RWTH AACHEN

Theory of Hybrid Systems
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Modeling and Analysis of Hybrid Systems - SS 2015

Series 8

Exercise 1

a) A 2-dimensional polyhedron P is defined by the following linear inequalities.

$$\begin{cases} -x & & \leq & 0 \\ x & + 2y & \leq & 6 \\ -x & - y & \leq & -2 \\ x & - y & \leq & 3 \\ & - y & \leq & 0 \end{cases}$$

Please give the vertices of P.

b) Given a 2-dimensional rectangle R which is defined by the convex hull of the points $(0,0), (0,1), (1,1), (1,0)$. Please give the vertices of the convex hull of R and P (given in the previous exercise), and the linear inequalities which define it.

Solution:

a) The given inequalities form the following polytope:

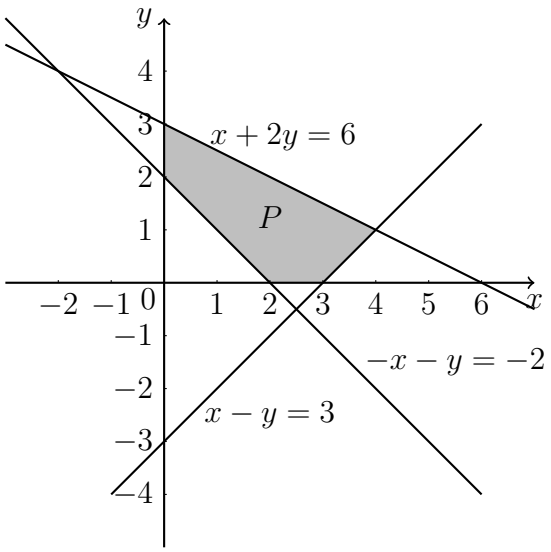

Figure 1: The polyhedron P

b) We use the vertices from the previous exercise and the given vertices to compute the convex hull: The vertices of the convex hull are $(0,0), (0,3), (4,1), (3,0)$, and

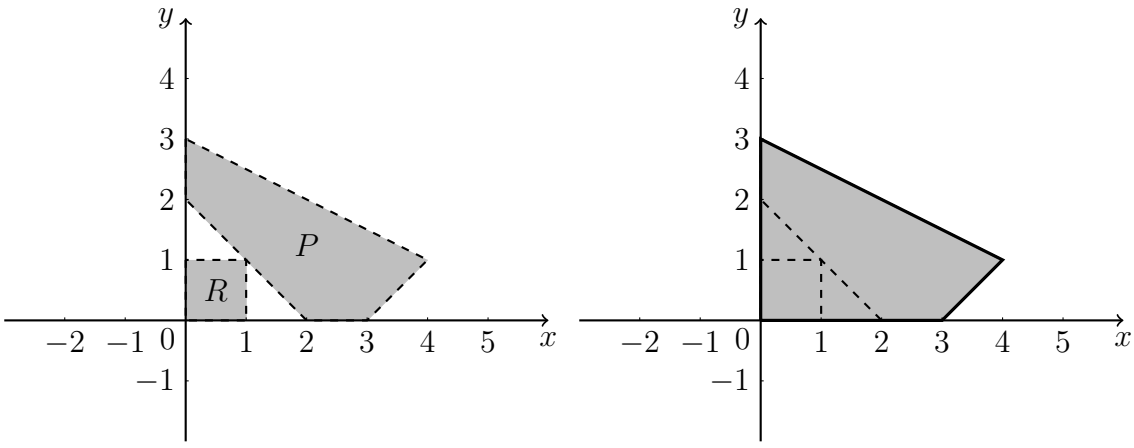

Figure 2: The convex hull of P and R

$conv(P,R)$ can be defined by the inequalities

$$-x \leq 0, -y \leq 0, x + 2y \leq 6, x - y \leq 3$$