RWTH AACHEN

Theory of Hybrid Systems
Prof. Dr. Erika Ábrahám
RWTH Aachen University, Informatik 2 · D-52056 Aachen · GERMANY
http://ths.rwth-aachen.de/

Modeling and Analysis of Hybrid Systems - SS 2015

Series 6

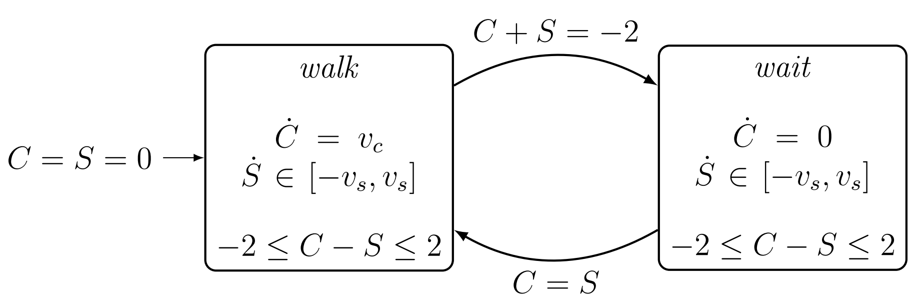
Exercise 1

Charlie Brown walks his dog Snoopy every day the same way:

- Both leave the house next to each other and start their walk.
- As Charlie (C) is thinking about important things (the girl with the red hair), he walks with continuous pace v_c.
- Curious Snoopy (S) is less steady and thus changes his pace between $-v_s$ and v_s, while $0 < v_c < v_s$ holds.
- The leach has only a length of 2 meters. Whenever Snoopy is left behind 2 meters, Charlie waits until Snoopy closes up to him and both continue the walk.

Please give a linear hybrid automaton, which models the position S and C of Snoopy and Charlie respectively.

<u>Solution:</u> The linear hybrid automaton which models the position of Charlie and Snoopy can be specified as follows:

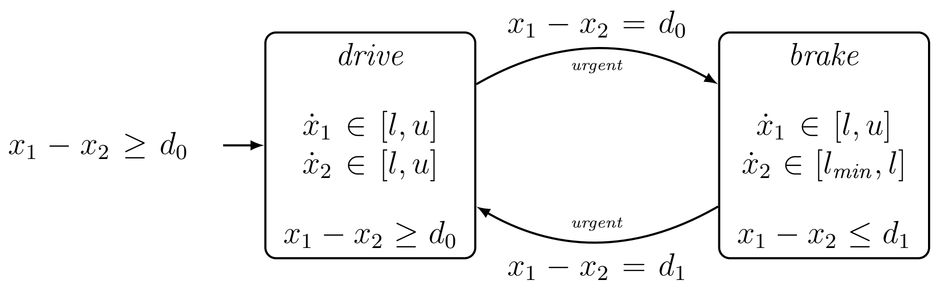


Exercise 2

We consider a vehicle platoon, where two cars are driving with speeds $\dot{x}_i \in [l, u], i \in \{1, 2\}, 0 < l < u$ on a road, such that the 1^{st} car is in front of the 2^{nd} car. The goal is to keep the distance between two cars above some constant $d_0 > 0$. When the distance is at its boundary d_0, the rear car brakes, which limits its speed to the interval $\dot{x}_2 \in [l_{min}, l], 0 < l_{min} < l$. Additionally we utilize a second constant $d_1 > d_0 > 0$ to prolong the braking process until this target distance d_1 is reached. Initially the goal condition is satisfied.

Note that both transitions are *urgent transitions*, which means that they are taken as soon as they are enabled.

A linear hybrid automaton of the above system is given as follows:



a) Please calculate the forward time closure as presented in the lecture.

Solution:

a) We use the algorithm presented in the lecture, where $e_1 = (l_0, x_1^{pre} - x_2^{pre} = d_0, x =$

$x^{pre} \wedge y = y^{pre}, l_1)$ and $e_2 = (l_1, x_1^{pre} - x_2^{pre} = d_1, x = x^{pre} \wedge y = y^{pre}, l_0)$:

$$
\begin{aligned}
P_0 =& \{(l_0, R_{l_0}^0), (l_1, R_{l_1}^0)\} \\
R_{l_0}^0 =& T_{l_0}^+(\varphi_{Init} \wedge \varphi_{Inv}) \\
=& T_{l_0}^+(x_1 - x_2 \geq d_0) \\
=& \exists t. \exists x_1^{pre}. \exists x_2^{pre}.\ x_1^{pre} - x_2^{pre} \geq d_0 \wedge t \geq 0 \wedge \\
& x_1 \leq x_1^{pre} + u \cdot t \wedge x_1 \geq x_1^{pre} + l \cdot t \wedge \\
& x_2 \leq x_2^{pre} + u \cdot t \wedge x_2 \geq x_2^{pre} + l \cdot t \wedge \\
& x_1 - x_2 \geq d_0 \\
\overset{(F.M.)}{=}& \exists t. \exists x_1^{pre}.\ t \geq 0 \wedge \\
& x_1 \leq x_1^{pre} + u \cdot t \wedge x_1 \geq x_1^{pre} + l \cdot t \wedge \\
& x_2 - u \cdot t \leq x_2 - l \cdot t \wedge x_2 - u \cdot t \leq x_1^{pre} - d_0 \wedge \\
& x_1 - x_2 \geq d_0 \\
\overset{(F.M.)}{=}& \exists t. t \geq 0 \wedge \\
& x_2 - u \cdot t + d_0 \leq x_1 - l \cdot t \wedge \\
& x_1 - u \cdot t \leq x_1 - l \cdot t \wedge \\
& x_2 - u \cdot t \leq x_2 - l \cdot t \wedge \\
& x_1 - x_2 \geq d_0 \\
\overset{(F.M.)}{=}& x_1 - x_2 \geq d_0 \\
R_{l_1}^0 =& T_{l_1}^+(\varphi_{Init} \wedge \varphi_{Inv}) = false \\
P_1 =& \{(l_0, R_{l_0}^1), (l_1, R_{l_1}^1)\} \\
R_{l_1}^1 =& T_{l_1}^+(D_{e_1}^+(R_0^{l_0})) \\
R_{l_1}^1 =& T_{l_1}^+(\exists x_1^{pre}. \exists x_2^{pre}. \\
& x_1^{pre} - x_2^{pre} \geq d_0 \wedge x_1^{pre} - x_2^{pre} = d_0 \wedge x_1 = x_1^{pre} \wedge x_2 = x_2^{pre} \wedge x_1^{pre} - x_2^{pre} \leq d_1) \\
=& T_{l_1}^+(x_1 - x_2 = d_0 \wedge x_1 - x_2 \leq d_1)
\end{aligned}
$$

$$
\begin{aligned}
R^1_{l_1} =& \exists t. \exists x_1^{pre}. \exists x_2^{pre}.\ t \geq 0 \wedge \\
& \underline{x_2^{pre} = x_1^{pre} - d_0} \wedge \\
& x_1 \leq x_1^{pre} + u \cdot t \wedge x_1 \geq x_1^{pre} + l \cdot t \wedge \\
& x_2 \leq x_2^{pre} + l \cdot t \wedge x_2 \geq x_2^{pre} + l_{min} \cdot t \wedge \\
& x_1 - x_2 \leq d_1 \\
=& \exists t. \exists x_1^{pre}.\ t \geq 0 \wedge \\
& x_1 \leq x_1^{pre} + u \cdot t \wedge x_1 \geq x_1^{pre} + l \cdot t \wedge \\
& x_2 \leq x_1^{pre} - d_0 + l \cdot t \wedge x_2 \geq x_1^{pre} - d_0 + l_{min} \cdot t \wedge \\
& x_1 - x_2 \leq d_1 \\
=& \exists t. \exists x_1^{pre}.\ t \geq 0 \wedge \\
& x_1 - u \cdot t \leq x_1^{pre} \wedge x_1^{pre} \leq x_1 - l \cdot t \wedge \\
& x_2 + d_0 - l \cdot t \leq x_1^{pre} \wedge x_1^{pre} \leq x_2 + d_0 - l_{min} \cdot t \wedge \\
& x_1 - x_2 \leq d_1 \\
\overset{(F.M.)}{=}& \exists t.\ t \geq 0 \wedge \\
& x_1 - u \cdot t \leq x_1 - l \cdot t \wedge x_1 - u \cdot t \leq x_2 + d_0 - l_{min} \cdot t \\
& \wedge x_2 + d_0 - l \cdot t \leq x_1 - l \cdot t \wedge x_2^{+} d_0 - l \cdot t \leq x_2 + d_0 - l_{min} \cdot t \\
& \wedge x_1 - x_2 \leq d_1 \\
=& \exists t.\ t \geq 0 \wedge \\
& \frac{x_1 - x_2 - d_0}{u - l_{min}} \leq t \wedge x_1 - x_2 \geq d_0 \wedge x_1 - x_2 \leq d_1 \\
\overset{(F.M.)}{=}& x_1 - x_2 \geq d_0 \wedge x_1 - x_2 \leq d_1 \\
R^2_{l_0} =& T^+_{l_0}(D^+_{e_2}(R^{l_1}_1)) \\
=& T^+_{l_0}(\exists x_1^{pre}. \exists x_2^{pre}. \wedge \\
& d_0 \leq x_1 - x_2 \wedge x_1 - x_2 \leq d_1 \wedge \\
& x_1^{pre} - x_2^{pre} = d_1 \wedge \underline{x_1 = x_1^{pre}} \wedge \underline{x_2 = x_2^{pre}} \wedge x_1 - x_2 \geq d_0) \\
=& T^+_{l_1}(d_0 \leq x_1 - x_2 \wedge x_1 - x_2 \leq d_1 \wedge x_1 - x_2 = d_1) \\
=& T^+_{l_1}(x_1 - x_2 = d_1) \subseteq R^0_{l_0}
\end{aligned}
$$

Reachable set: $R_0^{l_0} \cup R_1^{l_1}$