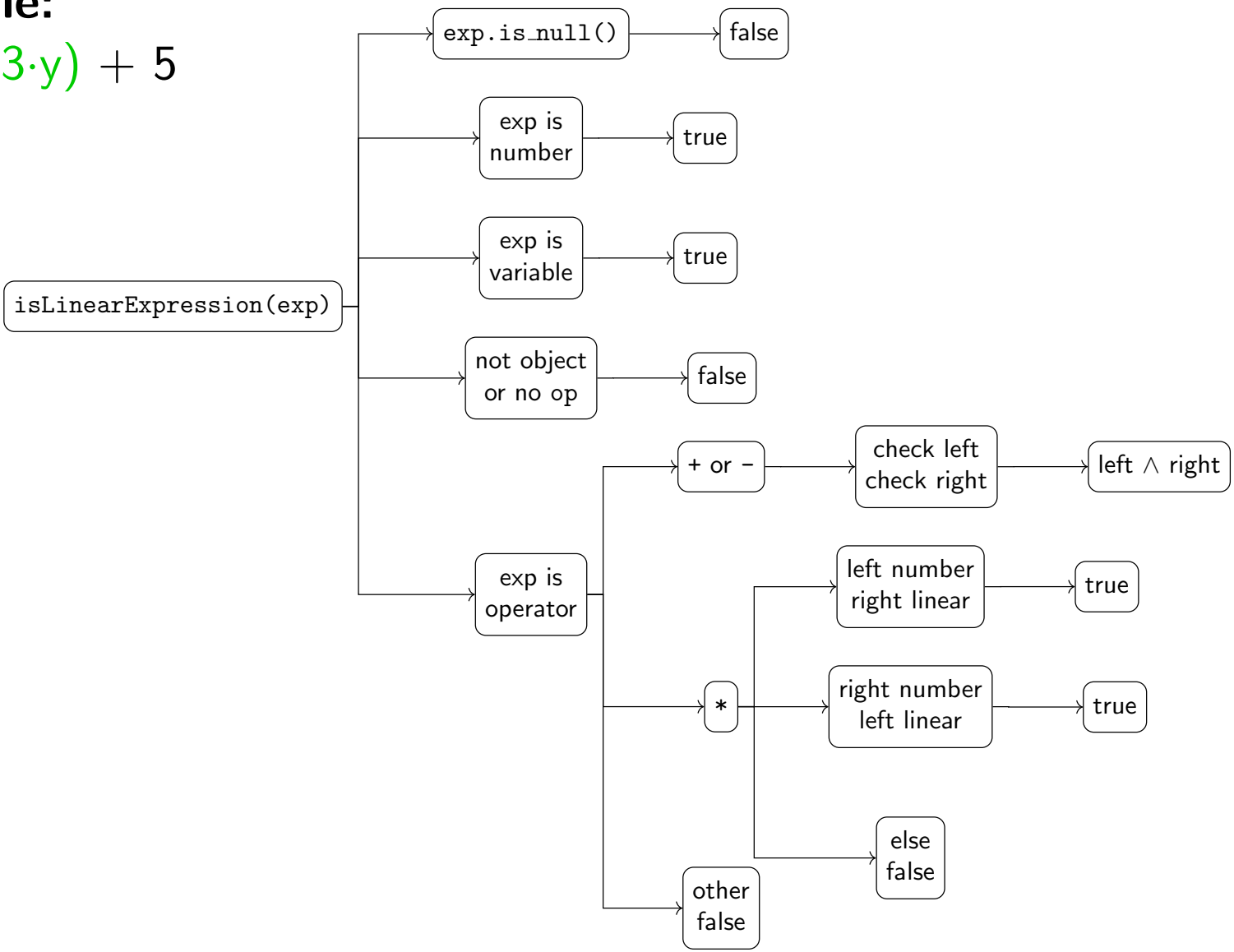

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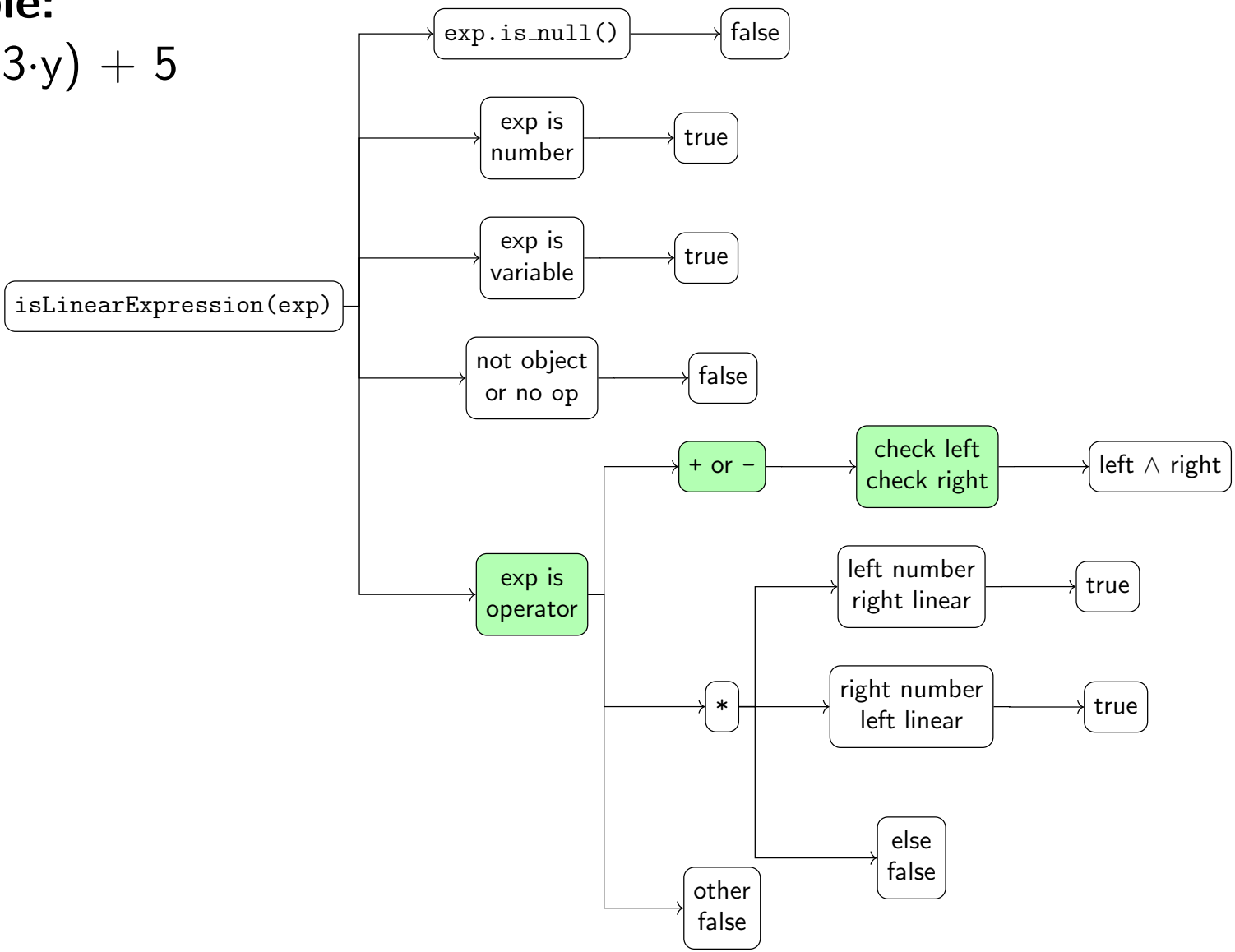

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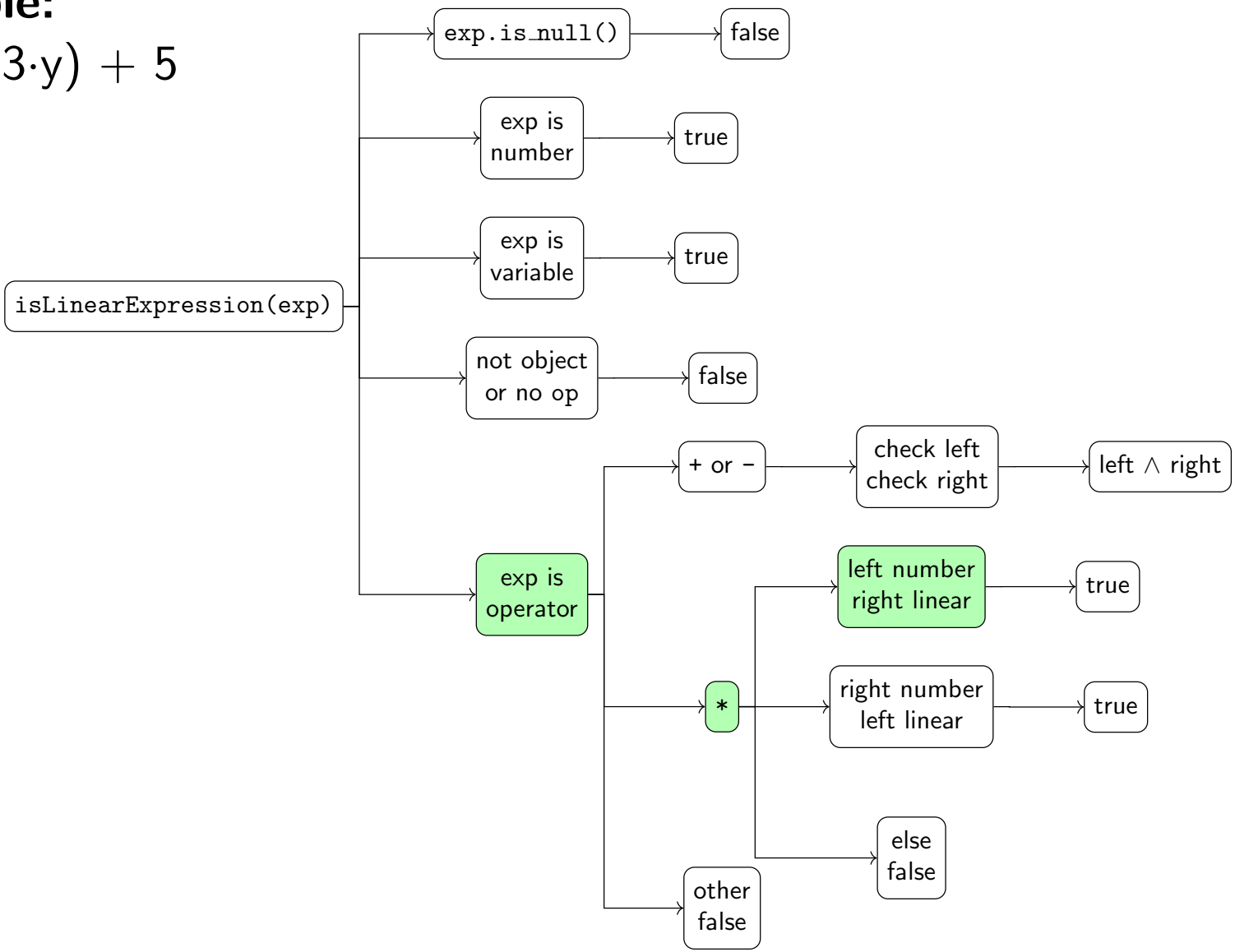

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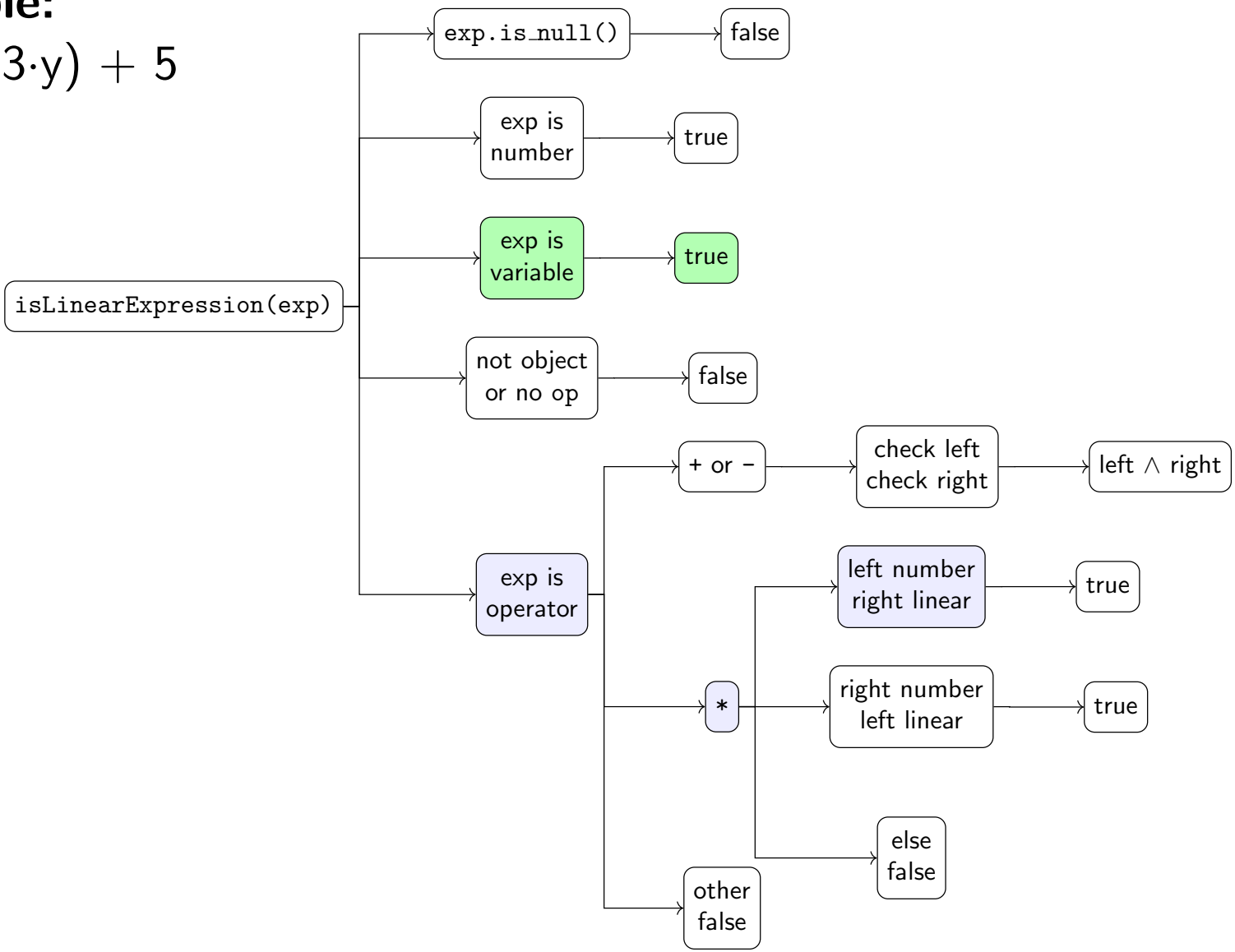

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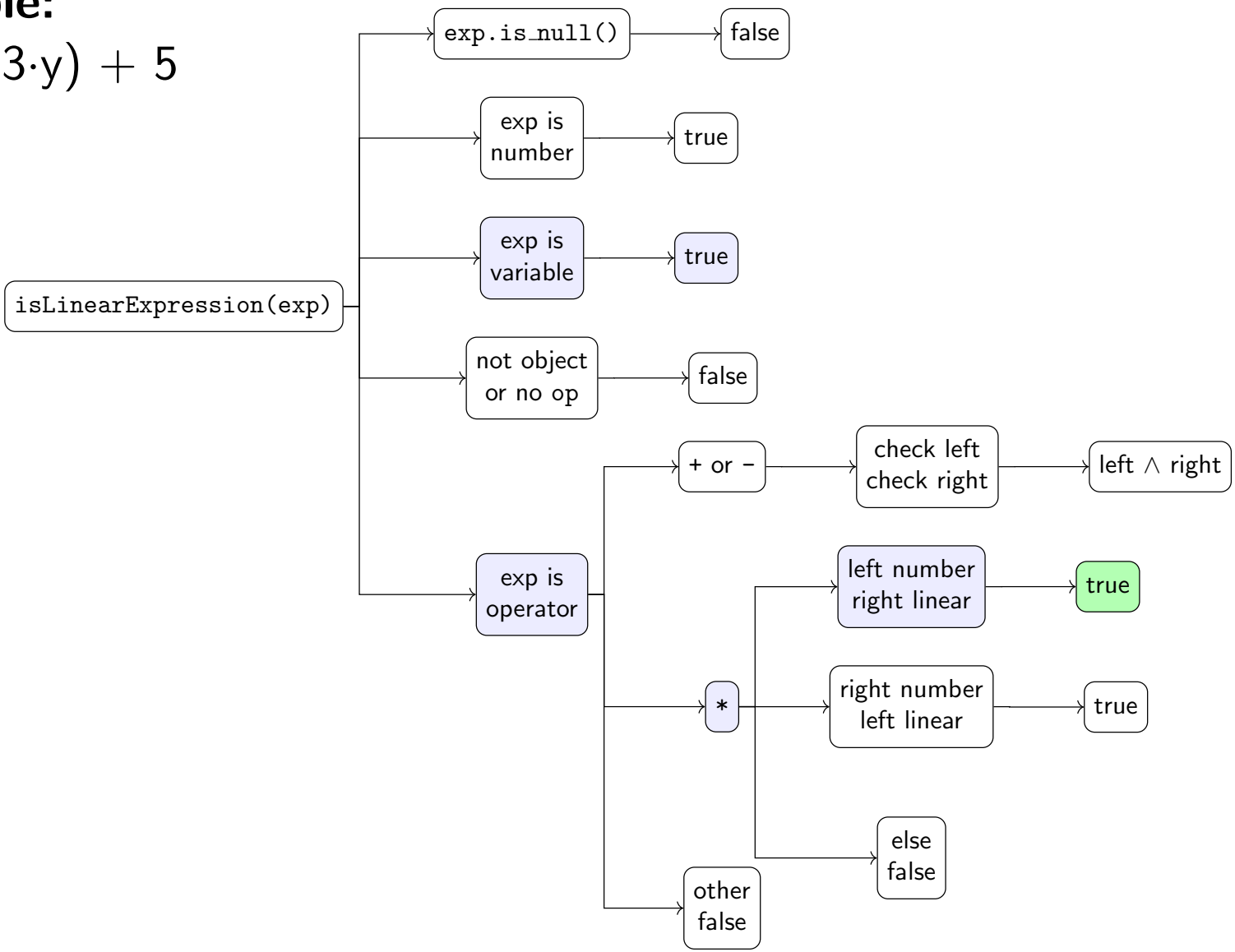

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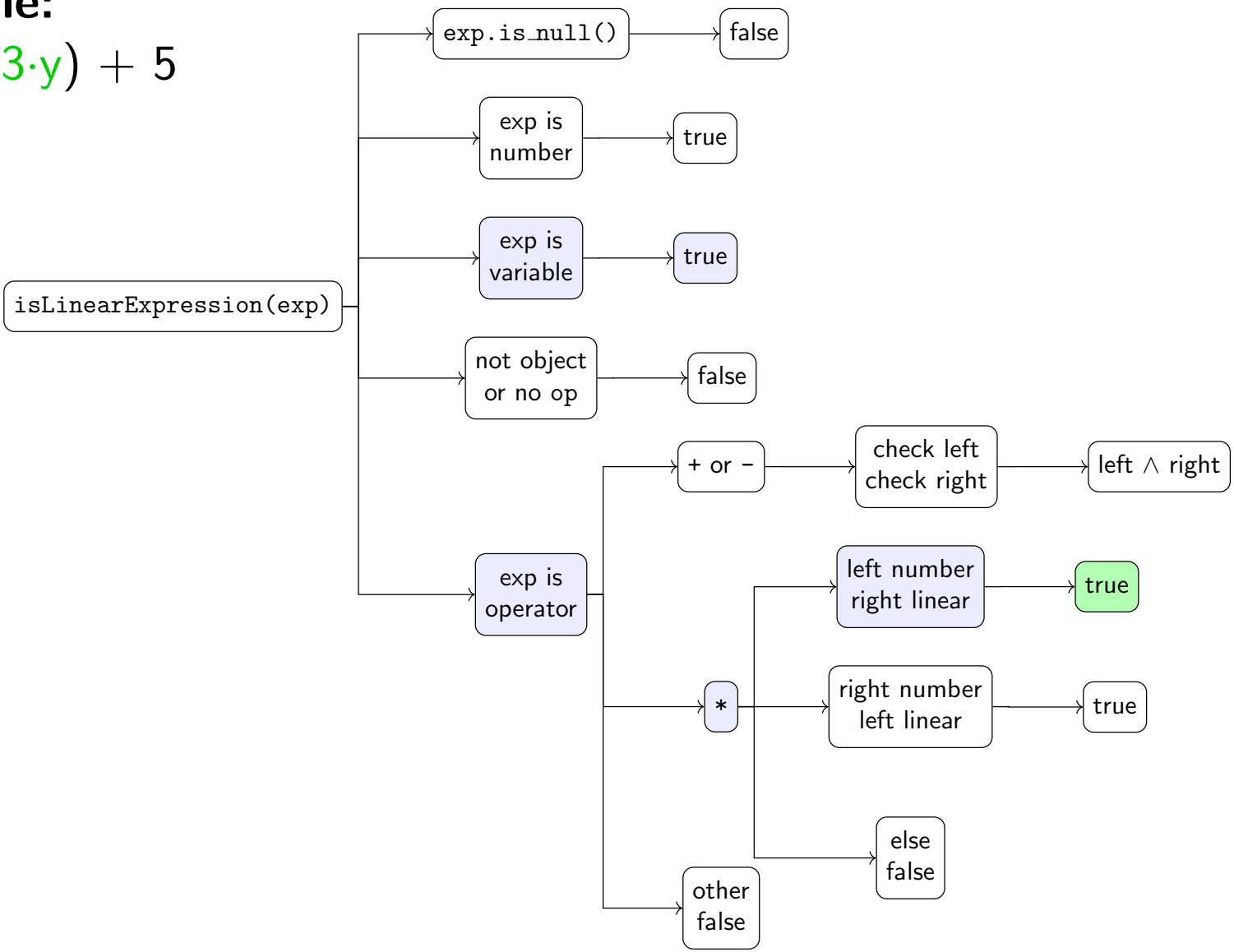
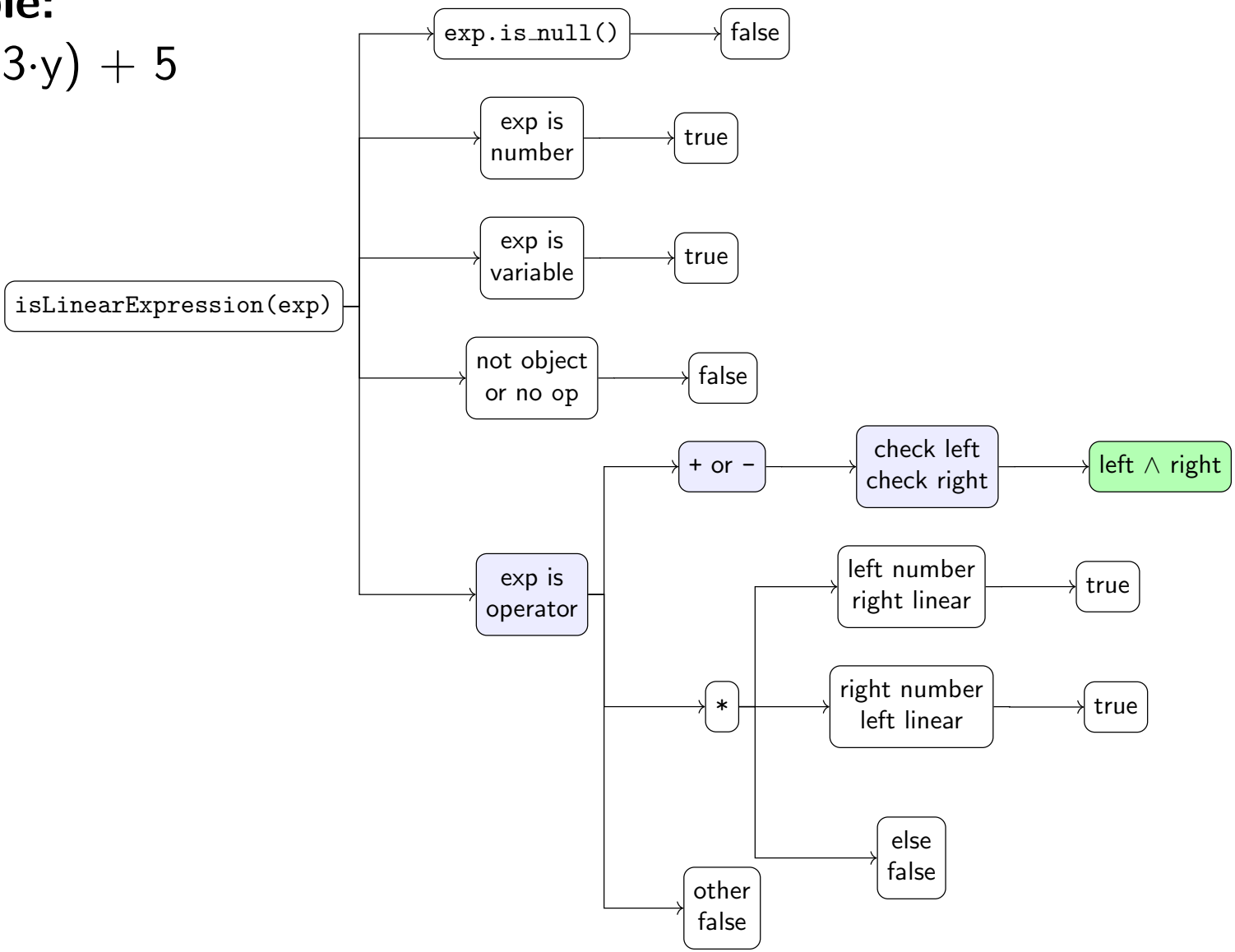


Implementation – Detecting Linear Expression

Example:

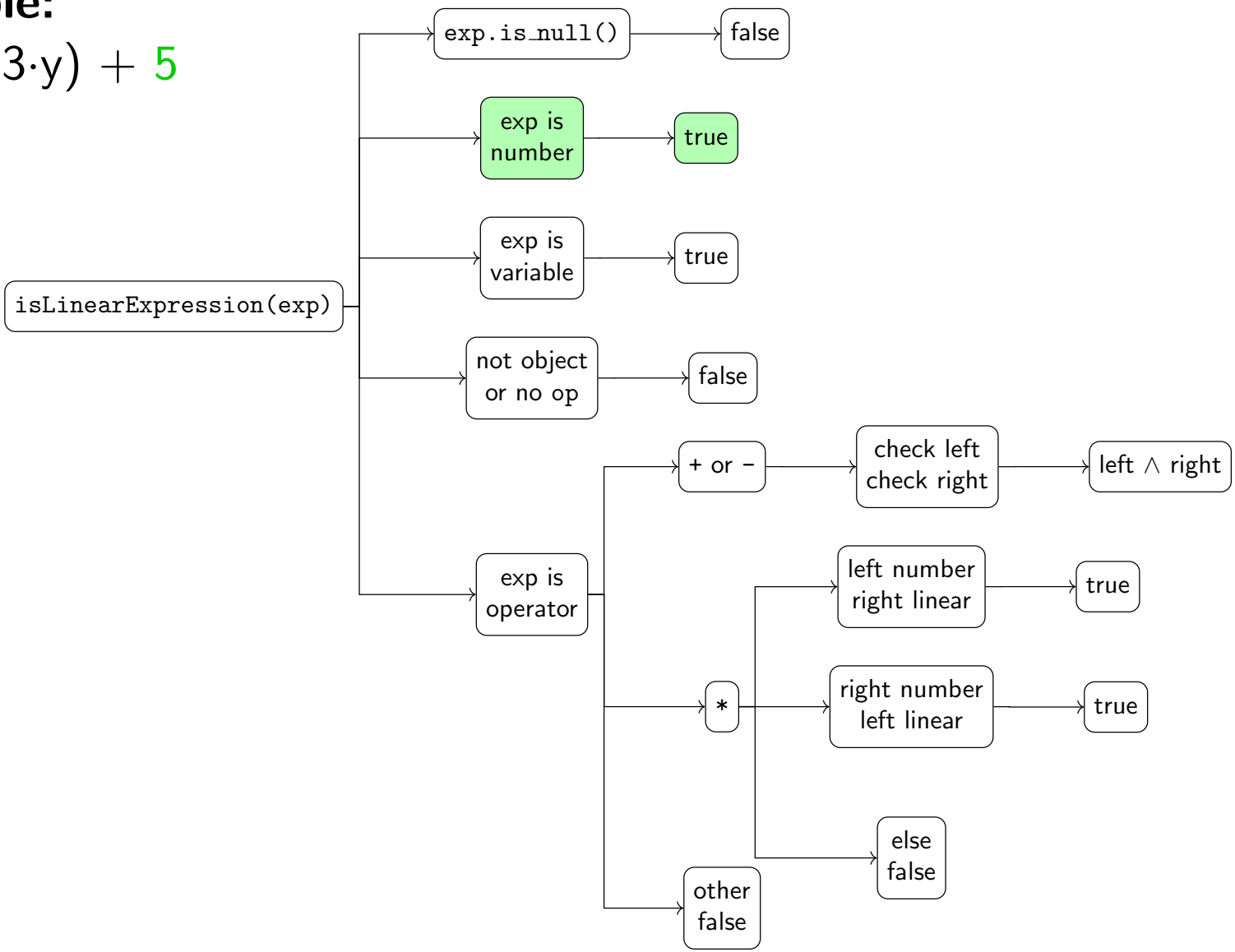
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Detecting Linear Expression

Example:

$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Detecting Linear Expression

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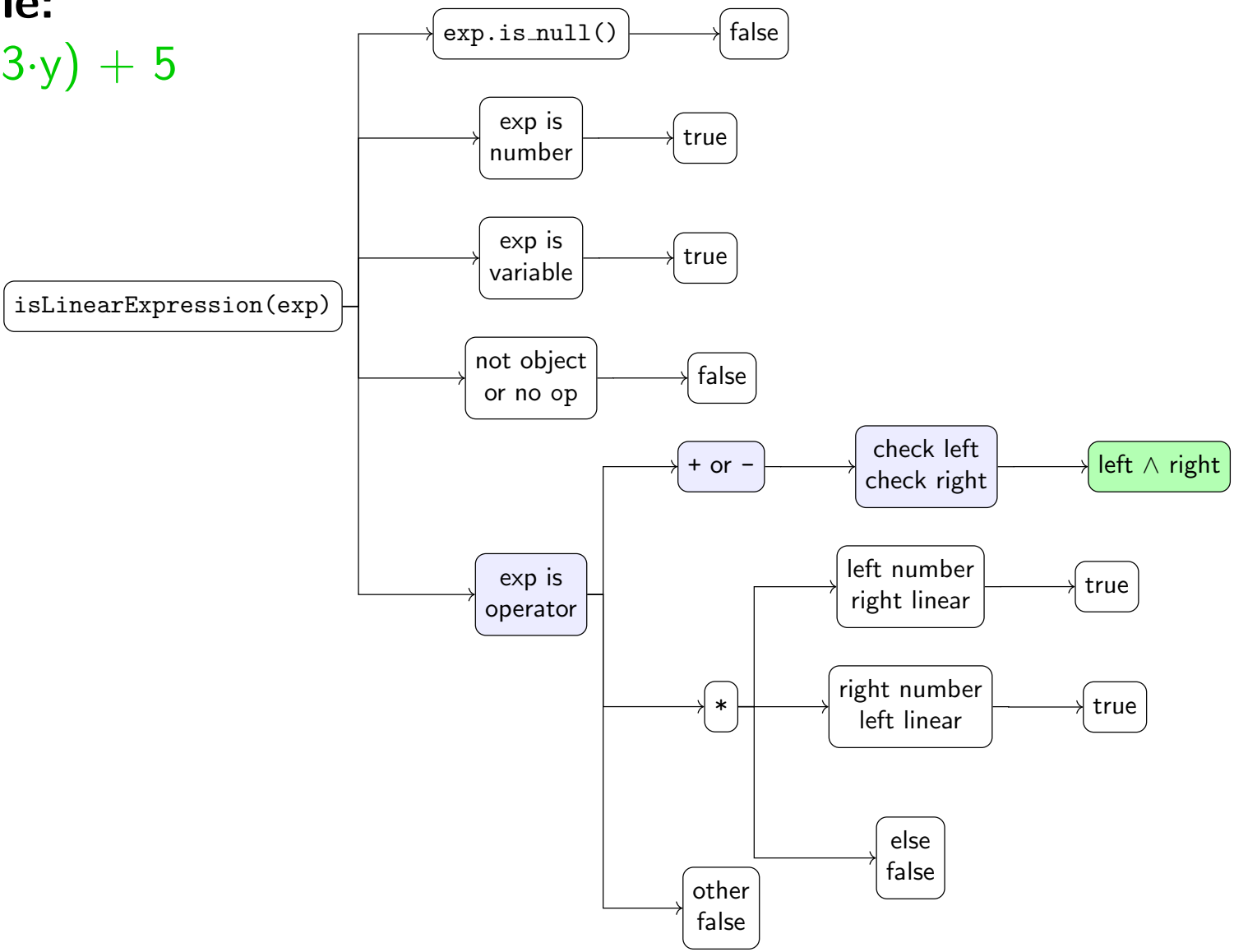
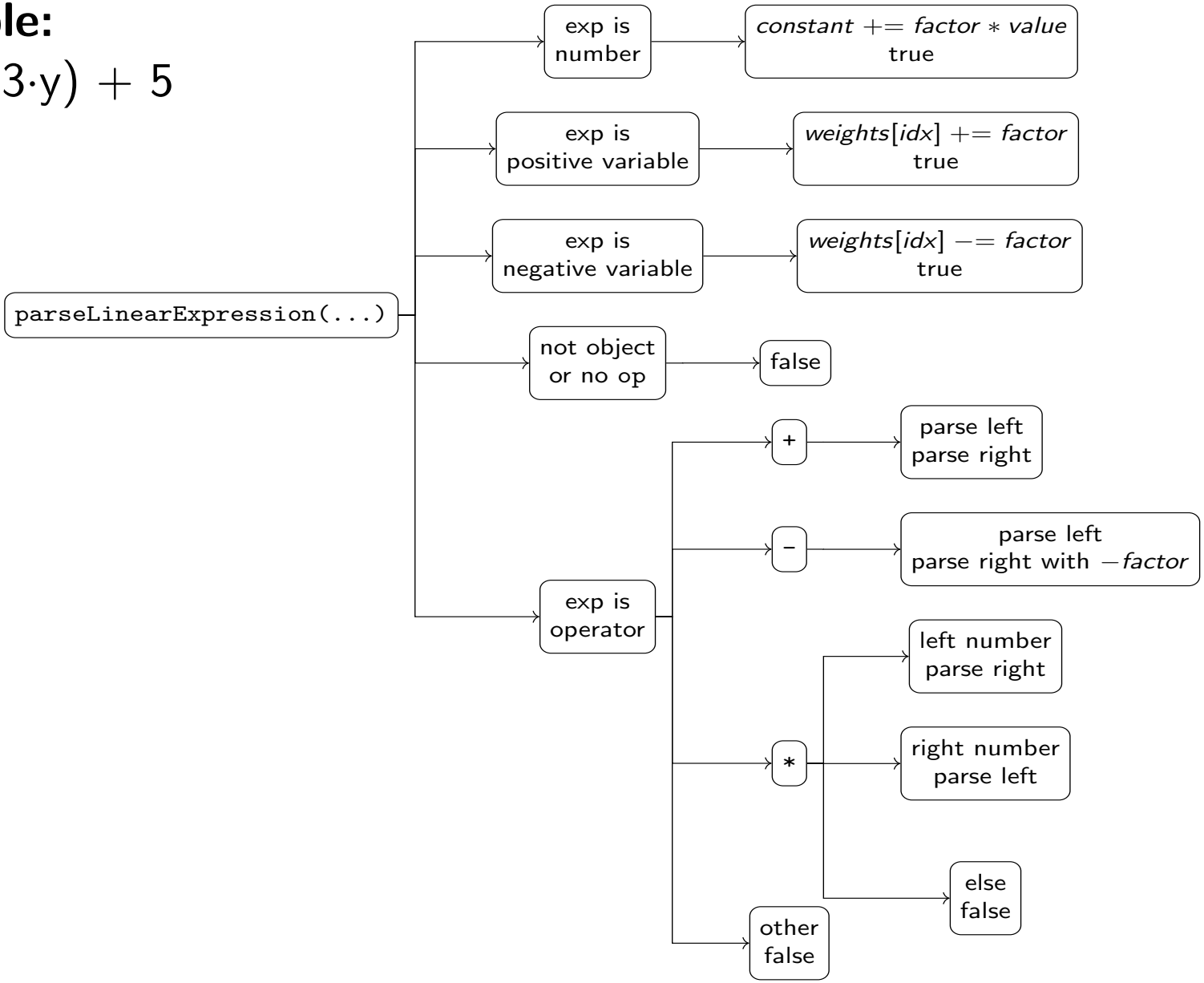
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

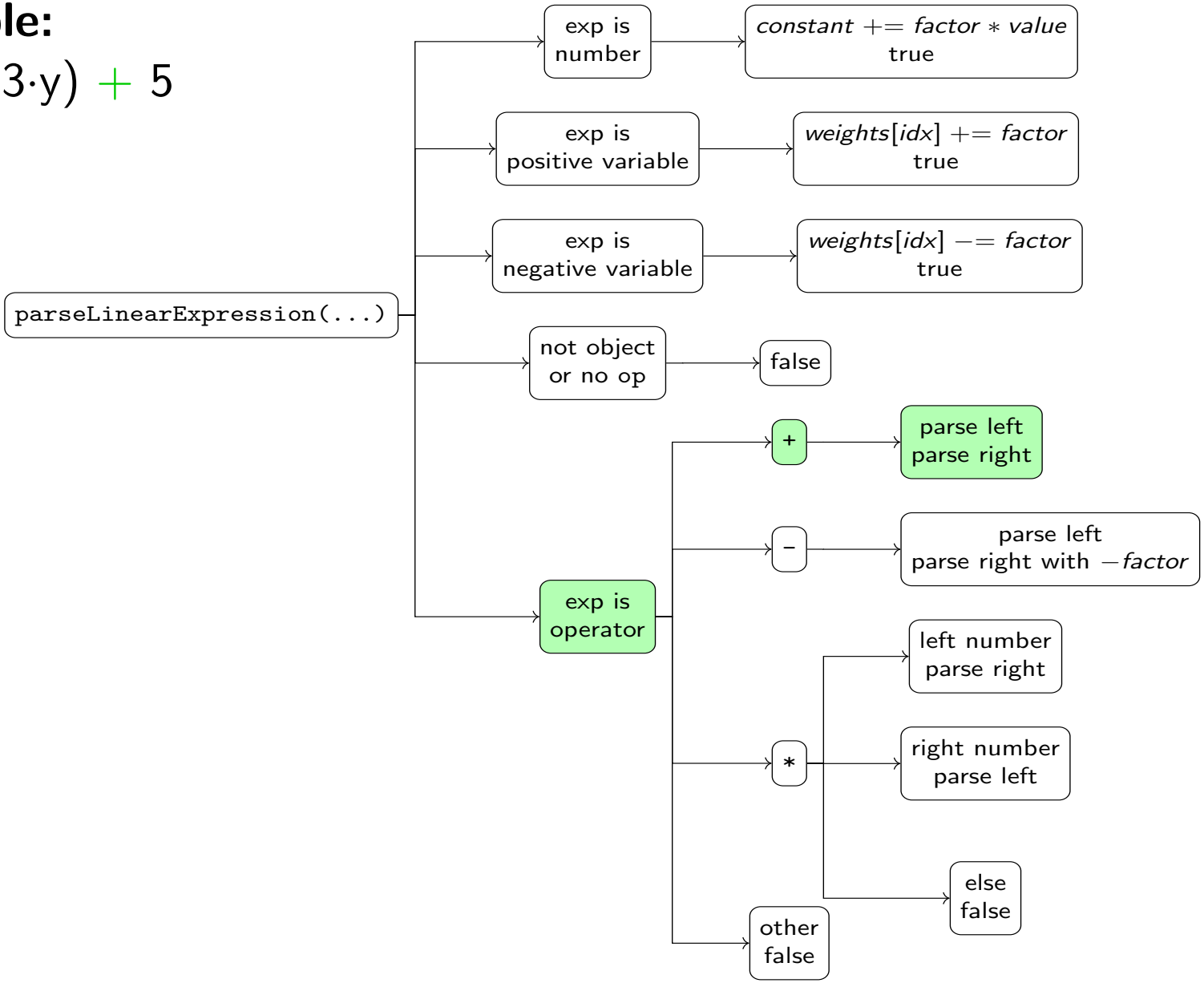
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

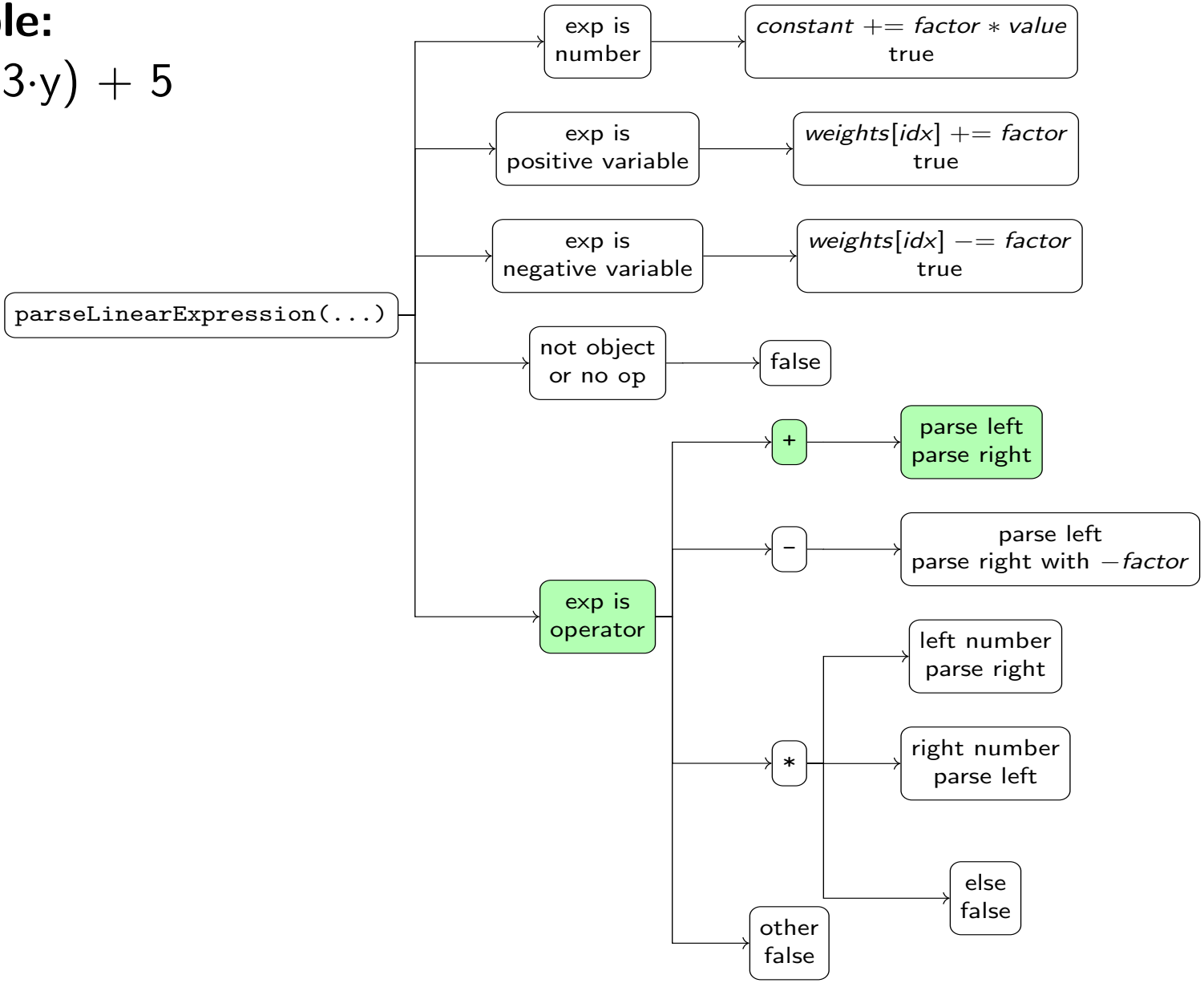
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

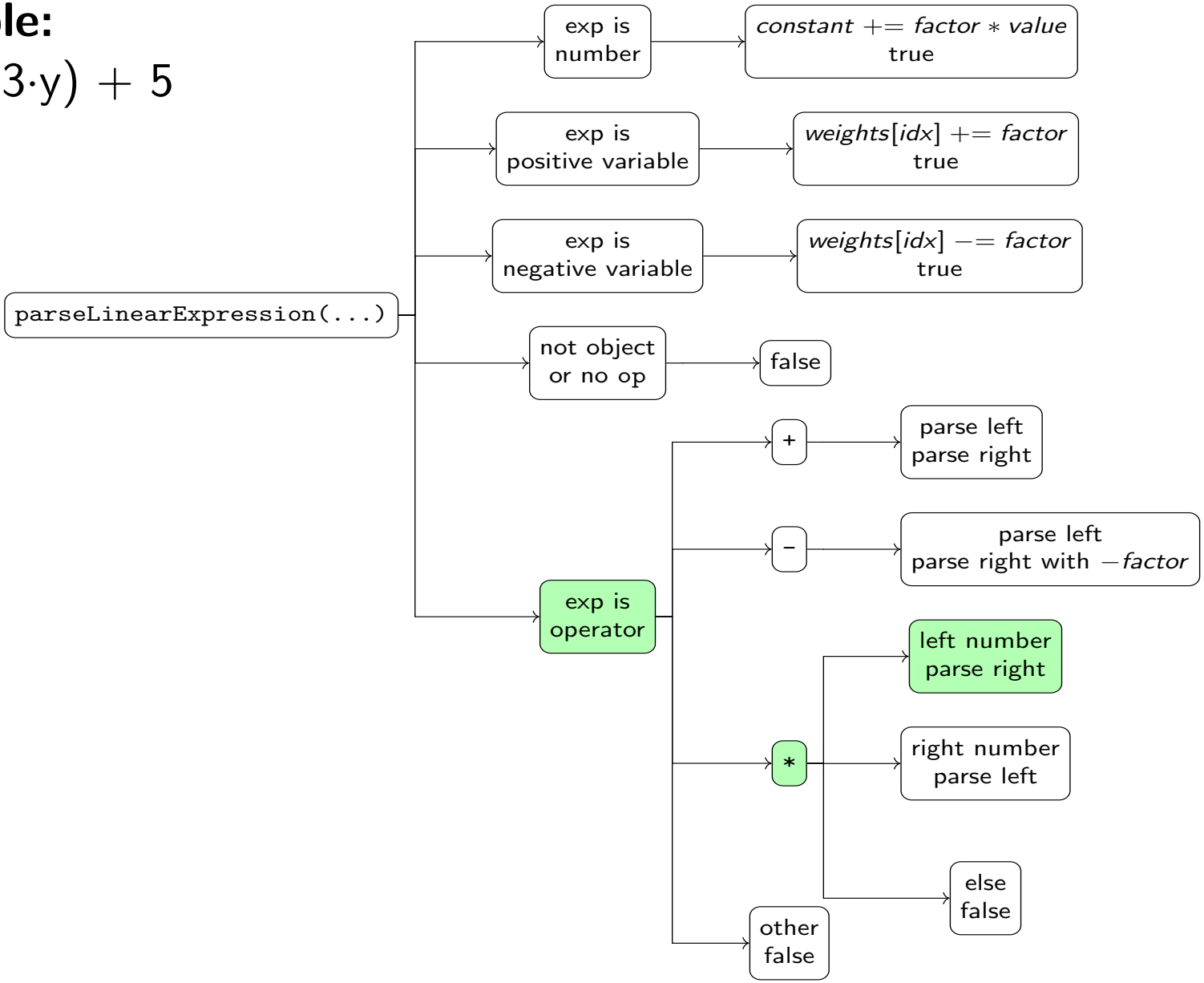
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

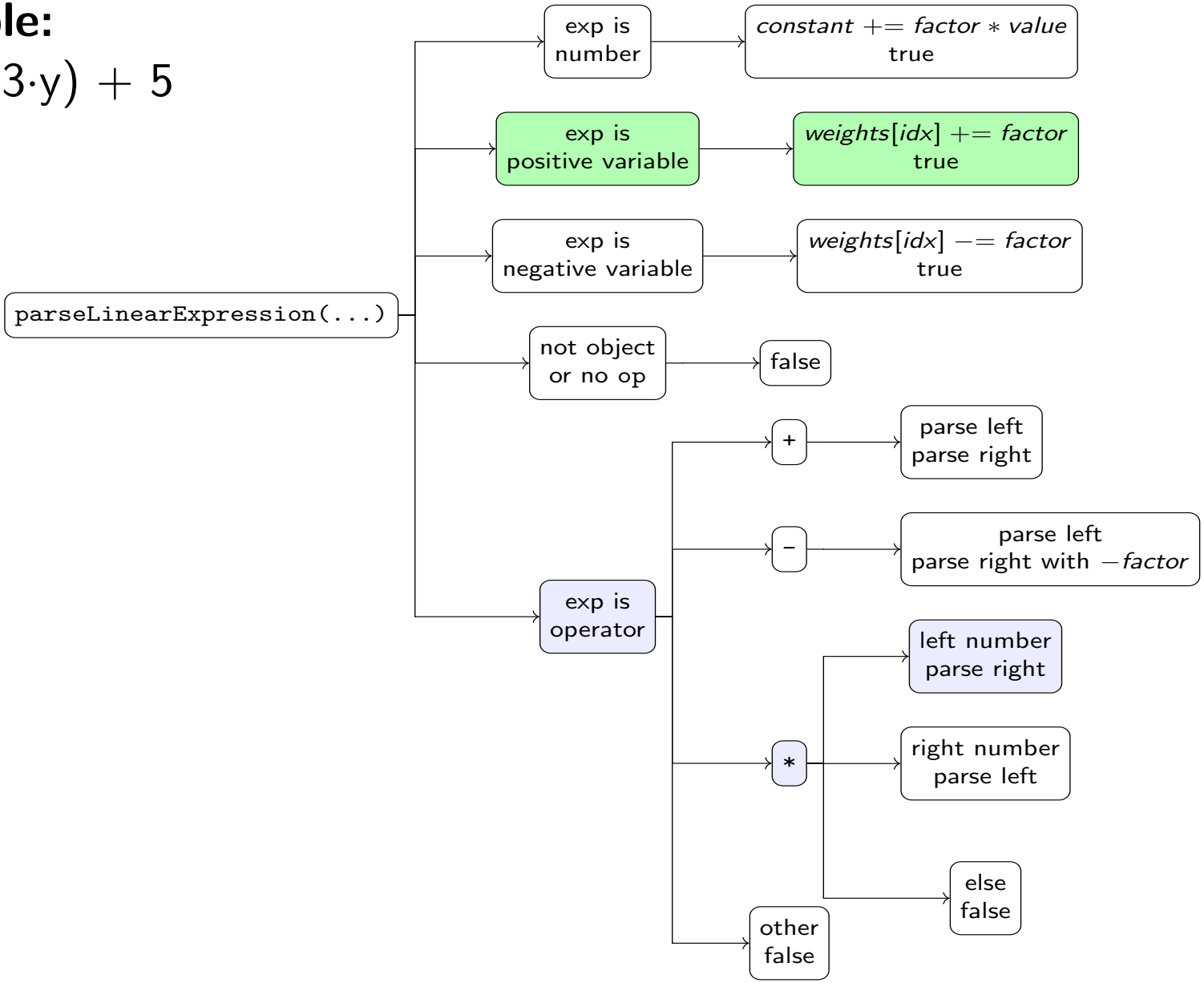
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

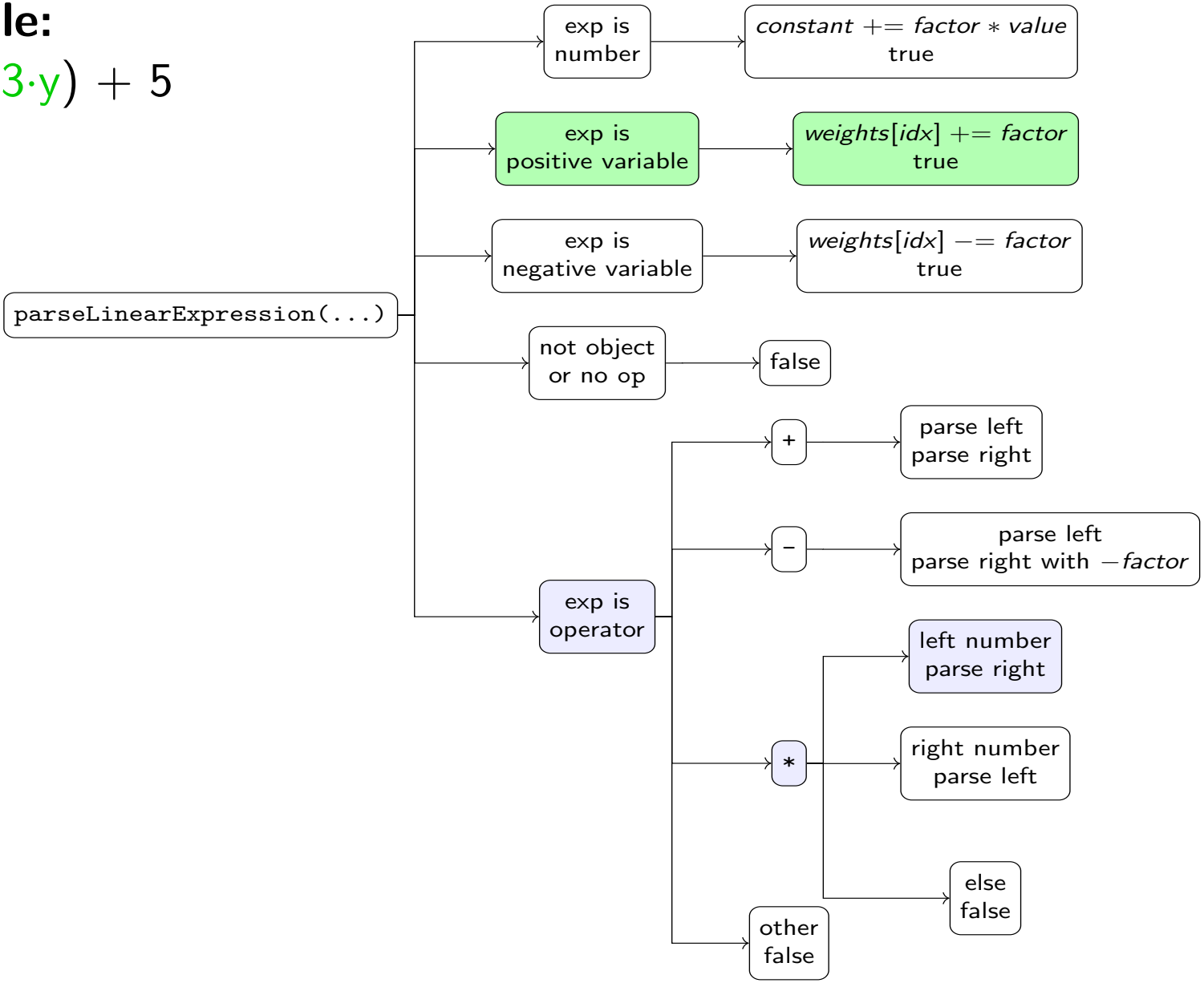
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

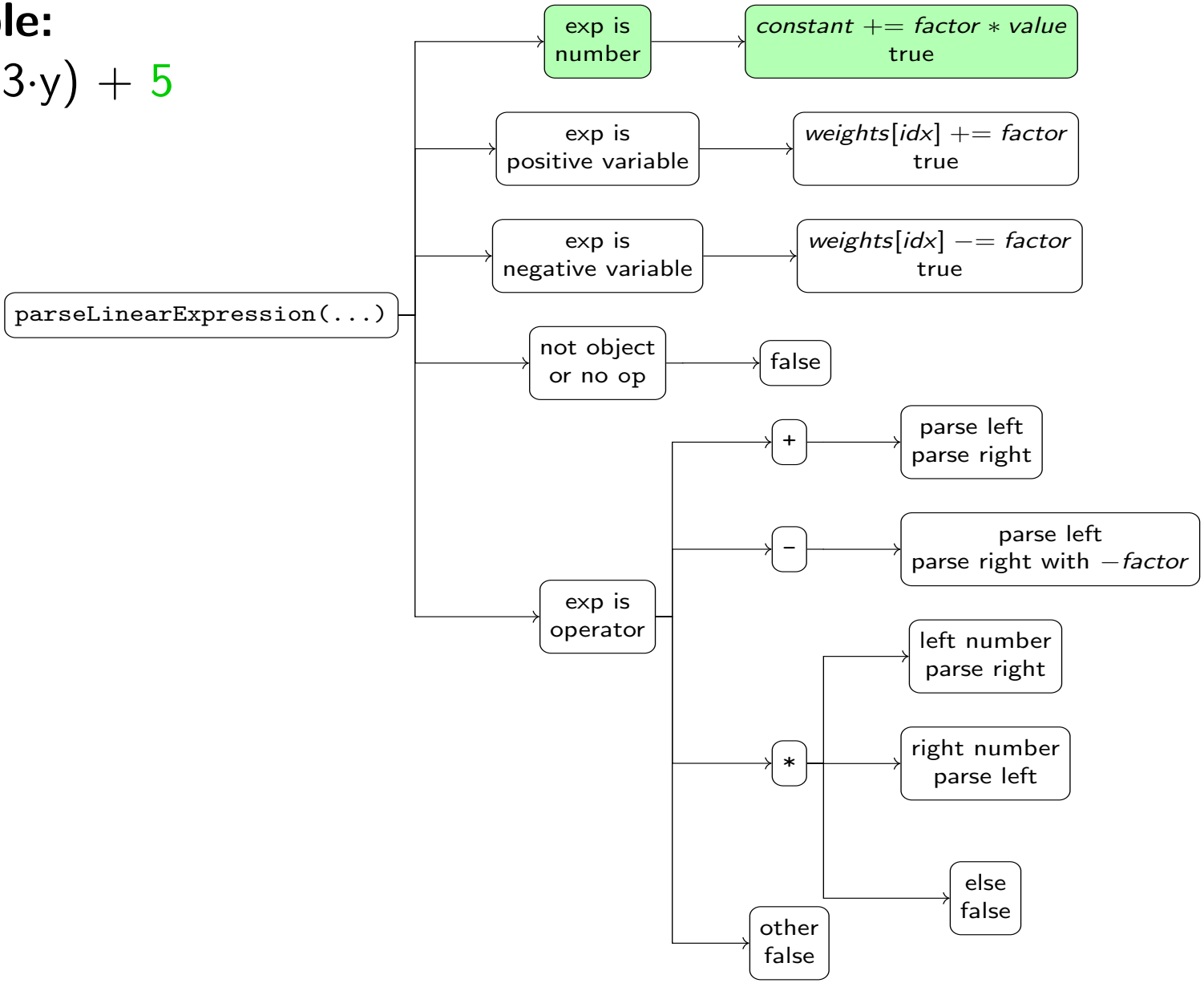
$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Parsing Linear Expression

Example:

$(2 \cdot x + 3 \cdot y) + 5$



Implementation – Saving Linear Expressions

Data Types

- `using WeightedVariables = std::map<VariableIndex, Number>;`
- `struct LinearFlow { LinearExpression linearTerms; Number constant = 0; }`
- `using GeneralFlowMap = std::variant<carl::Interval<Number>, LinearFlow>;`

Example

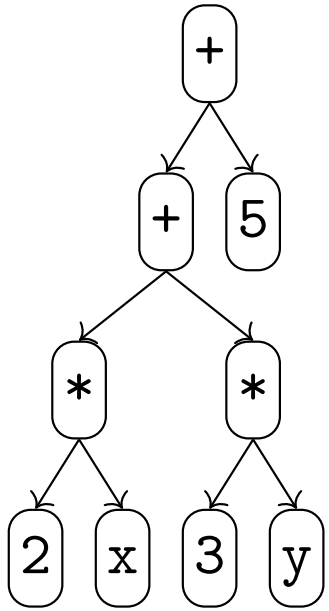
- `WeightedVariables: weights[x] = 2; weights[y] = 3;`
- `LinearFlow: flow.linearTerms = {{2,x},{3,y}}; flow.constant = 5;`
- `GeneralFlowMap: flows[z] = flow;`

Implementation – Linear Attributes

- Functions to detect and parse linear terms
- Extensions in `parseFlow()`, `parseGuards()`, `parseInvariant()` and functions for specifications
- New maps for each attribute with same structure
- `--LTIAnalysis` relevant for parseFlow()

Implementation – Creating Linear Terms

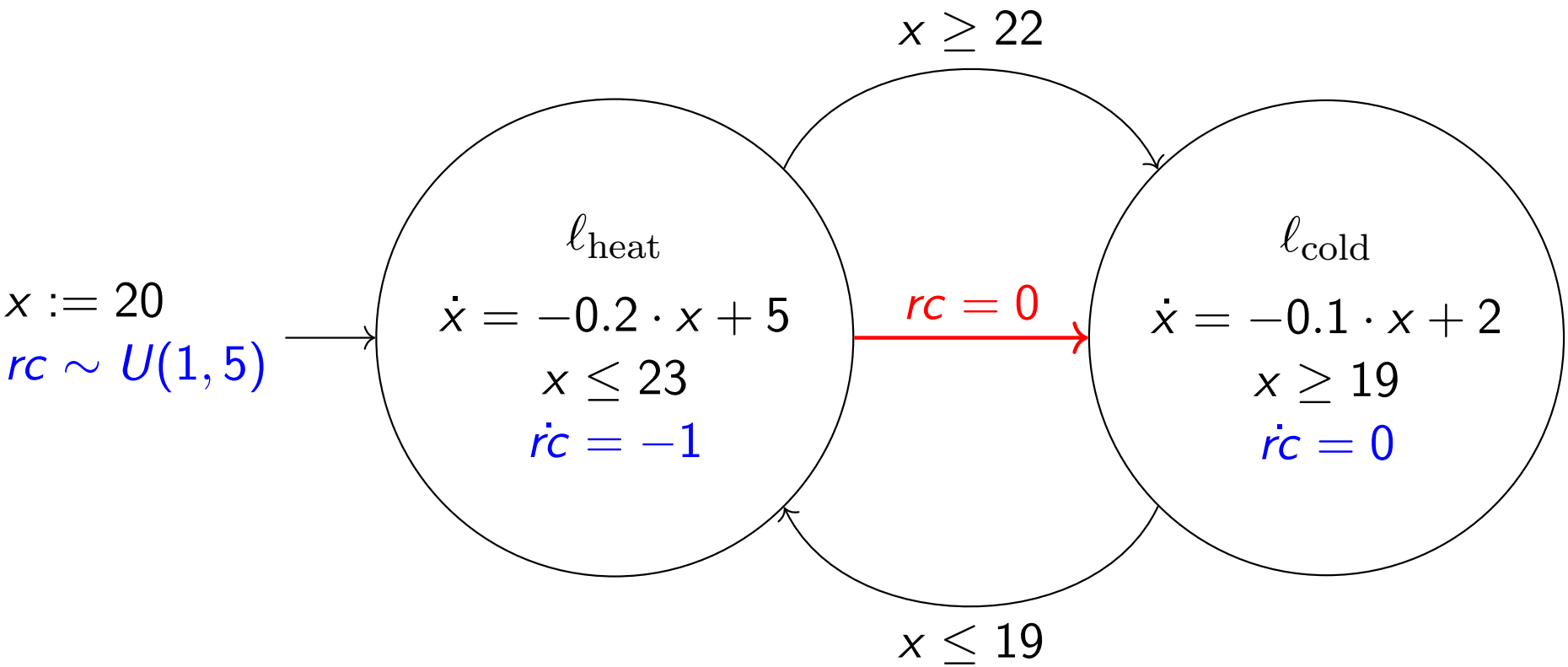
- Input: weighted variables; optional constant
- Example: $(2 \cdot x + 3 \cdot y) + 5$
- Construction process
  - create multiplication nodes $2 \cdot x$ and $3 \cdot y$
  - combine terms with $+$
  - append constant 5 with $+$ separately



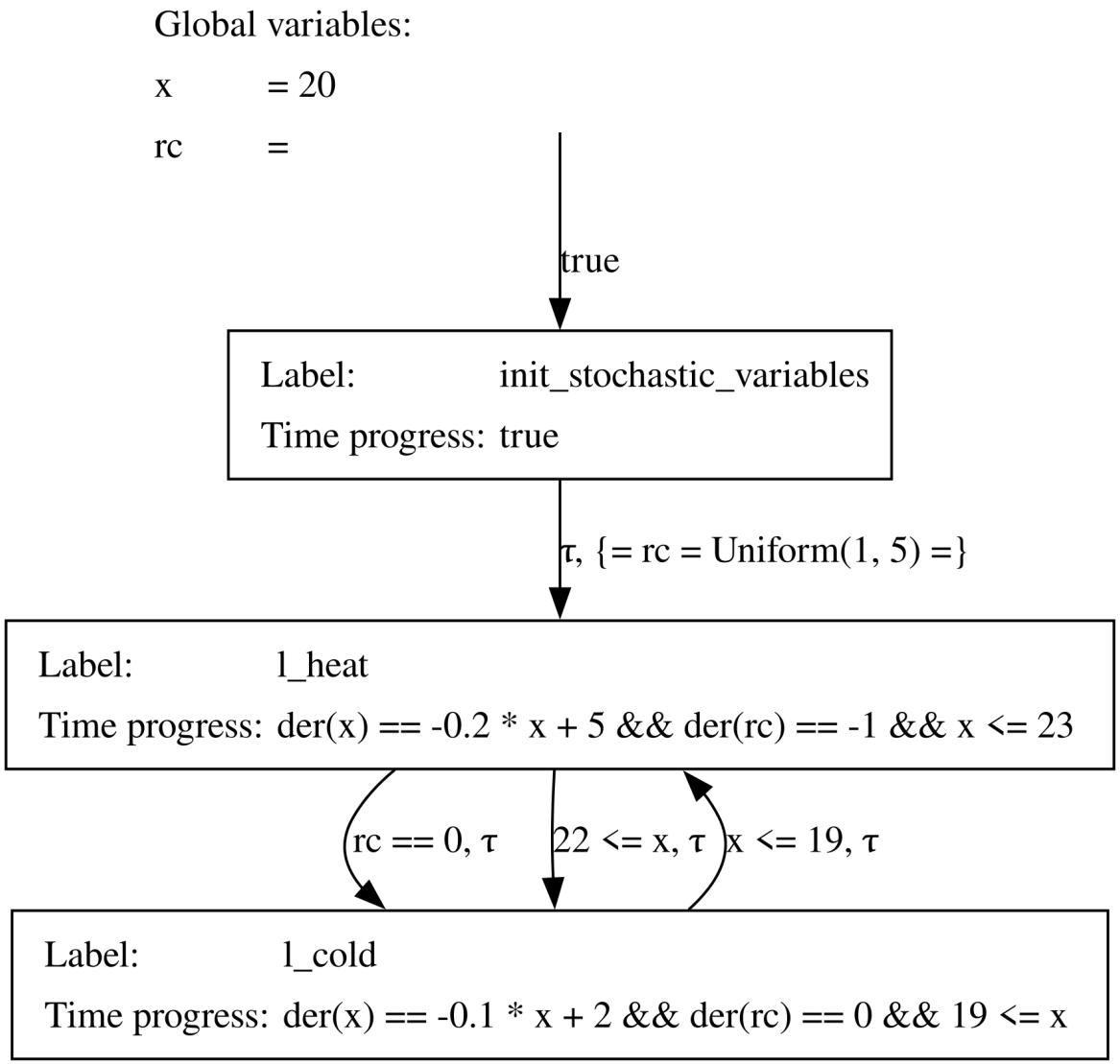
Implementation – Extension for other functionalities

- Specifications
  - Were already handled during the import
  - Added time-bounds in both directions
  - Extended for creating JANI-files
  - Support for defining goal locations for REALYST
- Resets
  - Resets were not handled in general
  - Only added support for linear resets

Plotting with Modest



Plotting with Modest



Results

- Linear Hybrid Automata can be parsed in RealySt
- Support for resets in general added
- Specifications included while creating Jani-files
- Approach of defining goal locations only compatible with RealySt
- Disjunction of specifications tested with Jani-files

Future Work

- Improve compatibility for goal locations in specifications
- Handling the $\vee$-operator
- Improve handling for multiple automata
- Further testing the parser with other tools