Satisfiability Checking

SMT Applications

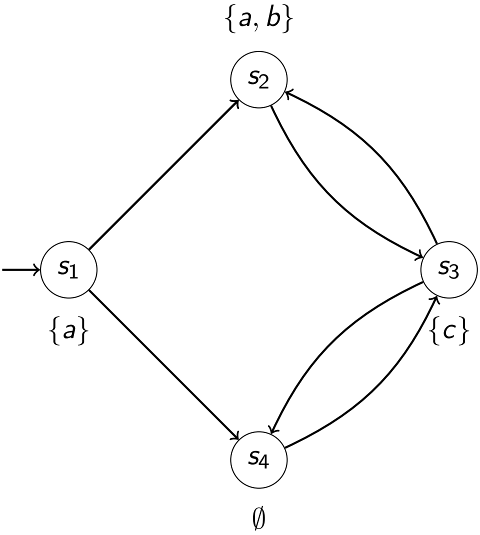
Prof. Dr. Erika Ábrahám

RWTH Aachen University
Informatik 2
LuFG Theory of Hybrid Systems

WS 14/15

Overview

Kripke structures



Kripke structure: Syntax

Definition

Let AP be a finite set of atomic propositions. A Kripke structure is a tuple $M = (S, s_{\text{init}}, T, L)$ with

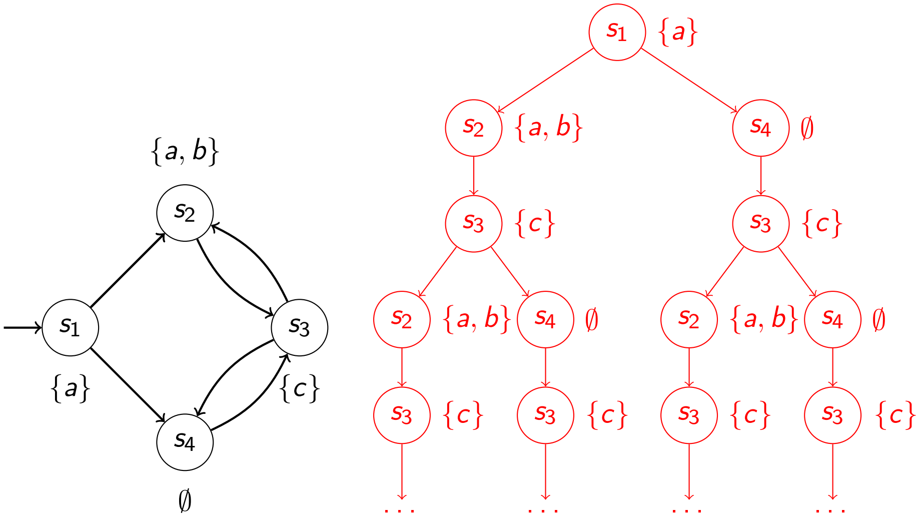
- S a finite set of states,
- $s_{\text{init}} \in S$ an initial state,
- $T \subseteq S \times S$ a transition relation,
- $L : S \to 2^{AP}$ a labeling function
  (2^{AP} denotes the powerset over AP).

The labeling function attaches information to the system: for a state $s \in S$ the set $L(s)$ consists of those atomic propositions that hold in s.

Kripke structure: Semantics

- An (infinite) path $\pi = s_0 s_1 s_2 \ldots$ of a Kripke structure $M = (S, s_{\text{init}}, T, L)$ is a sequence of states such that
  - $s_0 = s_{\text{init}}$ and
  - $(s_i, s_{i+1}) \in T$ for all $i \geq 0$.
- The behaviour of M is given by the set of all of its infinite paths.
- A finite path of M is a finite prefix of an infinite path of M.
- For a finite path $\pi = s_0 \ldots s_k$ we define $|\pi| = k$.
- We write $\pi(j)$ for the jth state (starting with 0) of the path π.
- By π_j we denote the postfix of π starting at $\pi(j)$.

Kripke structure: Semantics



Transition systems

```
     m(int u, int v){
l1:  while (u>0) do
l2:    v:=v+1;
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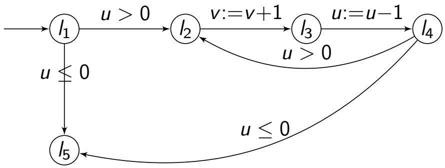
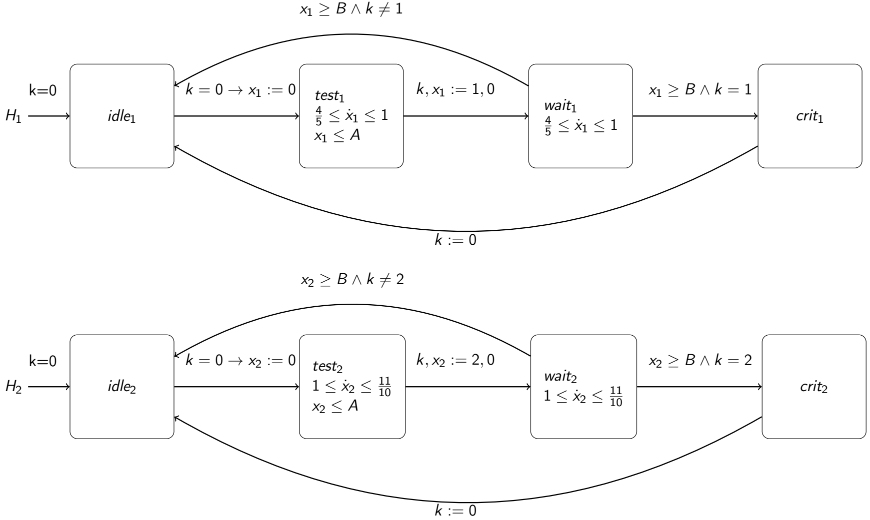


Fischer's mutual exclusion protocol

There are also more complex systems we can deal with similarly...



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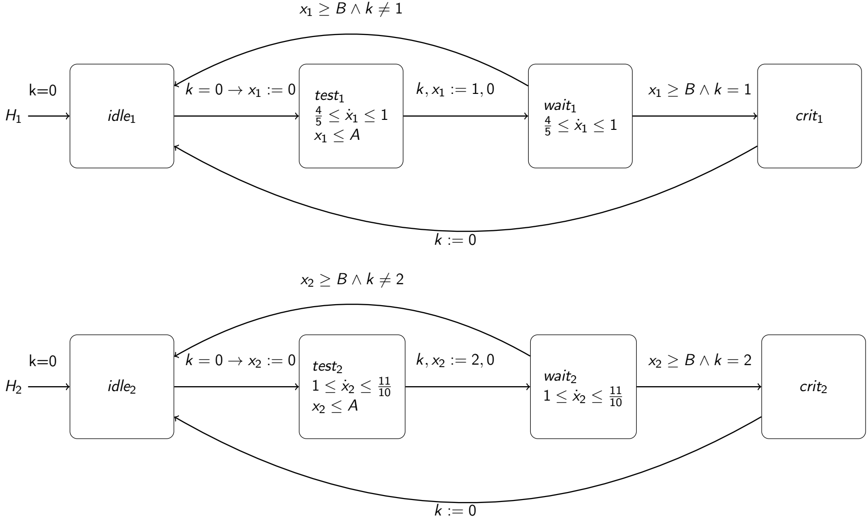

...but now we focus of Kripke structures and discrete transition systems.

Model checking

Early 1980s: First implementations of model checking as verification technique

- Explicit representations of the transition graphs
- Problem: Due to the state space explosion not applicable for complex industrial settings

1990: Symbolic model checking

- BDDs represent *characteristic functions* of state sets symbolically
- Problem: Building the BDD may be expensive

1999: Bounded model checking *[Biere et al.]*

- Check the existence of finite paths of incremental length by a SAT- or SMT-solver
- Problem: Incomplete (in general)
- Works for different logics, we consider only reachability

Model checking and counterexamples

- Given a model M and a logical description φ of safe states (invariant), a counterexample is a finite path of M reaching a non-φ state.
- If a system is buggy, counterexamples are extremely important for detecting and fixing the error.
- Bounded model checking (BMC) is a technique to search for finite counterexamples (also for more complex models and logics).

Bounded model checking

Algorithm:

1. Set $k = 0$
2. Construct a logical formula BMC_k describing a finite path
   - through the underlying model
   - of length k,
   - and violating a safety property φ after k steps.
3. Check BMC_k for satisfiability using a SAT or SMT solver
4. If SAT, the resulting assignment describes a counterexample $\rightarrow$ terminate
5. If UNSAT, increment k, goto 1.

Bounded model checking

$$BMC_k = I(s_0) \wedge T(s_0, s_1) \wedge \ldots \wedge T(s_{k-1}, s_k) \wedge \neg\varphi(s_k)$$

- I and T are (nearly) straightforward for Kripke structures and timeless transition systems
- $I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1})$ is called the unfolding of the transition relation

BMC example

```
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l1:  while (u>0) do
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Safety property: Return value = sum of the input values

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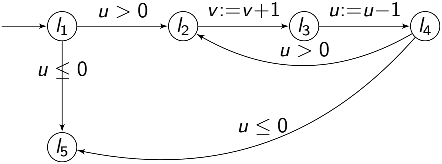

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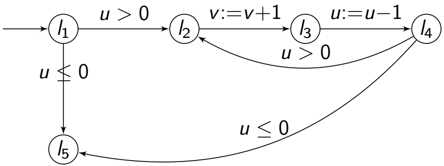

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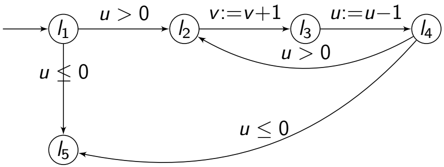

Safety property: Return value = sum of the input values

BMC for counterexamples: (encoding location l_j in step i by $at_i = j$)

$$
\begin{array}{ll}
I(i): & at_i = 1 \\
T_{1,2}(i, i+1): & at_i = 1 \wedge at_{i+1} = 2 \wedge u_i > 0 \wedge u_{i+1} = u_i \wedge v_{i+1} = v_i \\
T_{1,5}(i, i+1): & at_i = 1 \wedge at_{i+1} = 5 \wedge u_i \leq 0 \wedge u_{i+1} = u_i \wedge v_{i+1} = v_i \\
\ldots & \\
T(i, i+1): & T_{1,2}(i, i+1) \vee T_{1,5}(i, i+1) \vee \ldots \\
\varphi(i): & at_i \neq 5 \vee v_i = v_0 + u_0 \\
BMC_0: & I(0) \wedge \neg\varphi(0) \qquad \mathit{UNSAT} \\
BMC_0: & I(0) \wedge T(0,1) \wedge \neg\varphi(1) \qquad \mathit{SAT}
\end{array}
$$

BMC is not complete

- Termination is guaranteed iff counterexample exists
- If no counterexample exists, procedure does not terminate
- Upper bound for k to ensure property: Completeness threshold
- Another possibility to make BMC complete: k-induction (not content of this lecture)

Completeness threshold

- For each (finite state) system M, property p and given translation scheme there exists a number $\mathcal{CT}$, called completeness threshold.
- For reachability properties, $\mathcal{CT}$ is equal to the reachability diameter, i.e., the minimal distance required to reach all (reachable) states of the system.

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Definition (Reachability Diameter)

$$rd(M) := min\Big\{ i \mid \forall n > i.\ \forall s_0, \ldots, s_n.\ \exists t \leq i.\ \exists s'_0, \ldots, s'_t.$$

$$\left(I(s_0) \wedge \bigwedge_{j=0}^{n-1} T(s_j, s_{j+1}) \right) \rightarrow \left(I(s'_0) \wedge \bigwedge_{j=0}^{t-1} T(s'_j, s'_{j+1}) \wedge s'_t = s_n \right) \Big\}$$

"Every state that is reachable in n steps, is also reachable in i steps."

- This yields maximal shortest paths in the system.

Completeness threshold

- Problem: Show the property for all n
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"Every state that is reachable in $i + 1$ steps, is also reachable in at most i steps."

Completeness threshold

- Problem: Formula contains quantifier alternation

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- Solution: Over-approximation of $rd(M)$

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Definition (Recurrence Diameter)

$rdr(M) :=$

$$max\left\{ i \mid \exists s_0 \dots s_i : I(s_0) \wedge \bigwedge_{j=0}^{i-1} T(s_j, s_{j+1}) \wedge \bigwedge_{j=0}^{i-1} \bigwedge_{k=j+1}^{i} s_j \neq s_k \right\}$$

"Longest loop-free initial path in M."

- As every shortest path is a loop-free path, this is an over-approximation of $rd(M)$.

Overview

Deductive verification: Example

Overview

Schere, Stein, Papier

Overview

Hardware design