Modeling and Analysis of Hybrid Systems

1. Preliminaries

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Informatik 2 - LuFG Theory of Hybrid Systems
RWTH Aachen University

Szeged, Hungary, 27 September - 06 October 2017

Organizational

- email: abraham@cs.rwth-aachen.de
- 7 x 90 minutes lectures, 7 x 90 minutes exercises
- agree on time and place
- learning materials
- written exam mid of November 2017

Contents

1 Hybrid systems

2 Labeled state transition systems

3 Labeled transition systems

4 Temporal logics

5 CTL model checking

“Hybrid”

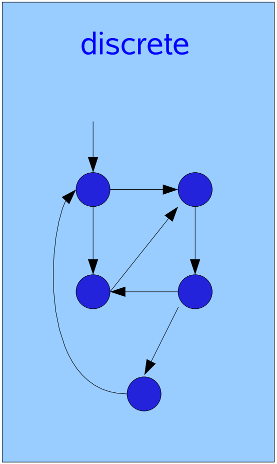
Wikipedia:

> “A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal.”

A hybrid rose

A hybrid car

Hybrid in computer science



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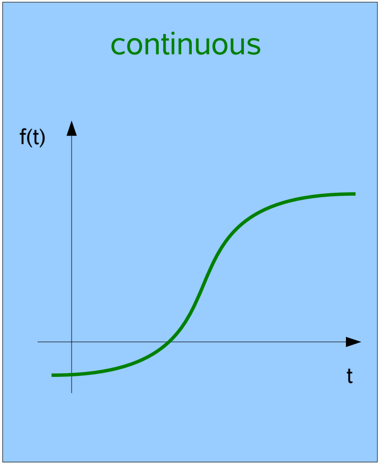
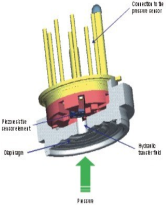
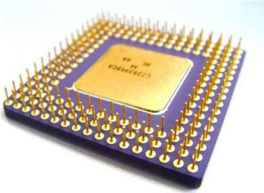


The discrete part


Connection to the pressure sensor
Piezoresistive sensorelement
Diaphragm
Hydraulic transfer fluid
Pressure



Combined with the continuous part

Example: Bouncing ball

Ball falls from a given height, bounces at the ground, raises, falls again...

- vertical position of the ball x_1
- velocity x_2

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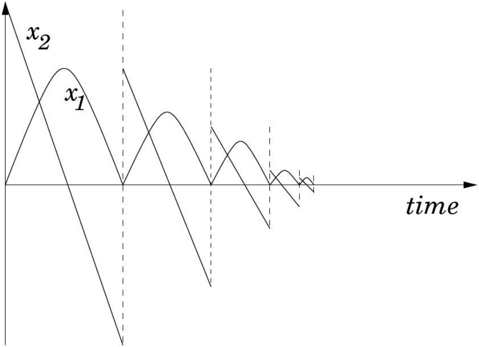

Example: Thermostat

- Temperature x is controlled by switching a heater on and off
- x is regulated by a thermostat:
  - $17^\circ \leq x \leq 18^\circ \rightsquigarrow$ "heater on"
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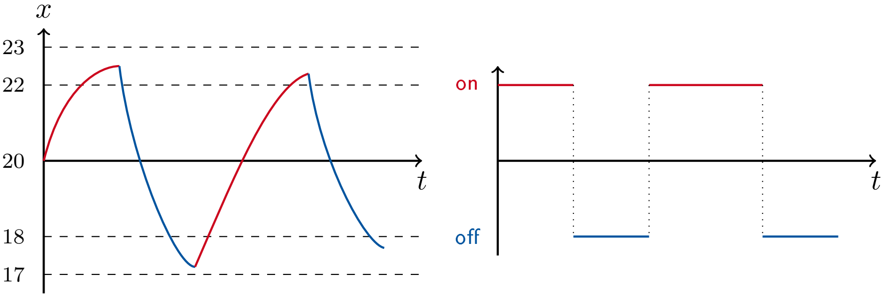
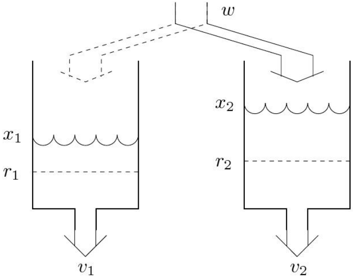


Example: Water tank system

- two constantly leaking tanks v_1 and v_2
- hose w refills exactly one tank at one point in time
- w can switch between tanks instantaneously



There are much more complex examples of hybrid systems, like e.g.

- automobiles, trains, etc.
- automated highway systems
- collision-avoidance and free flight for aircrafts
- digitally controlled chemical plants
- biological cell growth and division ...

In this course we learn how to model and analyse hybrid systems, considering a sequence of modeling languages with increasing expressive power.

- labeled state transition systems
- labeled transition systems
- timed automata
- initialized rectangular automata
- linear hybrid automata I
- linear hybrid automata II

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Labeled state transition systems

Definition

Labeled state transition systems

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A labeled state transition system (LSTS) is a tuple $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ with

- a (possibly infinite) state set Σ,
- a label set Lab (for synchronisation, we do not use it in this course),
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Pedestrian light

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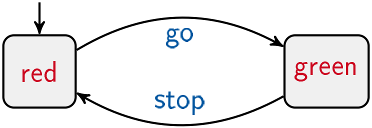


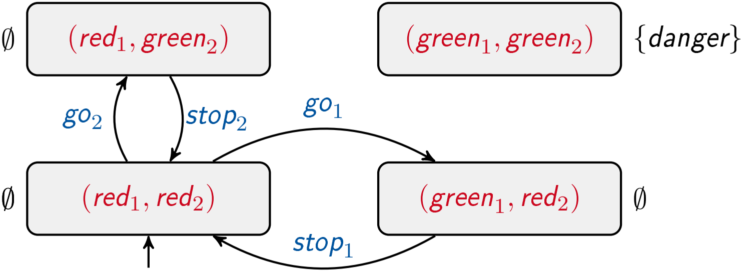
Labeling

To be able to formalize properties of LSTSs, it is common to define

- a set of atomic propositions AP and
- a state labeling function $L : \Sigma \rightarrow 2^{AP}$ assigning a set of atomic propositions to each state.

The set $L(\sigma)$ consists of all propositions that are defined to hold in σ. These propositional labels on states should not be mixed up with the synchronization labels on edges.

Two traffic lights



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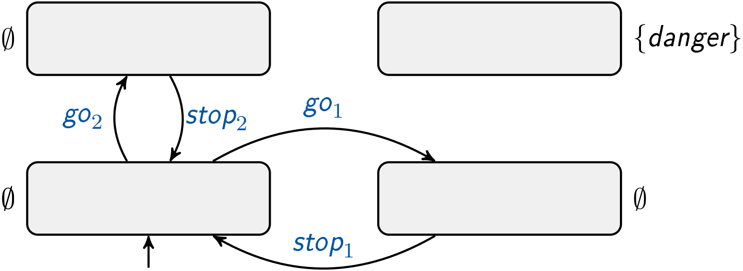

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A labeled transition system (LTS) is a tuple $\mathcal{LTS} = (Loc, Var, Lab, Edge, Init)$ with

- finite set of locations Loc,
- finite set of (typed) variables Var,
- finite set of synchronization labels Lab, $\tau \in Lab$ (stutter label)
- finite set of edges $Edge \subseteq Loc \times Lab \times 2^{V^2} \times Loc$ (including stutter transitions (l, τ, μ_τ, l) for each location $l \in Loc$),
- initial states $Init \subseteq \Sigma$.

with

- valuations $\nu : Var \to Domain$, V is the set of valuations
- state $\sigma = (l, \nu) \in Loc \times V$, Σ is the set of states

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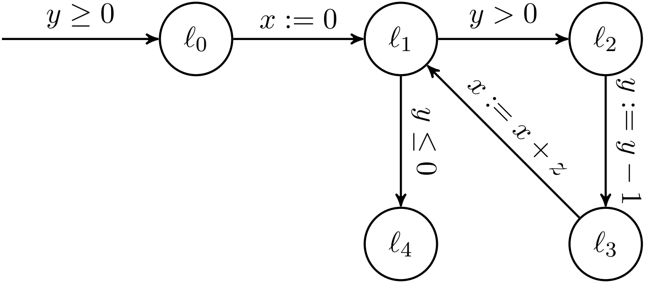
LSTS semantics of LTS

Each LTS $\mathcal{LTS} = (Loc, Var, Lab, Edge, Init)$ induces a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge', Init)$ with

- $\Sigma = Loc \times V$ and
- $Edge' = \{(\sigma, a, \sigma') \mid \sigma \overset{a}{\rightarrow} \sigma'\}$.

Semantics of the simple while-program



$$\frac{(l, a, \mu, l') \in Edge \quad (\nu, \nu') \in \mu}{(l, \nu) \xrightarrow{a} (l', \nu')}$$

Modeling a simple while-program

```
           method mult(int y, int z){
             int x;
$\ell_0$     x := 0;
$\ell_1$
             while( y > 0 ) {
$\ell_2$       y := y-1;
$\ell_3$       x := x+z;
             }
$\ell_4$   }
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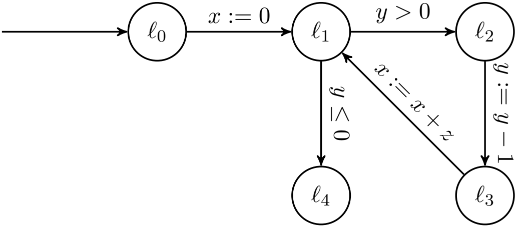

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Temporal logics

Assume

- a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$,
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- How can we describe properties of this system?
- We need a well-suited logic.

Propositional logic

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- Abstract syntax:

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$$\varphi \ ::= \ a \mid (\varphi \wedge \varphi) \mid (\neg\varphi)$$

  with $a \in AP$.
- Syntactic sugar: *true*, *false*, $\vee, \rightarrow, \leftrightarrow, \ldots$
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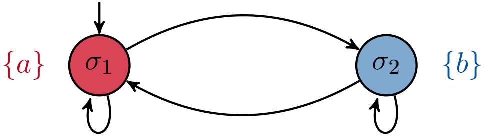
- Omit parentheses when no confusion
- Semantics (in the context of a state $\sigma \in \Sigma$):

$$\begin{array}{lll} \sigma \models a & \textit{iff} & a \in L(\sigma), \\ \sigma \models (\varphi_1 \land \varphi_2) & \textit{iff} & \sigma \models \varphi_1 \textit{ and } \sigma \models \varphi_2, \\ \sigma \models (\neg \varphi) & \textit{iff} & \sigma \not\models \varphi. \end{array}$$

Computation tree



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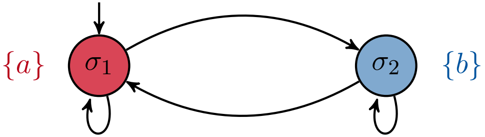
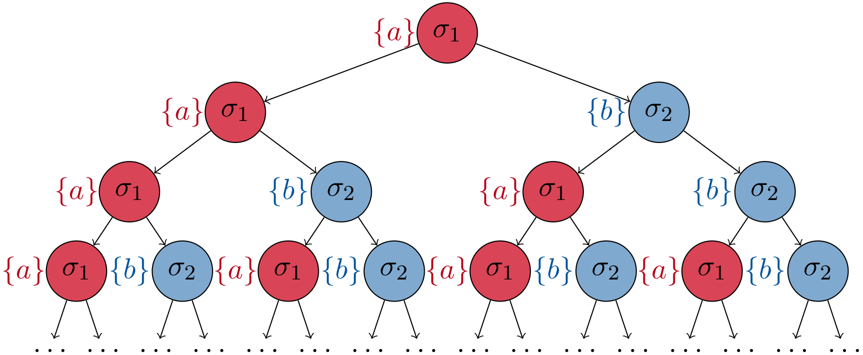
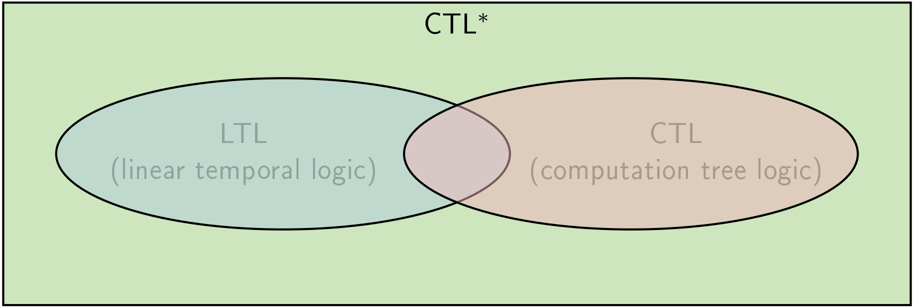




Temporal logics

In the computation tree, temporal logic formulas can describe

- a given path starting in a state (path formulae, “linear” properties) and
- quantified (universal/existential) properties over all paths starting in a given state (state formulae, “branching” properties).



Examples for path formulae

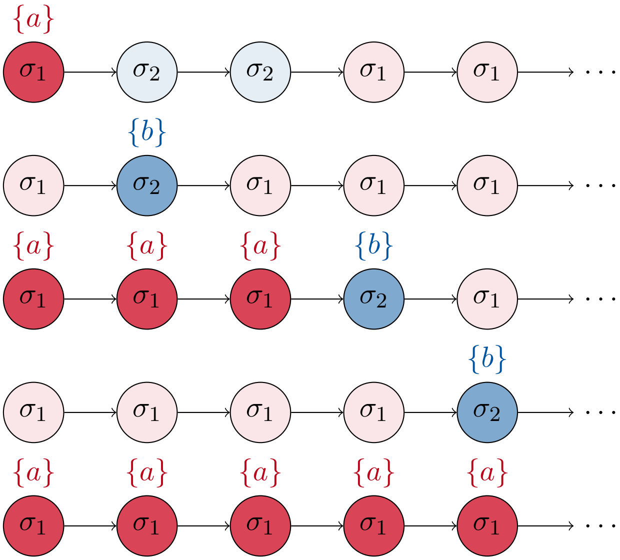
proposition a :

next $\mathcal{X} b$:

until $a \; \mathcal{U} \; b$:

finally $\mathcal{F} b$:

globally $\mathcal{G} a$:



$\{a\}$
σ_1 σ_2 σ_2 σ_1 σ_1 …
$\{b\}$
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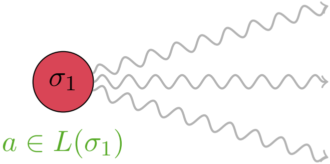
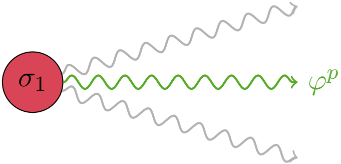
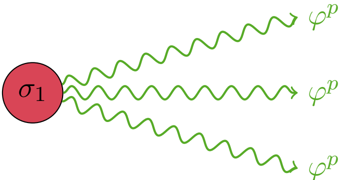


exists $\mathbf{E}\ \varphi^p$:



for all $\mathbf{A}\ \varphi^p$:



CTL* syntax

CTL* state formulae:

$$\varphi^s \quad ::= \quad a \mid (\varphi^s \land \varphi^s) \mid (\neg \varphi^s) \mid (\mathbf{E}\varphi^p)$$

with $a \in AP$ and φ^p are CTL* path formulae.

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Syntactic sugar:

- $\mathbf{A}\varphi^p := \neg\mathbf{E}\neg\varphi^p$ ("for all", state formula)
- $\mathcal{F}\varphi^p := \mathit{true}\,\mathcal{U}\varphi^p$ ("finally" or "eventually", path formula)
- $\mathcal{G}\varphi^p := \neg\mathcal{F}\neg\varphi^p$ ("globally" or "always", path formula)

CTL* semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.

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$\mathcal{L}, \sigma \models a$ *iff*

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Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.

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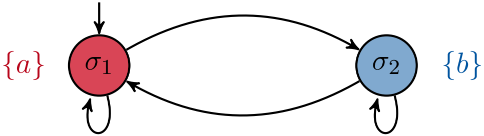
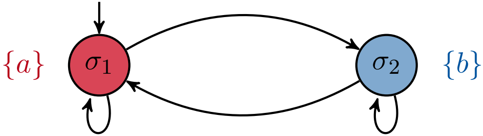
 & & \mathcal{L}, \pi^i \models \varphi_1^p \textit{ for all } 0 \leq i < j.
\end{array}
$$

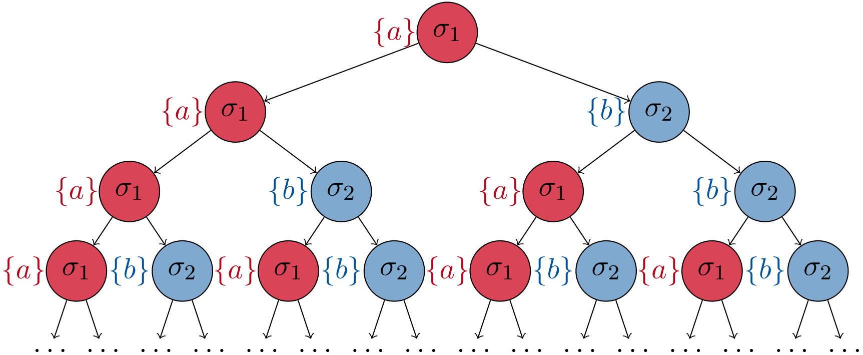
$\mathcal{L} \models \varphi^s$ iff $\mathcal{L}, \sigma_0 \models \varphi^s$ for all initial states σ_0 of $\mathcal{LSTS}$.

Computation tree



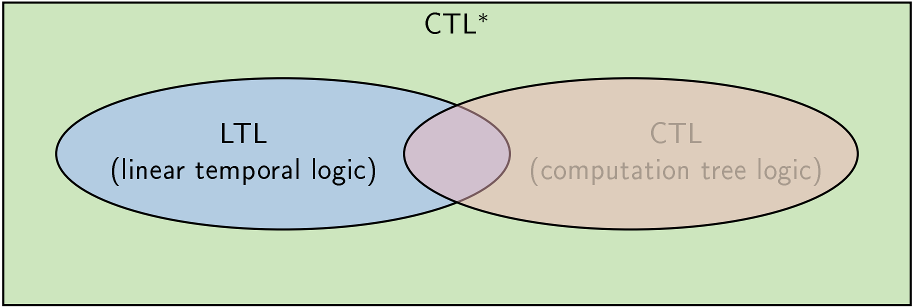
Computation tree





$\{a\}$ σ_1
$\{a\}$ σ_1
$\{b\}$ σ_2
$\{a\}$ σ_1
$\{b\}$ σ_2
$\{a\}$ σ_1
$\{b\}$ σ_2
$\{a\}$ σ_1 $\{b\}$ σ_2 $\{a\}$ σ_1 $\{b\}$ σ_2 $\{a\}$ σ_1 $\{b\}$ σ_2 $\{a\}$ σ_1 $\{b\}$ σ_2
$\dots$

The relation of LTL, CTL, and CTL*



LTL syntax

Linear Temporal Logic (LTL) is suited to argue about single (linear) paths in the computation tree.

- Abstract syntax:

$$\varphi^p \quad ::= \quad a \mid (\varphi^p \wedge \varphi^p) \mid (\neg \varphi^p) \mid (\mathcal{X} \varphi^p) \mid (\varphi^p \, \mathcal{U} \, \varphi^p)$$

  where $a \in AP$.

- Syntactic sugar: $\mathcal{F}$ ("finally" or "eventually"), $\mathcal{G}$ ("globally"), etc.
- Again, we sometimes omit parentheses using the same binding order as for CTL*.

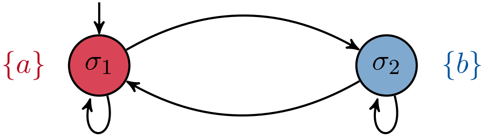
LTL semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.
For a path $\pi = \sigma_0 \to \sigma_1 \to \ldots$ of $\mathcal{LSTS}$, let $\pi(i)$ denote σ_i, and let π^i denote $\sigma_i \to \sigma_{i+1} \to \ldots$.

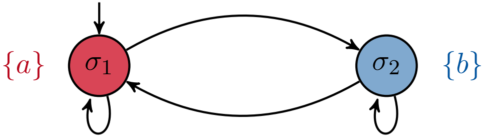
$$
\begin{array}{lll}
\mathcal{L}, \pi \models a & \textit{iff} & a \in L(\pi(0)), \\
\mathcal{L}, \pi \models \varphi_1^p \wedge \varphi_2^p & \textit{iff} & \mathcal{L}, \pi \models \varphi_1^p \textit{ and } \mathcal{L}, \pi \models \varphi_2^p, \\
\mathcal{L}, \pi \models \neg\varphi^p & \textit{iff} & \mathcal{L}, \pi \not\models \varphi^p, \\
\mathcal{L}, \pi \models \mathcal{X}\varphi^p & \textit{iff} & \pi^1 \models \varphi^p, \\
\mathcal{L}, \pi \models \varphi_1^p \, \mathcal{U} \, \varphi_2^p & \textit{iff} & \exists j \geq 0. \pi^j \models \varphi_2^p \wedge \forall 0 \leq i < j. \pi^i \models \varphi_1^p.
\end{array}
$$

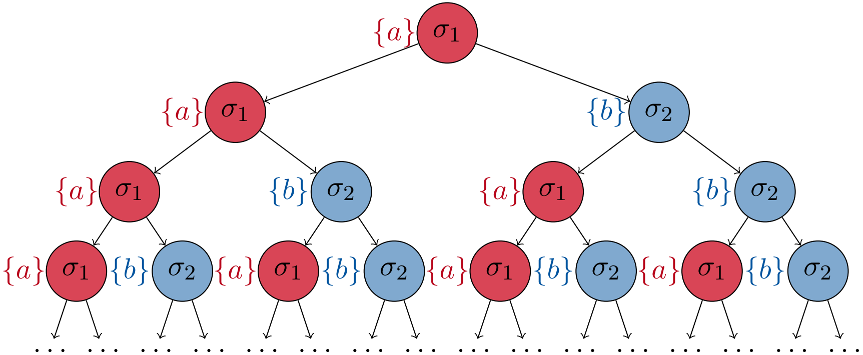
$\mathcal{LSTS} \models \varphi^p$ iff $\pi \models \varphi^p$ for all paths π of $\mathcal{LSTS}$ starting in an initial state.

Computation tree

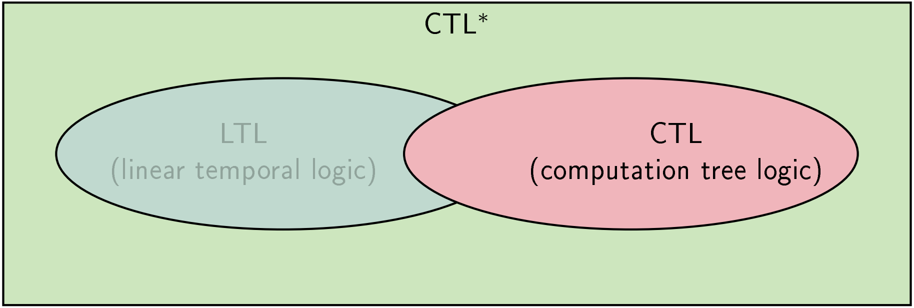


Computation tree





The relation of LTL, CTL, and CTL*



CTL syntax

CTL state formulae:

$$\varphi^s \quad ::= \quad a \mid (\varphi^s \wedge \varphi^s) \mid (\neg\varphi^s) \mid (\mathbf{E}\varphi^p) \mid (\mathbf{A}\varphi^p)$$

with $a \in AP$ and φ^p are CTL path formulae.

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CTL formulae are CTL state formulae.

As before, we sometimes omit parentheses.

CTL semantics

Assume $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ to be a labeled state transition system $\mathcal{LSTS} = (\Sigma, Lab, Edge, Init)$ along with a labeling function $L : \Sigma \to 2^{AP}$, where AP is a finite set of atomic propositions.
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\begin{array}{lll}
\mathcal{L}, \sigma \models a & \textit{iff} & a \in L(\sigma) \\
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\mathcal{L}, \sigma \models \mathbf{E}\varphi^p & \textit{iff} & \mathcal{L}, \pi \models \varphi^p \textit{ for some path } \pi = \sigma \to \ldots \textit{ of } \mathcal{LSTS} \\
\mathcal{L}, \sigma \models \mathbf{A}\varphi^p & \textit{iff} & \mathcal{L}, \pi \models \varphi^p \textit{ for all } \pi = \sigma_0 \to \sigma_1 \to \ldots \textit{ with } \sigma_0 = \sigma
\end{array}
$$

$$
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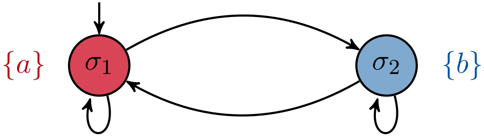

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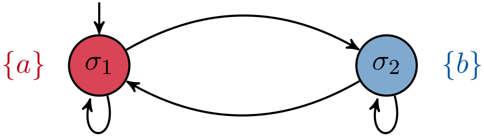

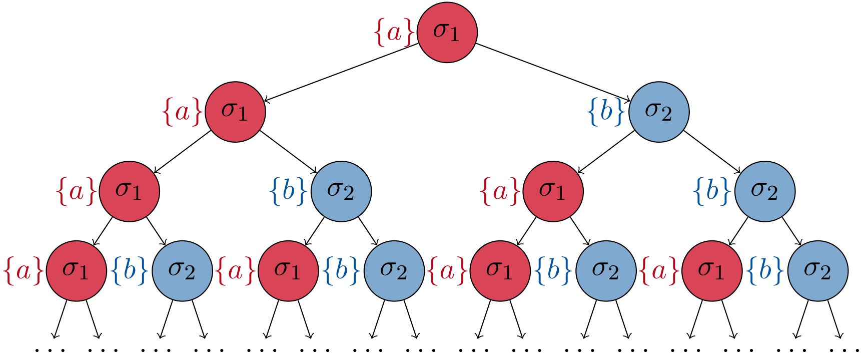

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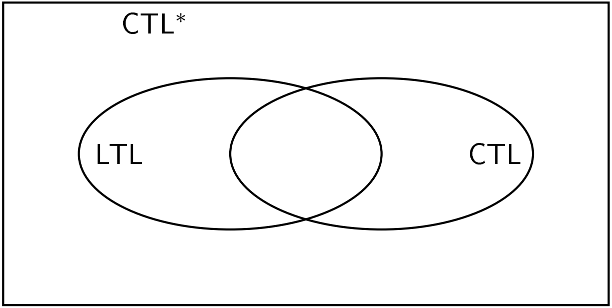

- The LTL formula $\mathcal{F}\mathcal{G}a$ is not expressible in CTL.
- The CTL formula $\mathbf{A}\mathcal{F}\mathbf{A}\mathcal{G}a$ is not expressible in LTL.

Contents

1 Hybrid systems

2 Labeled state transition systems

3 Labeled transition systems

4 Temporal logics

5 CTL model checking

CTL (explicit) model checking

Next learn a model checking algorithm to decide whether a labeled state transition system satisfies a CTL formula.

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For $\mathcal{L} = (\Sigma, Lab, Edge, Init, L)$ (being a labeled state transition system $(\Sigma, Lab, Edge, Init)$ with a labeling function L) and for a CTL formula ψ^s, CTL model checking labels the states of $\mathcal{L}$ recursively with the sub-formulae of ψ^s inside-out, such that exactly those states are labeled with each sub-formula at which the given sub-formula holds.

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