Modeling and Analysis of Hybrid Systems

Introduction

Prof. Dr. Erika Ábrahám

Informatik 2 - Theory of Hybrid Systems
RWTH Aachen University

SS 2013

Organizational

Lecture:

- Tuesday 13:15-14:15 in 5056
- Friday 13:15-14:30 in 5056

Exercise:

- Tuesday 14:15-15:00 in 5056

Exam dates will be chosen by Doodle vote:

- 1st: 26.07.2013 09:45-12:15
  01.08.2013 13:45-16:15
  02.08.2013 09:45-12:15
- 2nd: 18.09.2013 15:45-18:15
  19.09.2013 10:45-13:15

Learning materials and contact persons

Learning materials available in L2P:

- Slides
- Lecture notes
- Video recordings
- Some research publications
- Exercise sheets, solutions

Lecture:

Erika Ábrahám
room: 2U07 (Hauptbau, basement), phone: 0241/80-21242
email: abraham@informatik.rwth-aachen.de

Exercise:

Xin Chen
room: 2U08 (Hauptbau, basement), phone: 0241/80-21243
email: xin.chen@informatik.rwth-aachen.de

Further information (topic, evaluations etc.):`http://www-i2.informatik.rwth-aachen.de/i2/hybrid_lecture/`

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1. Hybrid systems
2. Modeling
3. Specification
4. Analysis

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1 Hybrid systems

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“Hybrid”

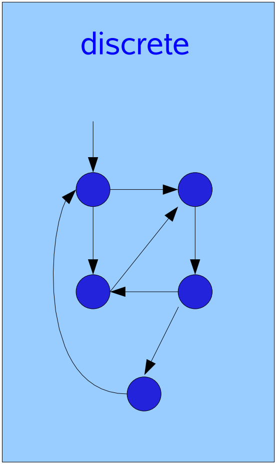
Wikipedia:

> “A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal.”

A hybrid rose

A hybrid car

Hybrid in computer science



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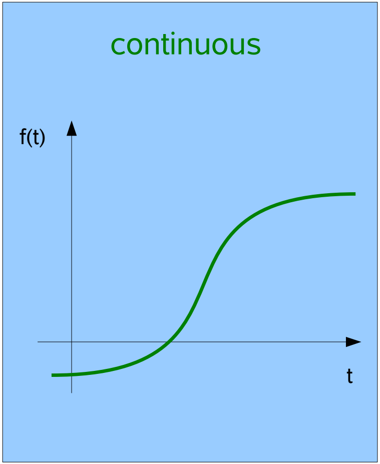
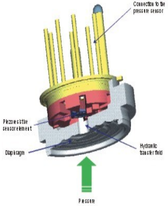
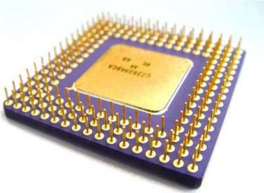


The discrete part


Connection to the pressure sensor
Piezoresistive sensorelement
Diaphragm
Hydraulic transfer fluid
Pressure



Combined with the continuous part

Example: Thermostat

Example: Thermostat

- Temperature x is controlled by switching a heater on and off
- x is regulated by a thermostat:
  - $17° \leq x \leq 18°$ $\rightsquigarrow$ "heater on"
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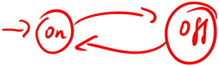
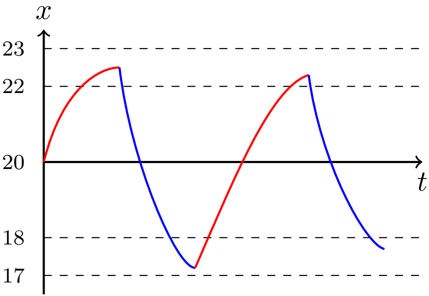
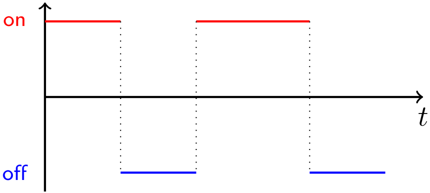
Continuous: temperature

Discrete: switching







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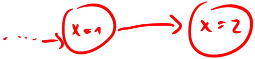
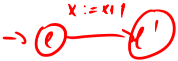
Modeling

Modeling

ℓ: $x := x + 1$ ℓ':





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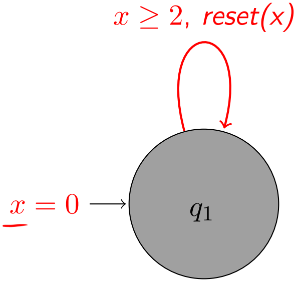
- What you probably also know: Transition systems
- What you perhaps know: Timed automata

Example: Timed automaton



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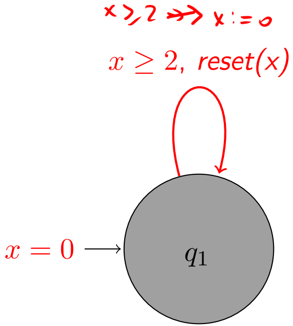

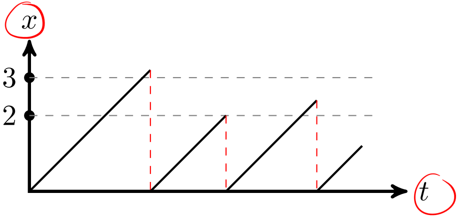

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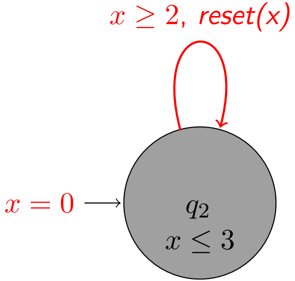

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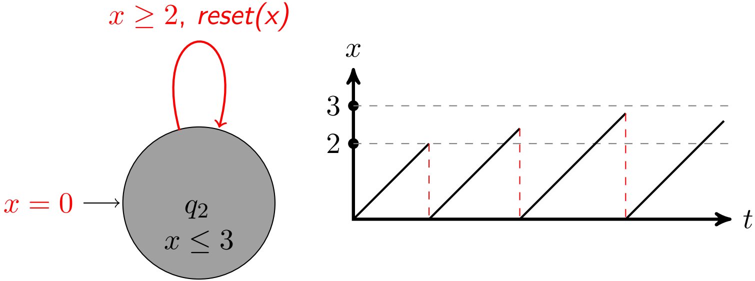

Modeling general hybrid systems: Hybrid automata

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Let's take again the thermostat as an example.

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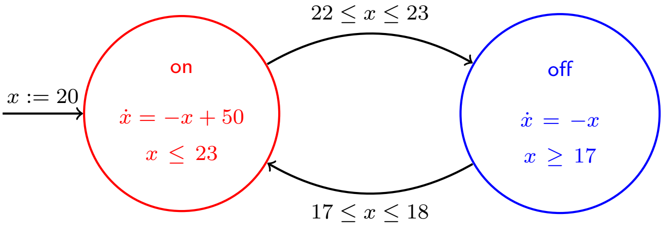

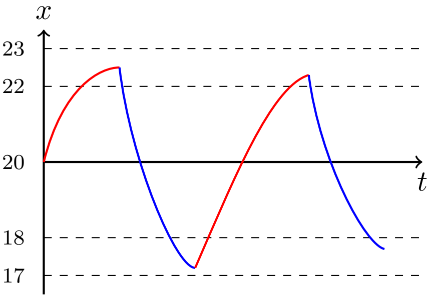

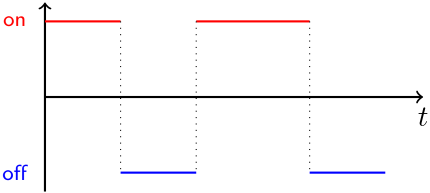

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  "The temperature is always below $20°C$."
- Or

  "If the temperature is above $20°C$ it will get below $20°C$ within 5 seconds."
- Or

  "It is always the case that the temperature will somewhen in the future get above $20°C$."

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- Well, it depends...
- ...on the fact if the logic is decidable for the underlying modeling language.
- We will see for which classes of hybrid automata the reachability question is decidable.
- We will deal with
    - (unbounded) reachability for timed automata.
    - (unbounded) reachability for initialized rectangular automata.
    - bounded reachability for linear hybrid automata.
    - reachability approximation for general hybrid automata.

Method for timed automata: Finite abstraction

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Constructive proof of decidability via finite abstraction:

Method for timed automata: Finite abstraction

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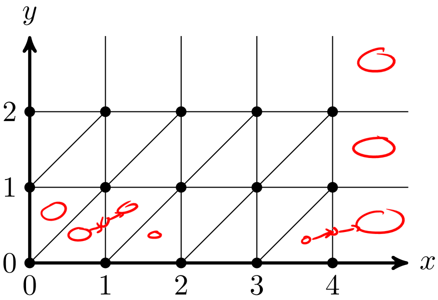
Method for timed automata: Finite abstraction

Constructive proof of decidability via finite abstraction:



Method for initialized rectangular automata: Transformation

Method for initialized rectangular automata: Transformation

Leading back the proof of decidability to a known problem:

Method for initialized rectangular automata: Transformation

$\dot{x} = 1$ $\dot{x} \in [2,3]$

Leading back the proof of decidability to a known problem:

Timed automaton

↑

Initialized stopwatch automaton

↑

Initialized singular automaton

↑

→ Initialized rectangular automaton

Method for linear hybrid automata: Fixedpoint computation

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Forward reachability computation:

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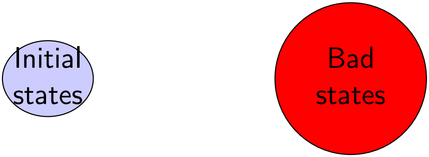
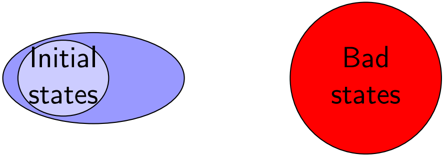


Method for linear hybrid automata: Fixedpoint computation

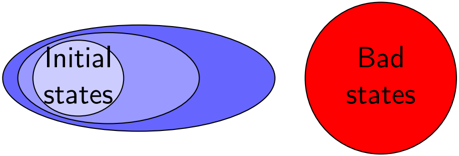
Forward reachability computation:



Method for linear hybrid automata: Fixedpoint computation

$$X \supseteq Reach(X)$$

Forward reachability computation:



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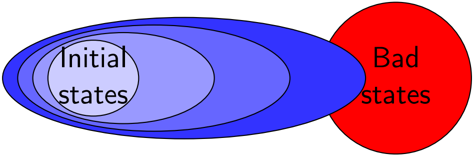

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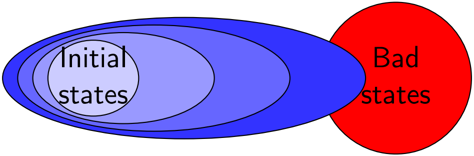


$x = 0$

$\downarrow$

$x \in \{0,1\}$

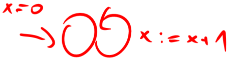
$\downarrow$

$x \in \{0,1,2\}$

$\vdots$

Note: the method is incomplete

$x=0 \rightarrow$ while (true) { $x := x+1$ }



Method for linear hybrid automata: Fixedpoint computation

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Backward reachability computation:

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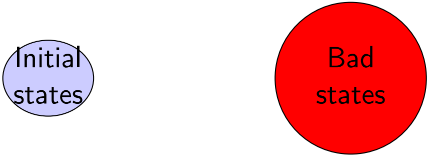

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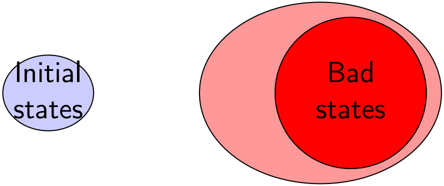

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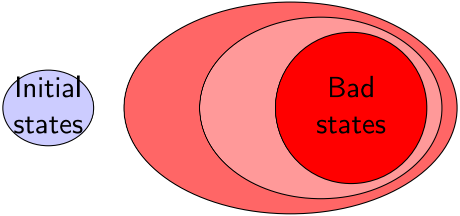

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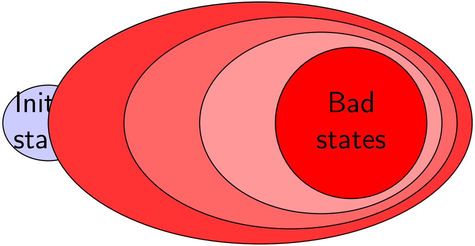

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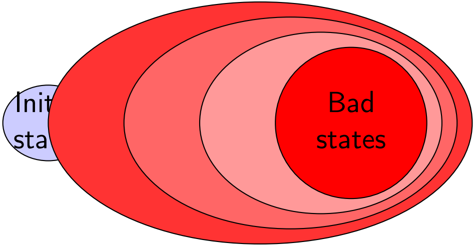

Note: also the backward method is incomplete

Method for hybrid automata: Approximation

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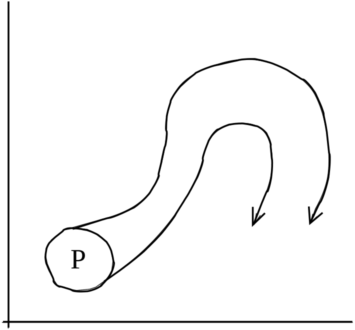
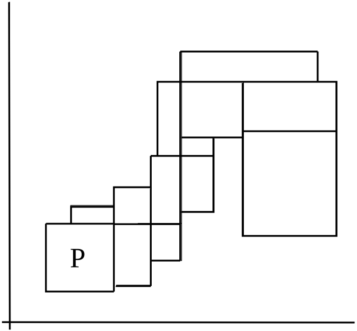
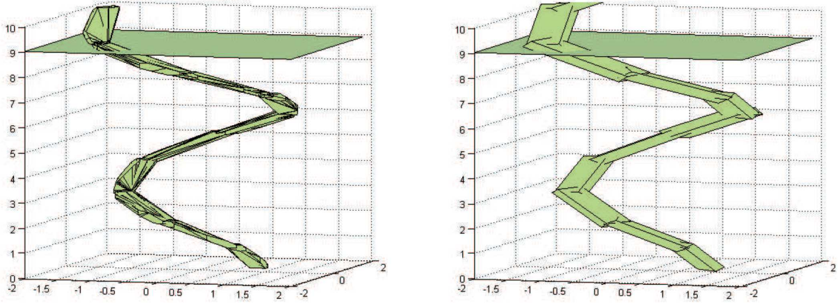
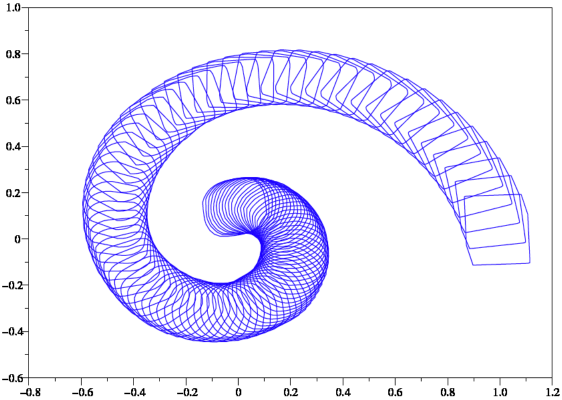




Polyhedra (left) and oriented rectangular hulls (right) in reachability computation



Zonotopes in reachability computation



1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1.0
1.2