Modeling and Analysis of Hybrid Systems

1. Preliminaries

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Informatik 2 - LuFG Theory of Hybrid Systems
RWTH Aachen University

Szeged, Hungary, 27 September - 06 October 2017

Organizational

- email: abraham@cs.rwth-aachen.de
- 7 x 90 minutes lectures, 7 x 90 minutes exercises
- agree on time and place
- learning materials
- written exam mid of November 2017

Contents

"Hybrid"

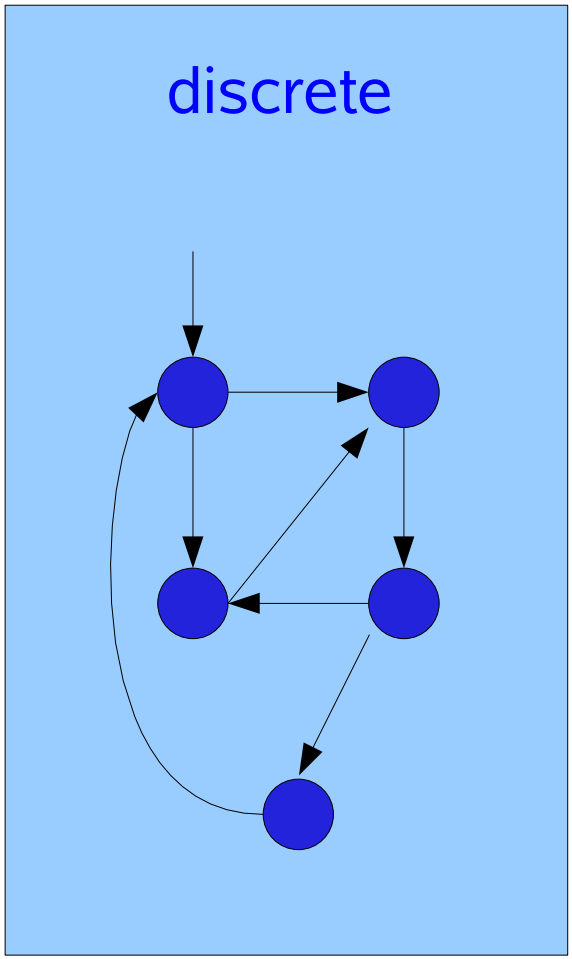
Wikipedia:

> "A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal."

A hybrid rose

A hybrid car

Hybrid in computer science



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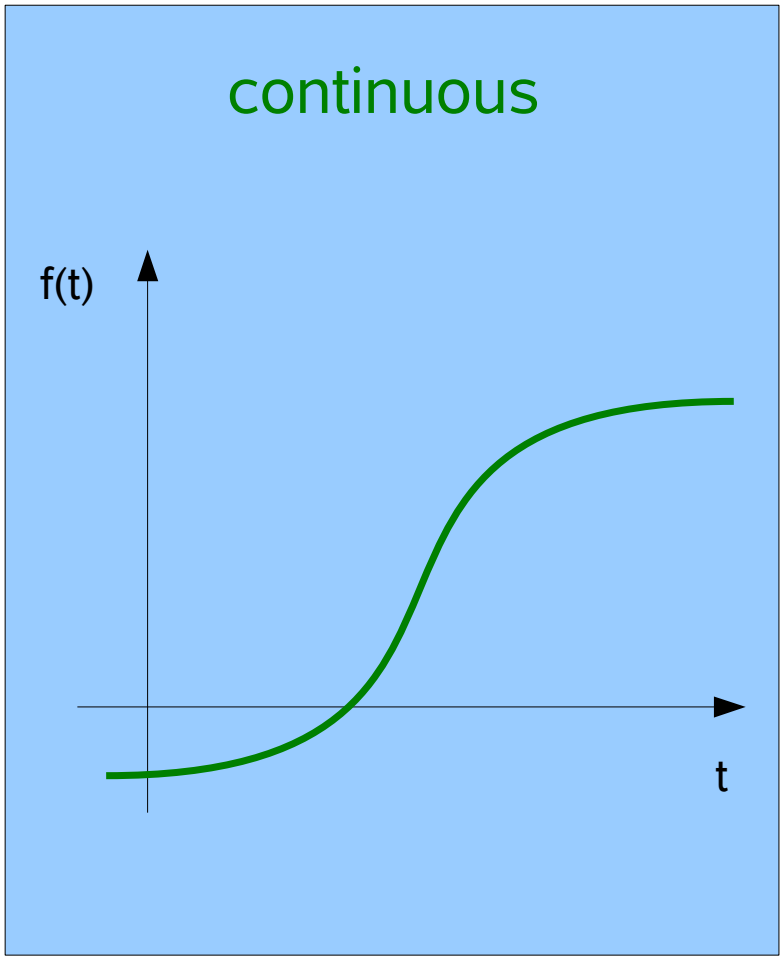
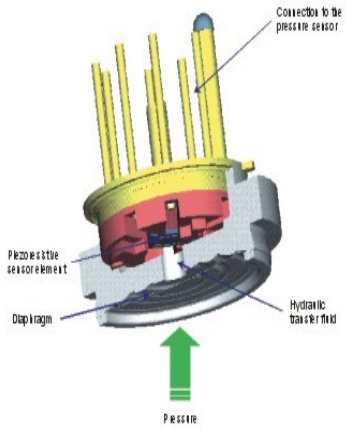
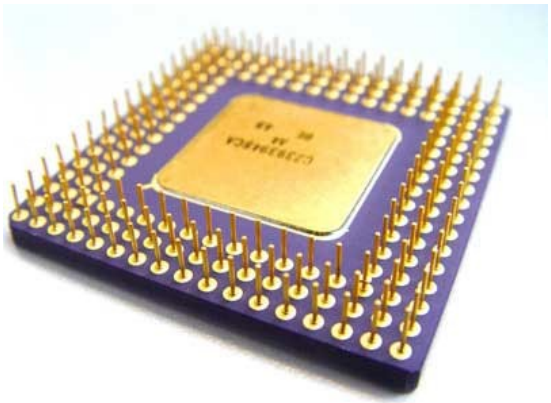


The discrete part





Combined with the continuous part

Example: Bouncing ball

Ball falls from a given height, bounces at the ground, raises, falls again...

- vertical position of the ball x_1
- velocity x_2

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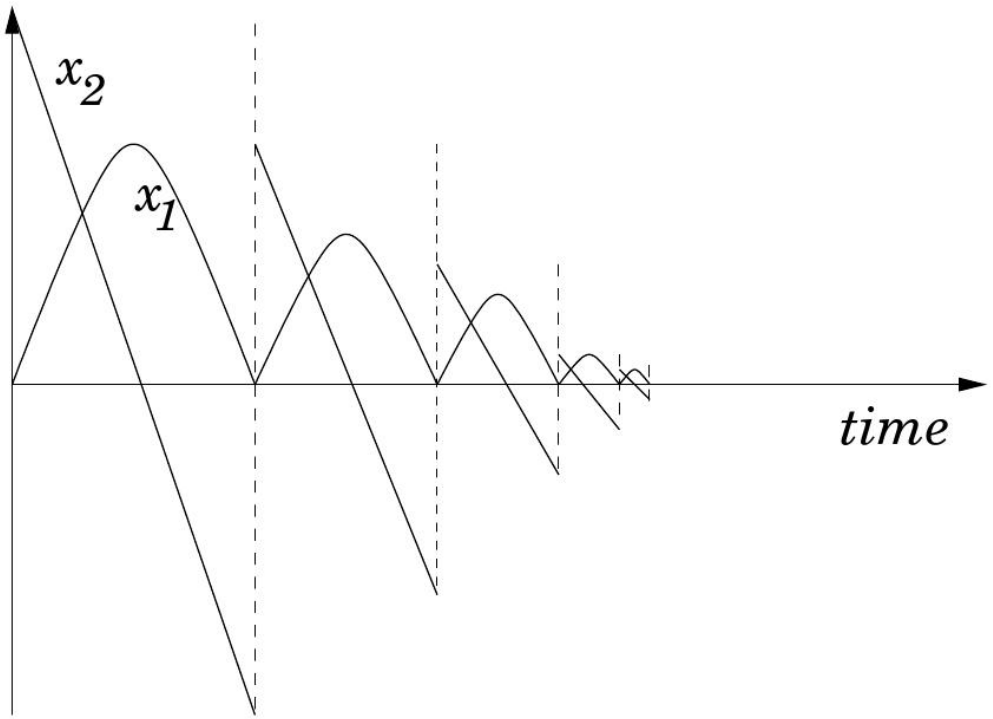

Example: Thermostat

- Temperature x is controlled by switching a heater on and off
- x is regulated by a thermostat:
  - $17° \leq x \leq 18° \rightsquigarrow$ "heater on"
  - $22° \leq x \leq 23° \rightsquigarrow$ "heater off"

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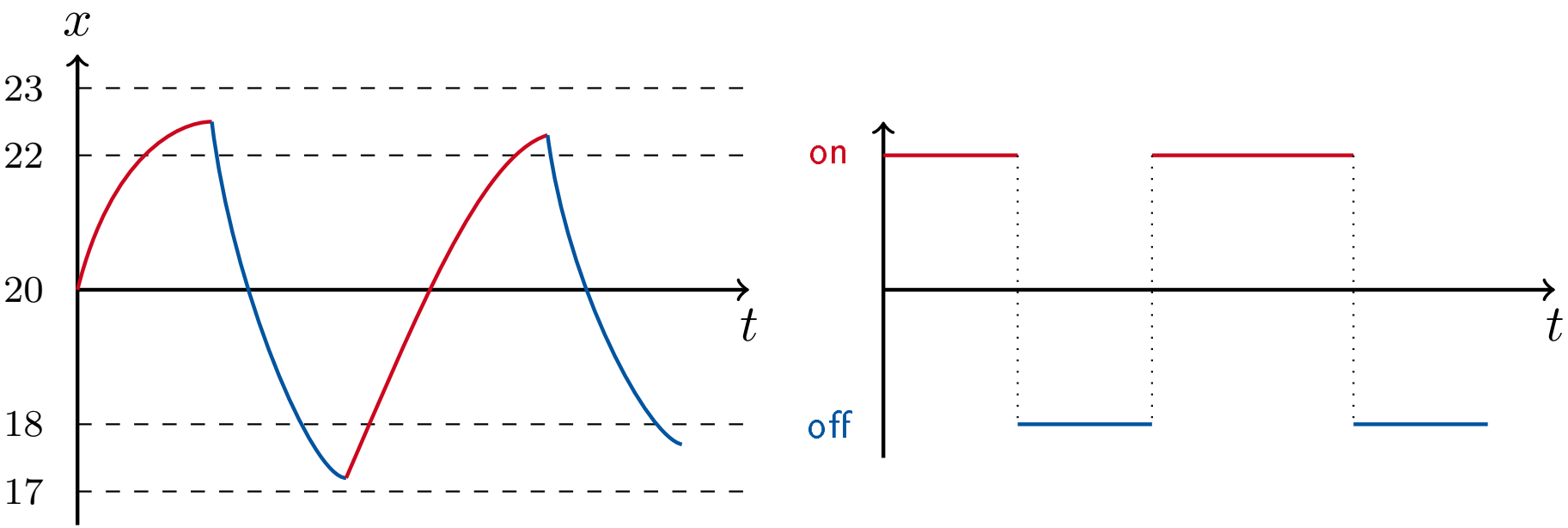
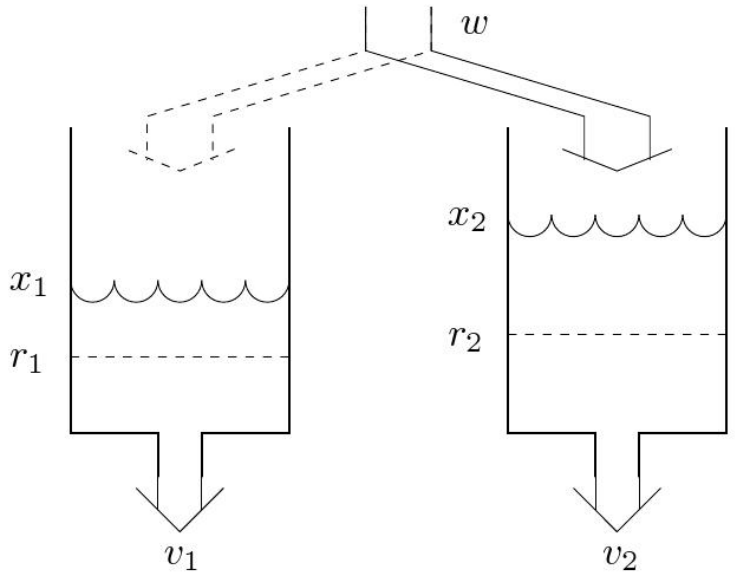


Example: Water tank system

- two constantly leaking tanks v_1 and v_2
- hose w refills exactly one tank at one point in time
- w can switch between tanks instantaneously



There are much more complex examples of hybrid systems, like e.g.

- automobiles, trains, etc.
- automated highway systems
- collision-avoidance and free flight for aircrafts
- digitally controlled chemical plants
- biological cell growth and division ...

In this course we learn how to model and analyse hybrid systems, considering a sequence of modeling languages with increasing expressive power.

- labeled state transition systems
- labeled transition systems
- timed automata
- initialized rectangular automata
- linear hybrid automata I
- linear hybrid automata II

Contents

Labeled state transition systems

Definition

Labeled state transition systems

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A labeled state transition system (LSTS) is a tuple $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ with

- a (possibly infinite) state set Σ,
- a label set Lab (for synchronisation, we do not use it in this course),
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Pedestrian light

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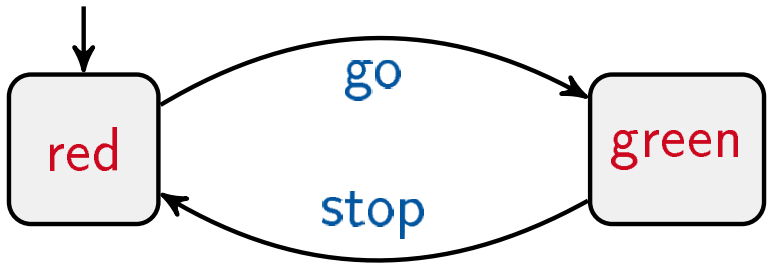


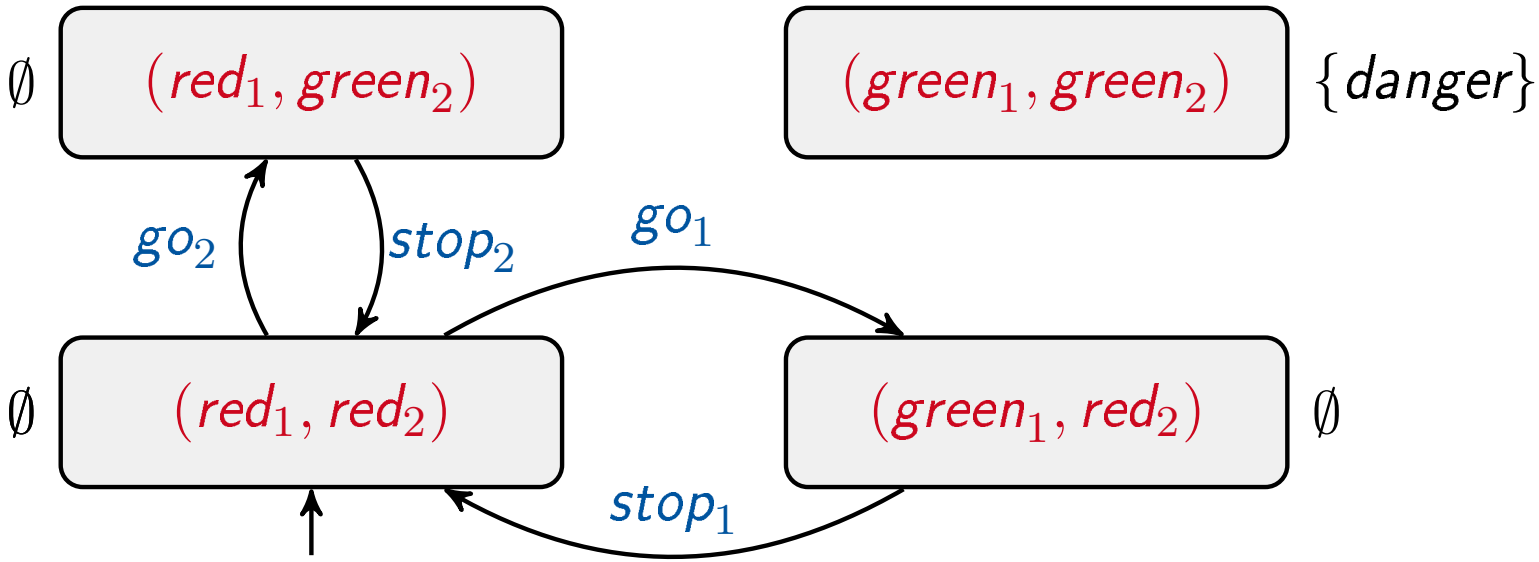
Labeling

To be able to formalize properties of LSTSs, it is common to define

- a set of atomic propositions AP and
- a state labeling function $L : \Sigma \to 2^{AP}$ assigning a set of atomic propositions to each state.

The set $L(\sigma)$ consists of all propositions that are defined to hold in σ. These propositional labels on states should not be mixed up with the synchronization labels on edges.

Two traffic lights



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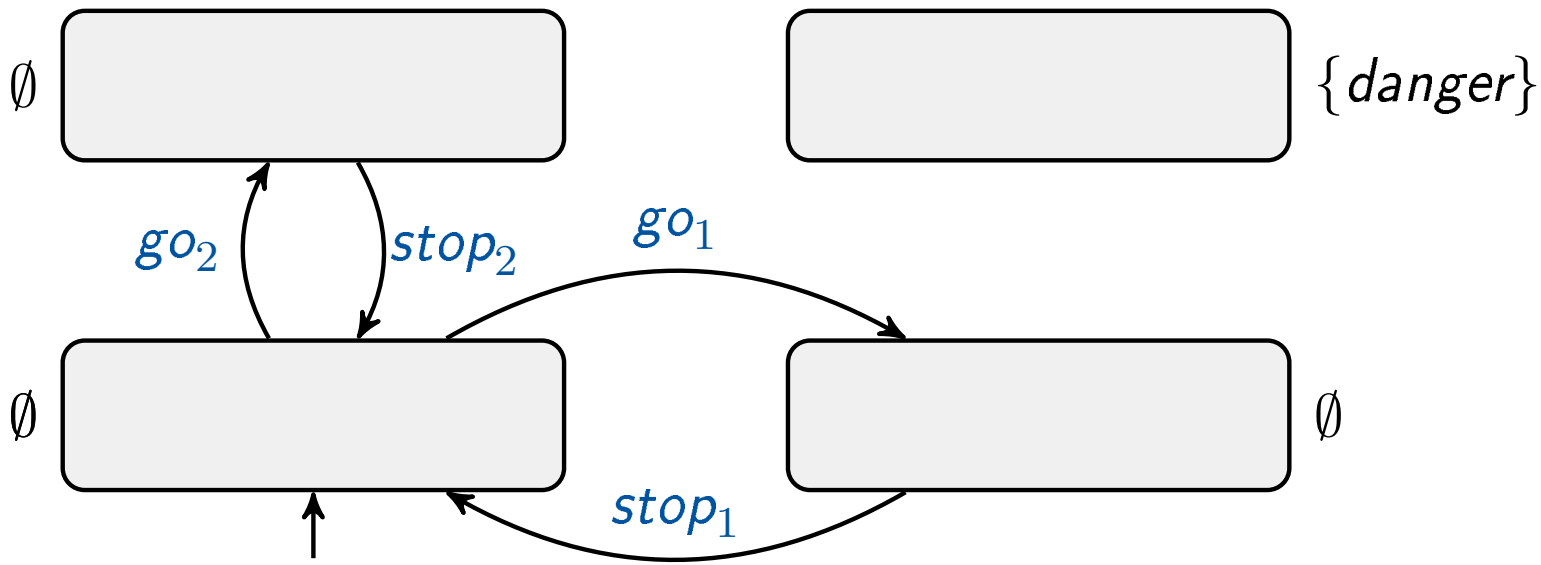

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A labeled transition system (LTS) is a tuple $\mathcal{LTS} = (Loc, Var, Lab, Edge, Init)$ with

- finite set of locations Loc,
- finite set of (typed) variables Var,
- finite set of synchronization labels Lab, $\tau \in Lab$ (stutter label)
- finite set of edges $Edge \subseteq Loc \times Lab \times 2^{V^2} \times Loc$ (including stutter transitions (l, τ, μ_τ, l) for each location $l \in Loc$),
- initial states $Init \subseteq \Sigma$.

with

- valuations $\nu : Var \to Domain$, V is the set of valuations
- state $\sigma = (l, \nu) \in Loc \times V$, Σ is the set of states

Semantics of LTS

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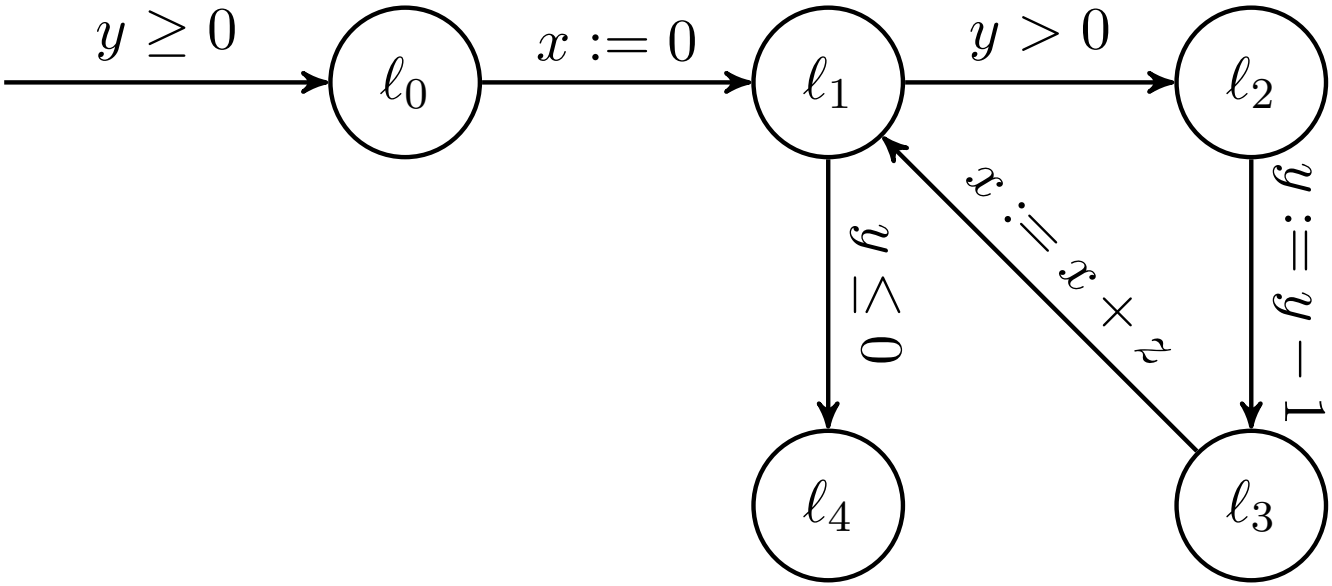
LSTS semantics of LTS

Each LTS $\mathcal{LTS} = (Loc, Var, Lab, Edge, Init)$ induces a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge', Init)$ with

- $\Sigma = Loc \times V$ and
- $Edge' = \{(\nu, a, \nu') \mid \nu \xrightarrow{a} \nu'\}$.

Semantics of the simple while-program



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Modeling a simple while-program

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     method mult(int y, int z){
       int x;
ℓ0     x := 0;
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       while( y > 0 ) {
ℓ2       y := y-1;
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ℓ4   }
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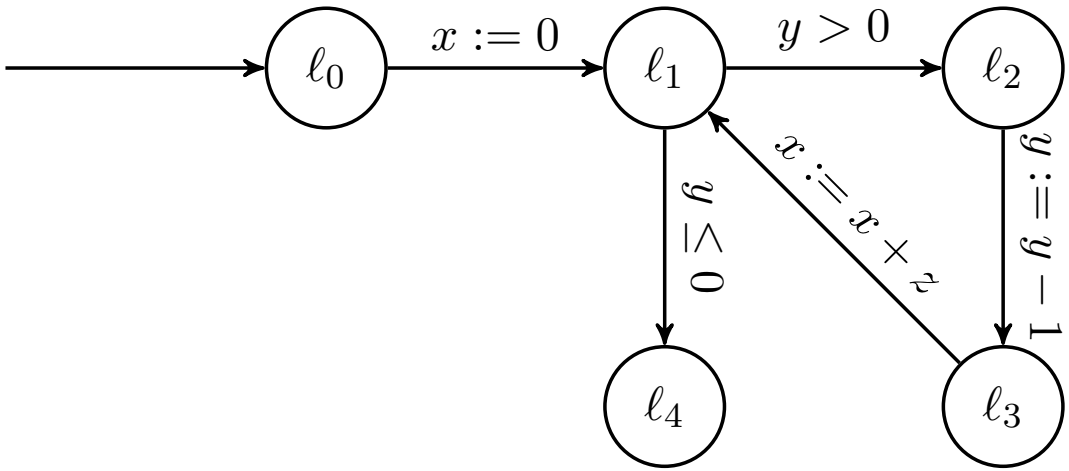

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Temporal logics

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- How can we describe properties of this system?
- We need a well-suited logic.

Propositional logic

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$$\varphi \ ::= \ a \mid (\varphi \wedge \varphi) \mid (\neg\varphi)$$

  with $a \in AP$.
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- Syntactic sugar: *true*, *false*, $\vee, \rightarrow, \leftrightarrow, \ldots$
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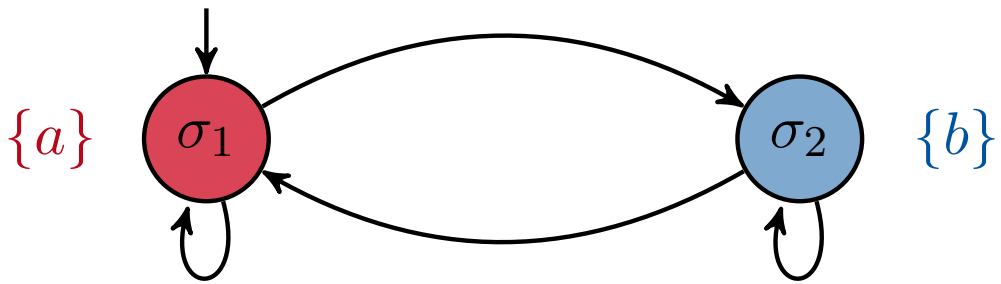
- Semantics (in the context of a state $\sigma \in \Sigma$):

$$\begin{array}{lll} \sigma \models a & \textit{iff} & a \in L(\sigma), \\ \sigma \models (\varphi_1 \wedge \varphi_2) & \textit{iff} & \sigma \models \varphi_1 \textit{ and } \sigma \models \varphi_2, \\ \sigma \models (\neg\varphi) & \textit{iff} & \sigma \not\models \varphi. \end{array}$$

Computation tree



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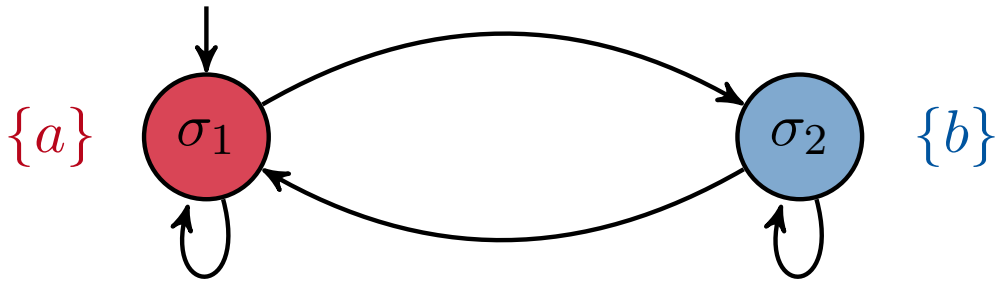
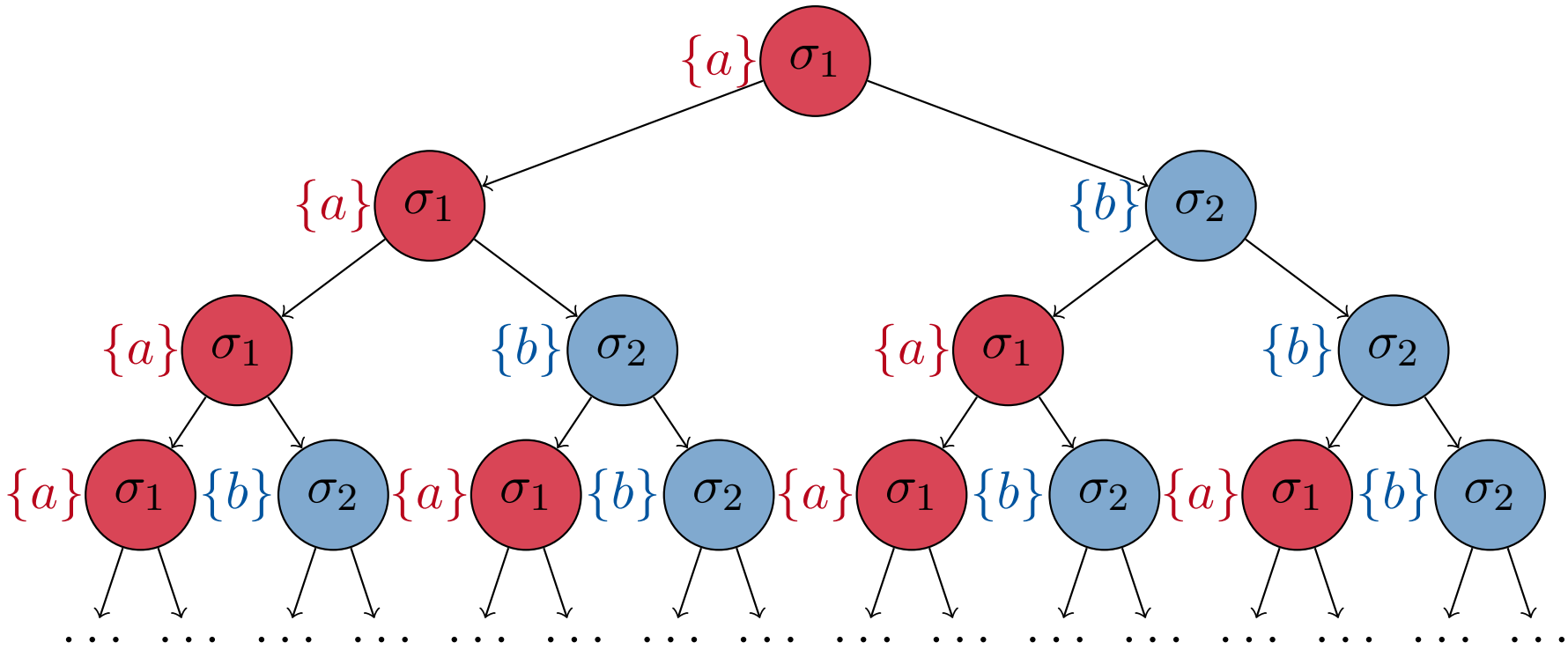
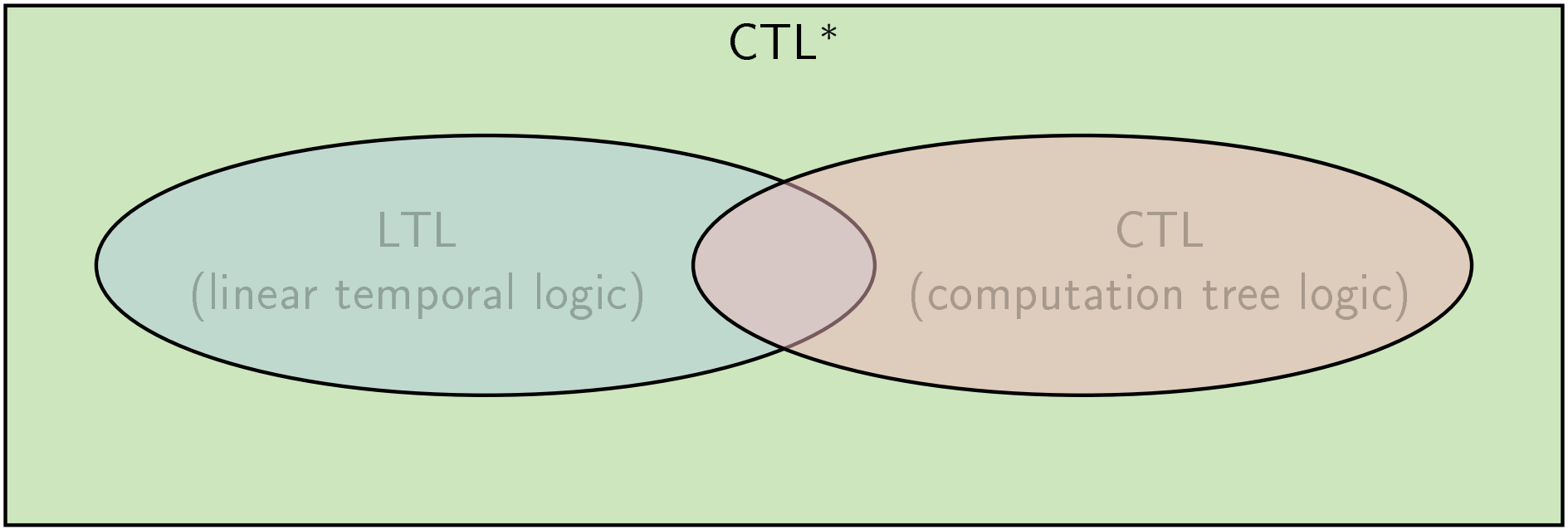




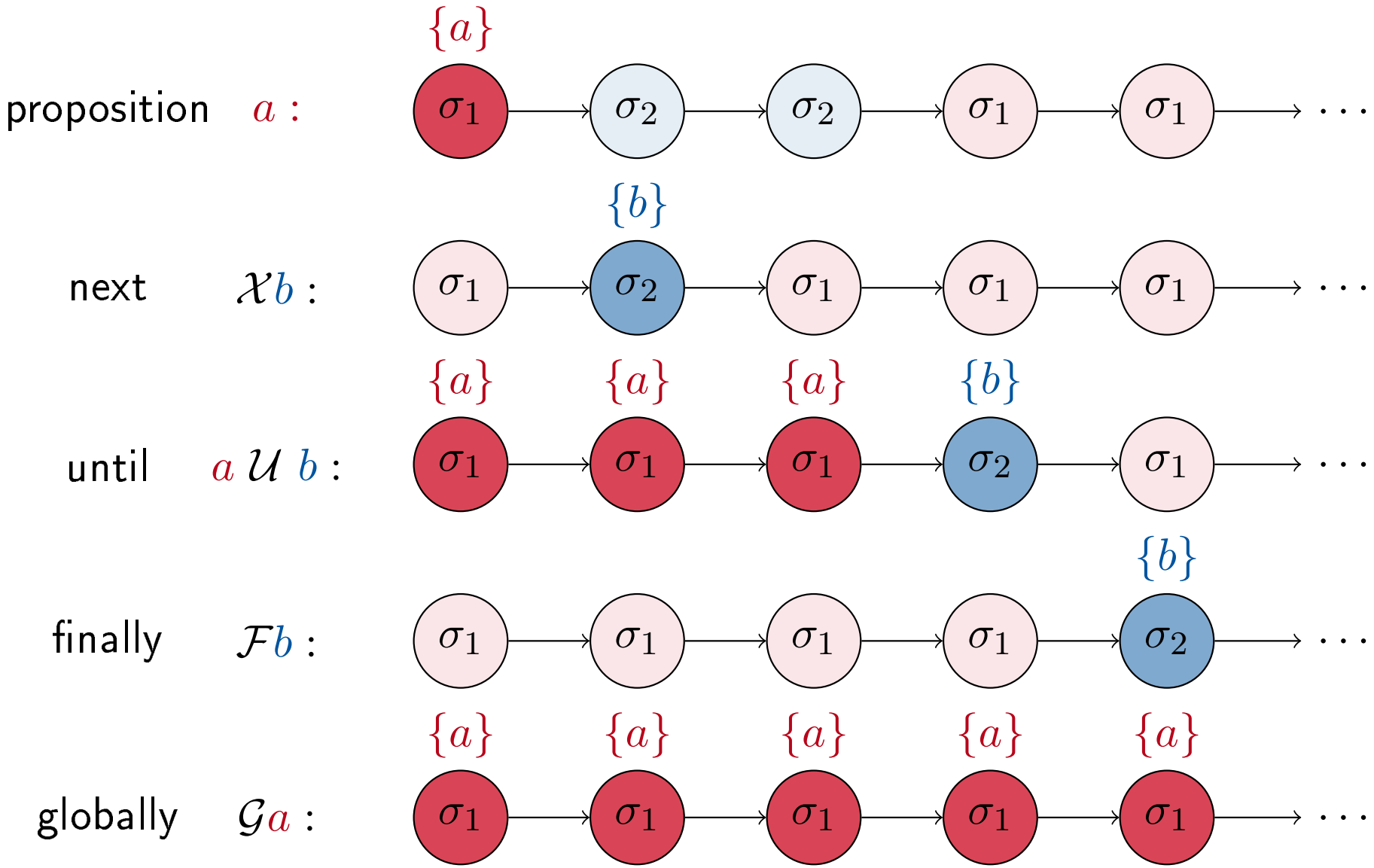
Temporal logics

In the computation tree, temporal logic formulas can describe

- a given path starting in a state (path formulae, "linear" properties) and
- quantified (universal/existential) properties over all paths starting in a given state (state formulae, "branching" properties).

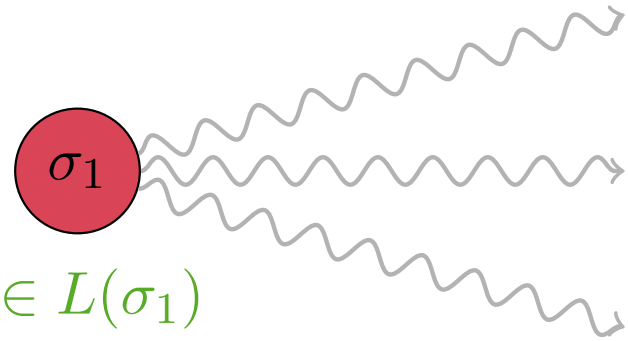


Examples for path formulae

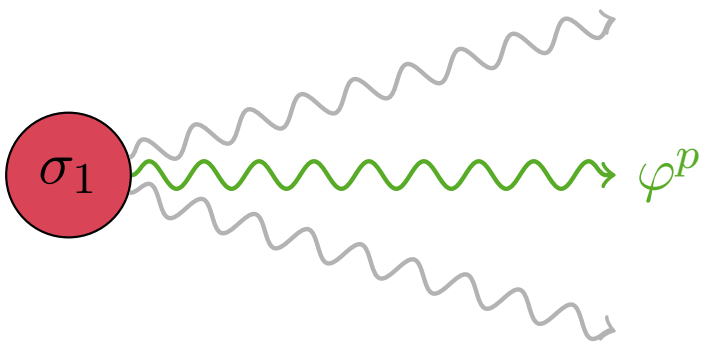


Examples for state formulae

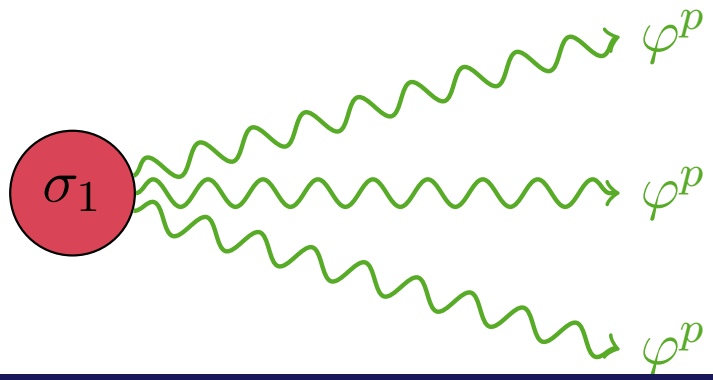
proposition a :



exists $\mathbf{E}\ \varphi^p$:



for all $\mathbf{A}\ \varphi^p$:



CTL* syntax

CTL* state formulae:

$$\varphi^s \quad ::= \quad a \mid (\varphi^s \wedge \varphi^s) \mid (\neg \varphi^s) \mid (\mathbf{E} \varphi^p)$$

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Syntactic sugar:

- $\mathbf{A}\varphi^p := \neg\mathbf{E}\neg\varphi^p$ ("for all", state formula)
- $\mathcal{F}\varphi^p := true\,\mathcal{U}\varphi^p$ ("finally" or "eventually", path formula)
- $\mathcal{G}\varphi^p := \neg\mathcal{F}\neg\varphi^p$ ("globally" or "always", path formula)

CTL* semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.

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For a path $\pi = \sigma_0 \to \sigma_1 \to \ldots$ of $\mathcal{LSTS}$, let $\pi(i)$ denote σ_i, and let π^i denote $\sigma_i \to \sigma_{i+1} \to \ldots$.

CTL* semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.
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\mathcal{L}, \pi \models \varphi_1^p \,\mathcal{U}\, \varphi_2^p & \textit{iff} & \textit{exists } 0 \leq j \textit{ with } \mathcal{L}, \pi^j \models \varphi_2^p \textit{ and} \\
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$\mathcal{L} \models \varphi^s$ iff

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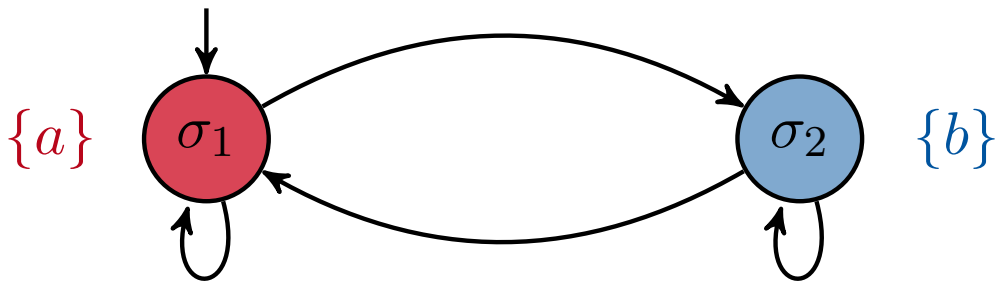
$\mathcal{L} \models \varphi^s$ iff $\mathcal{L}, \sigma_0 \models \varphi^s$ for all initial states σ_0 of $\mathcal{LSTS}$.

Computation tree



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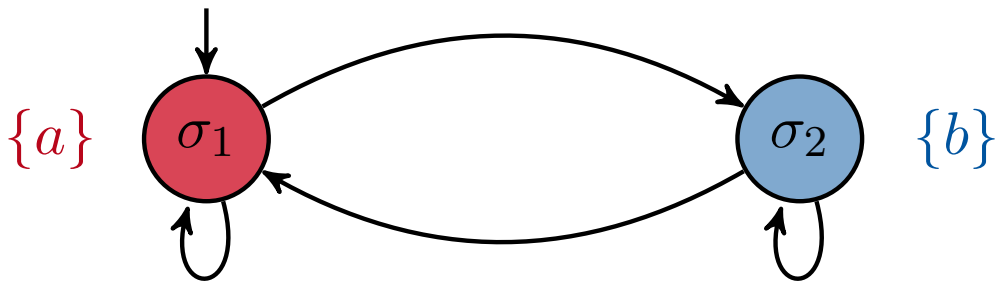
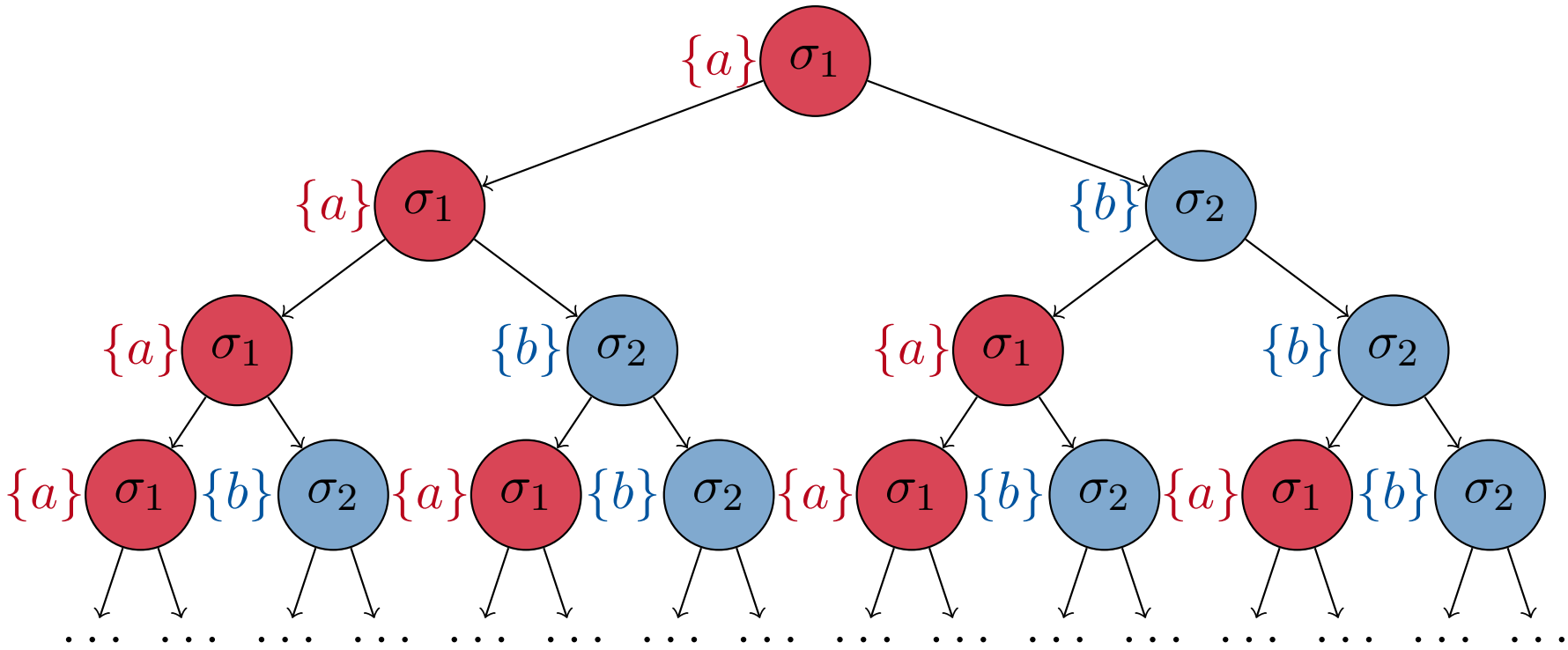
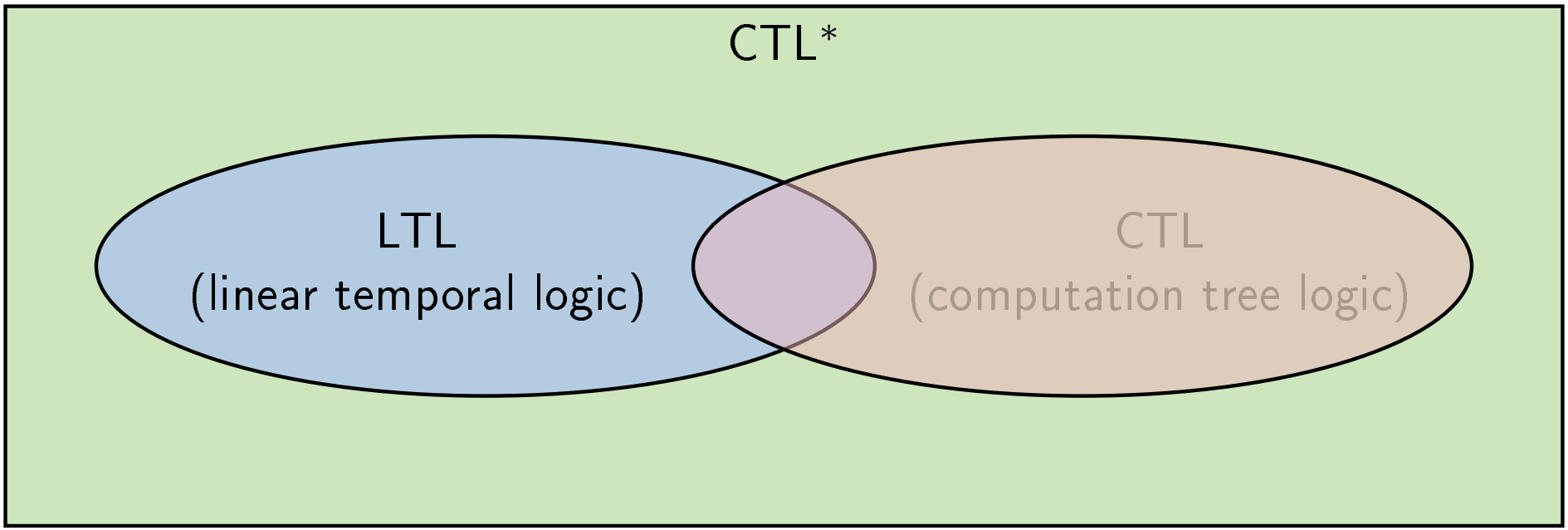




The relation of LTL, CTL, and CTL*



LTL syntax

Linear Temporal Logic (LTL) is suited to argue about single (linear) paths in the computation tree.

- Abstract syntax:

$$\varphi^p \quad ::= \quad a \mid (\varphi^p \wedge \varphi^p) \mid (\neg\varphi^p) \mid (\mathcal{X}\varphi^p) \mid (\varphi^p \,\mathcal{U}\, \varphi^p)$$

  where $a \in AP$.
- Syntactic sugar: $\mathcal{F}$ ("finally" or "eventually"), $\mathcal{G}$ ("globally"), etc.
- Again, we sometimes omit parentheses using the same binding order as for CTL*.

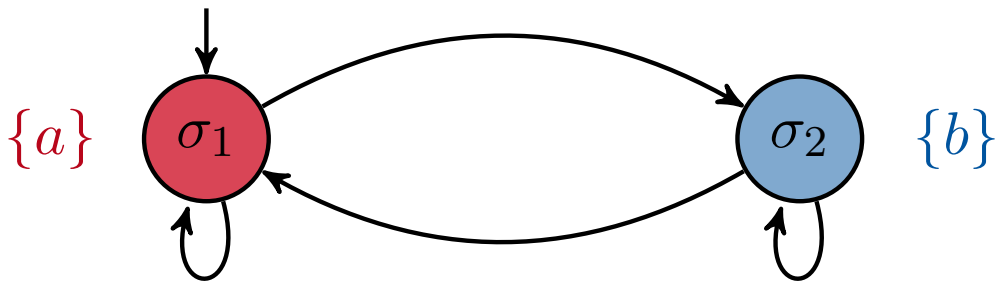
LTL semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.
For a path $\pi = \sigma_0 \to \sigma_1 \to \ldots$ of $\mathcal{LSTS}$, let $\pi(i)$ denote σ_i, and let π^i denote $\sigma_i \to \sigma_{i+1} \to \ldots$.

$$
\begin{array}{lll}
\mathcal{L}, \pi \models a & \textit{iff} & a \in L(\pi(0)), \\
\mathcal{L}, \pi \models \varphi_1^p \wedge \varphi_2^p & \textit{iff} & \mathcal{L}, \pi \models \varphi_1^p \textit{ and } \mathcal{L}, \pi \models \varphi_2^p, \\
\mathcal{L}, \pi \models \neg\varphi^p & \textit{iff} & \mathcal{L}, \pi \not\models \varphi^p, \\
\mathcal{L}, \pi \models \mathcal{X}\varphi^p & \textit{iff} & \pi^1 \models \varphi^p, \\
\mathcal{L}, \pi \models \varphi_1^p \,\mathcal{U}\, \varphi_2^p & \textit{iff} & \exists j \geq 0 . \pi^j \models \varphi_2^p \wedge \forall 0 \leq i < j . \pi^i \models \varphi_1^p.
\end{array}
$$

$\mathcal{LSTS} \models \varphi^p$ iff $\pi \models \varphi^p$ for all paths π of $\mathcal{LSTS}$ starting in an initial state.

Computation tree



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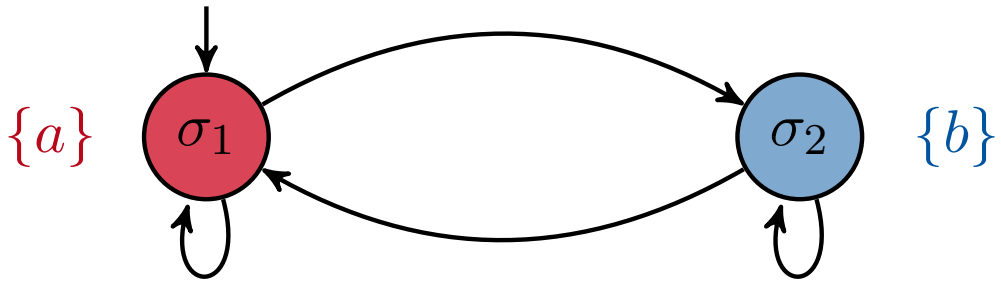

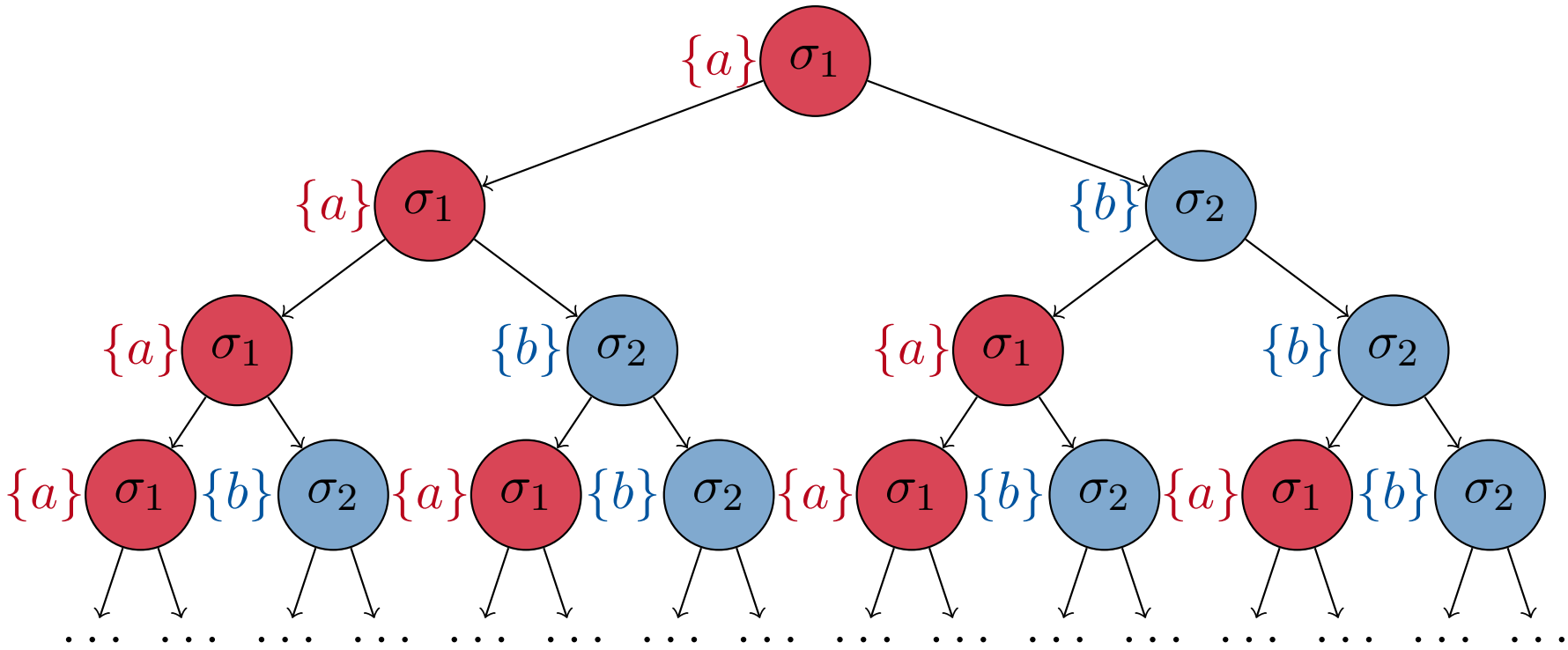

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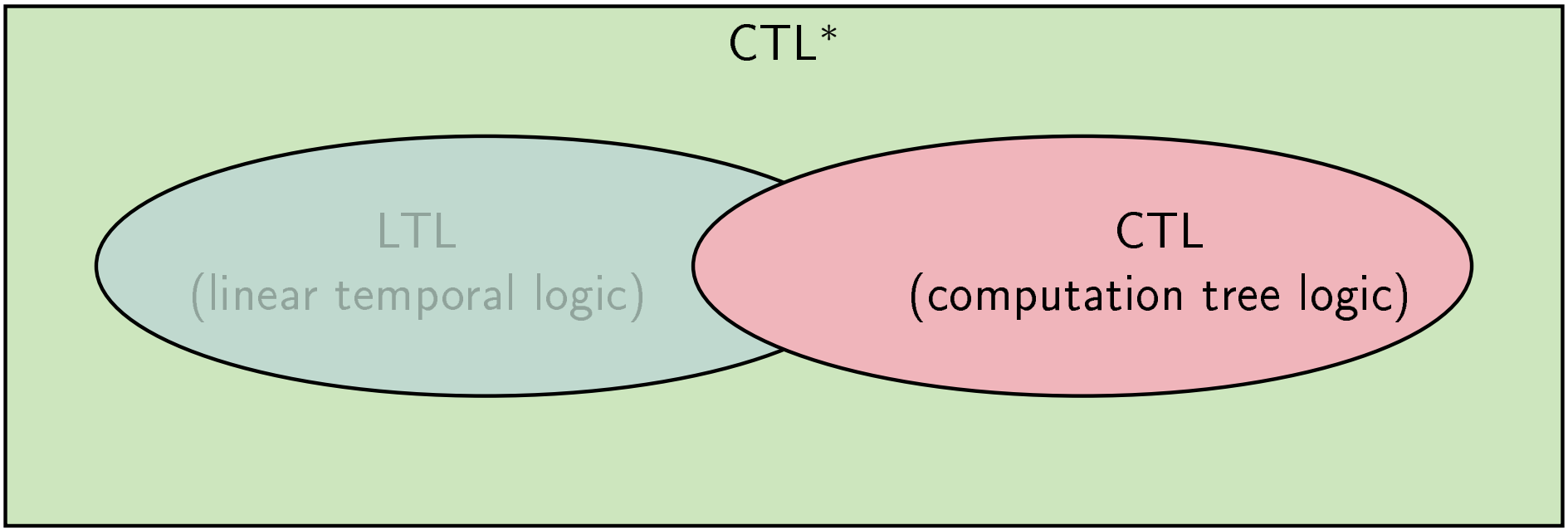

CTL syntax

CTL state formulae:

$$\varphi^s \quad ::= \quad a \mid (\varphi^s \wedge \varphi^s) \mid (\neg\varphi^s) \mid (\mathbf{E}\varphi^p) \mid (\mathbf{A}\varphi^p)$$

with $a \in AP$ and φ^p are CTL path formulae.

CTL path formulae:

$$\varphi^p \quad ::= \quad \mathcal{X}\varphi^s \mid \varphi^s \,\mathcal{U}\, \varphi^s$$

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CTL formulae are CTL state formulae.

As before, we sometimes omit parentheses.

CTL semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.
For a path $\pi = \sigma_0 \to \sigma_1 \to \ldots$ of $\mathcal{LSTS}$, let $\pi(i)$ denote σ_i, and let π^i denote $\sigma_i \to \sigma_{i+1} \to \ldots$.

$\mathcal{L}, \sigma \models a$ *iff* $a \in L(\sigma)$

$\mathcal{L}, \sigma \models \varphi_1^s \wedge \varphi_2^s$ *iff* $\mathcal{L}, \sigma \models \varphi_1^s$ *and* $\mathcal{L}, \sigma \models \varphi_2^s$

$\mathcal{L}, \sigma \models \neg\varphi^s$ *iff* $\mathcal{L}, \sigma \not\models \varphi^s$

$\mathcal{L}, \sigma \models \mathbf{E}\varphi^p$ *iff* $\mathcal{L}, \pi \models \varphi^p$ *for some path* $\pi = \sigma \to \ldots$ *of* $\mathcal{LSTS}$

$\mathcal{L}, \sigma \models \mathbf{A}\varphi^p$ *iff* $\mathcal{L}, \pi \models \varphi^p$ *for all* $\pi = \sigma_0 \to \sigma_1 \to \ldots$ *with* $\sigma_0 = \sigma$

$\mathcal{L}, \pi \models \mathcal{X}\varphi^s$ *iff* $\mathcal{L}, \pi(1) \models \varphi^s$

$\mathcal{L}, \pi \models \varphi_1^s \,\mathcal{U}\, \varphi_2^s$ *iff* *exists* $0 \leq j$ *with* $\mathcal{L}, \pi(j) \models \varphi_2^s$ *and* $\mathcal{L}, \pi(i) \models \varphi_1^s$ *for all* $0 \leq i < j$.

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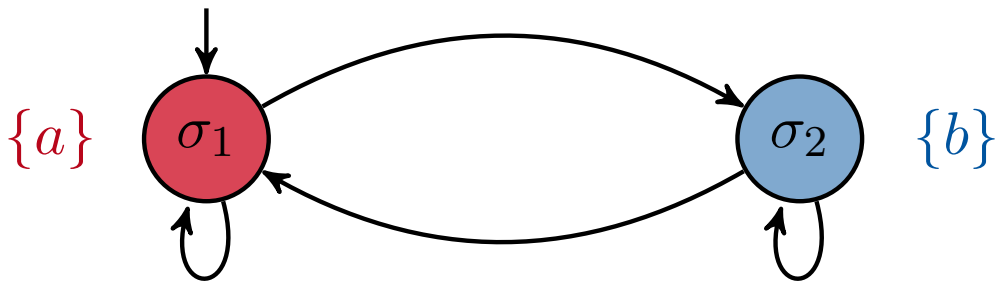

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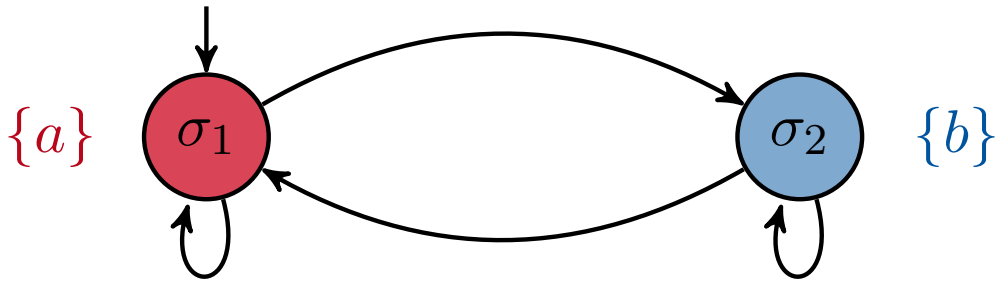

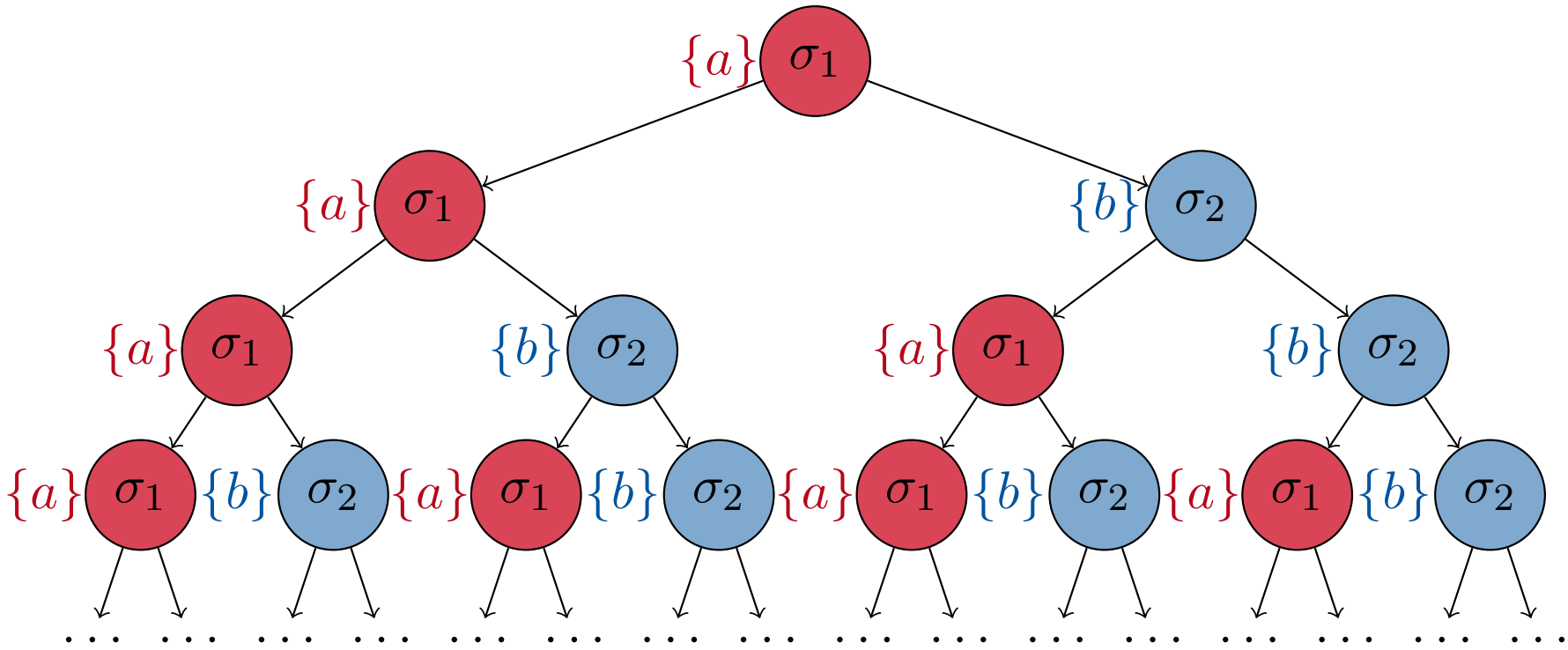

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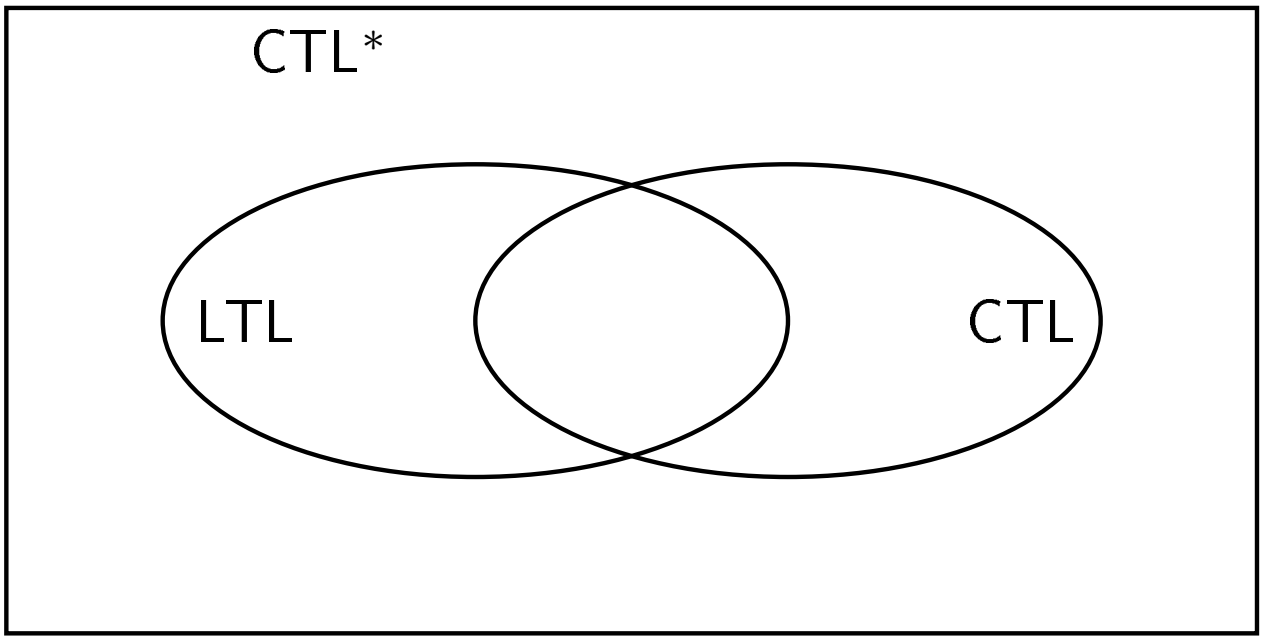

- The LTL formula $\mathcal{F}\mathcal{G}a$ is not expressible in CTL.
- The CTL formula $\mathbf{A}\mathcal{F}\mathbf{A}\mathcal{G}a$ is not expressible in LTL.

Contents

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