Modeling and Analysis of Hybrid Systems - SS 2015

Series 9

Exercise 1

What are two reasons we require bloating in the computation of the reachable states?

Solution:
We need bloating on the one hand for the dynamic itself and on the other hand to cope with external input, if specified. The bloating factor thereby depends on the specification on the dynamics, the initial set and the length of the time step. In a polytopal setup, the bloating is realized by adding (Minkowski sum) a box (hyperrectangle) of where the edge length corresponds to the bloating factor. In a setup, where support functions are used, we simply can add a ball with corresponding diameter.

Exercise 2

Please describe the difference between the two presented approaches towards flowpipe computation.

Solution:
The difference between both approaches is the way of computing the flowpipe segments:

- The first presented approach computes one timestep and creates the according flowpipe segment. The succeeding flowpipe segments can be computed iteratively by using the current set and applying a time step.
- The second presented approach computes from the initial set each time step (without any recurrence) separately.

Exercise 3

Why is the choice of the state set representation crucial?

Solution:
The choice of the used state set representation has a strong impact on the computational time and space required for the reachability analysis. On the other hand it also has an effect on the overappoximation error we introduce by the abstraction. In general this is a trade of between complexity and precision.

Exercise 4

What types of state set representations do you know? Please give a short description of each mentioned representation.

Solution:

- Boxes: Boxes are a very simple set representation where each dimension is limited by an interval such that the product of the intervals results in a hyperrectangle.
- Convex polytopes: A convex polytope can be described as the convex hull of a finite set of points or the intersection of a finite set of hyperplanes, which also can be specified as a conjunction of linear inequalities. We can realize polyhedra (not necessarily closed polytopes) in V-representation by adding a cone to the set of vertices.
- Taylor models: A Taylor model is a non-convex representation suitable to represent nonlinear dynamics. It can be computed by creating a Taylor polynomial of suitable degree, which approximates the dynamics at some point. The error of the approximation is compensated by adding an inverval to the polynomial, which results in an envelope holding the real dynamics of the system for a specified time step.
- (Zonotopes: A zonotope is the Minkowski sum of a finite set of generators, which are represented as vectors. Each generator influences the result with a coefficient $\lambda \in [-1; 1]$. This results in a representation which is point symmetric.)
- (Support functions: A support function is an implicit representation. Each operation on it is stored and an evaluation in a specified direction is executed as the evaluation of the inverted operations on the direction in the original object.)

Exercise 5

What are the main operations on state set representations we require during the computation of the reachable states? Which one do you think has the most impact on the computational time and why?

Solution:

- Convex hull of the union: In the approaches presented in the lecture this operation is used to compute the first time-step.
- Intersection: Used whenever the guards are checked.
- Membership: Is used to check, whether the current flowpipe segment still fulfills the invariant of the current location.

- Linear transformation: Used to compute the next flowpipe segment when computing the reachable states for linear hybrid automata. Can also be used for the reset function.
- Minkowski sum: Used for overapproximation of the dynamics and external input.