

# Modeling and Analysis of Hybrid Systems

Reachability analysis for linear hybrid automata I using polyhedra

Prof. Dr. Erika Ábrahám

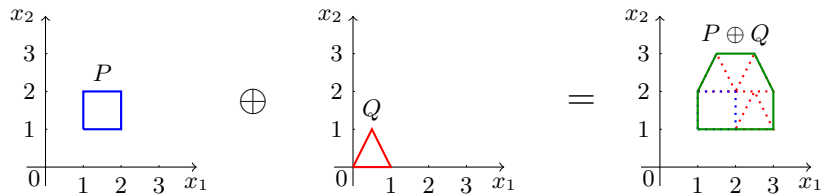
Informatik 2 - Theory of Hybrid Systems  
RWTH Aachen University

SS 2015



- Uppaal [Behrmann et al., 2004]
- HyTech [Henzinger et al., 1997]
- PHAVer [Frehse, 2005]
- SpaceEx [Frehse et al., 2011]
- d/dt [Asarin et al., 2002]
- Ellipsoidal toolbox [Kurzhan et al., 2006]
- MATISSE [Girard et al., 2007]
- Multi-Parametric Toolbox [Kvasnica et al., 2004]
- Flow\* [Chen et al., 2012]

# Reminder: Minkowski sum



$$P \oplus Q = \{p + q \mid p \in P \text{ and } q \in Q\}$$

# Most well-known state set representations

## Geometric objects:

- hyperrectangles [Moore et al., 2009]
- oriented rectangular hulls [Stursberg et al., 2003]
- convex polyhedra [Ziegler, 1995] [Chen et al., 2011]
- orthogonal polyhedra [Bournez et al., 1999]
- template polyhedra [Sankaranarayanan et al., 2008]
- ellipsoids [Kurzbaniski et al., 2000]
- zonotopes [Girard, 2005]

## Other symbolic representations:

- support functions [Le Guernic et al., 2009]
- Taylor models [Berz and Makino, 1998, 2009] [Chen et al., 2012]

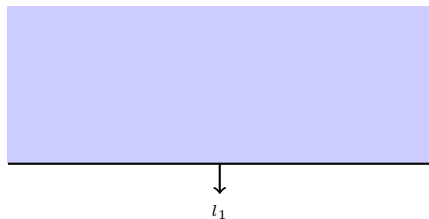
# Example: Polytopes

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- **Halfspace:** set of points satisfying  $l \cdot x \leq z$

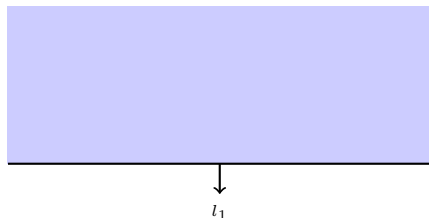
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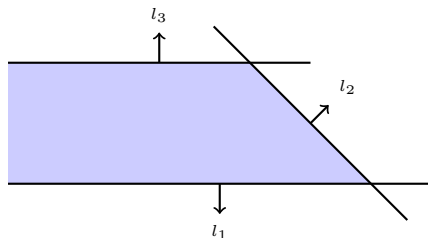
- **Halfspace:** set of points satisfying  $l \cdot x \leq z$
- **Polyhedron:** an intersection of finitely many halfspaces





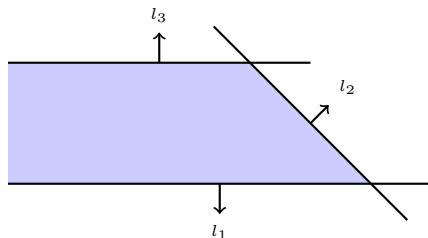
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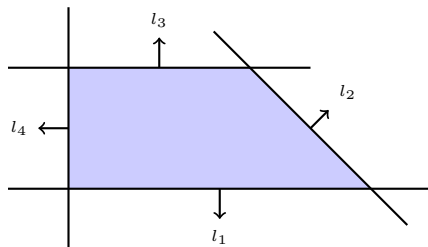
# Example: Polytopes

- **Halfspace:** set of points satisfying  $l \cdot x \leq z$
- **Polyhedron:** an intersection of finitely many halfspaces
- **Polytope:** a bounded polyhedron



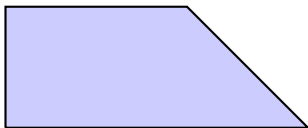
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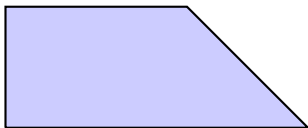
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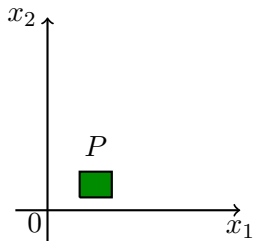
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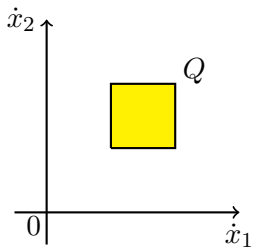
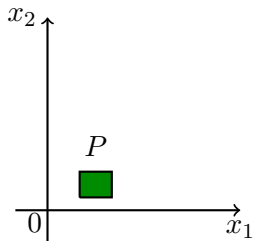
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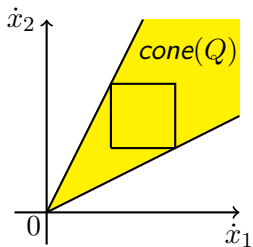
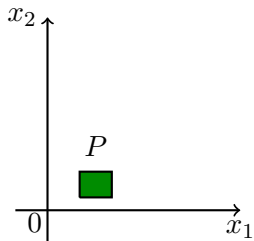
representation	union	intersection	Minkowski sum
$\mathcal{V}$ -representation by <b>vertices</b>	easy	hard	easy
$\mathcal{H}$ -representation by <b>facets</b>	hard	easy	hard

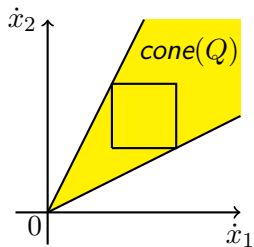
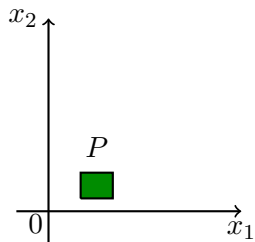


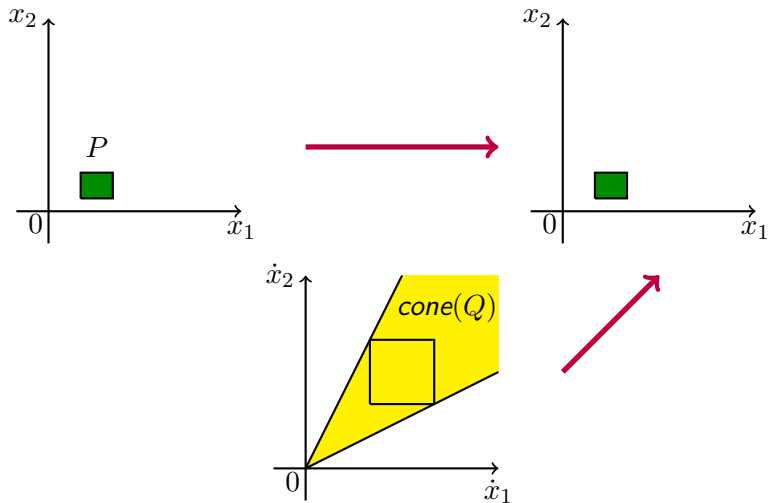


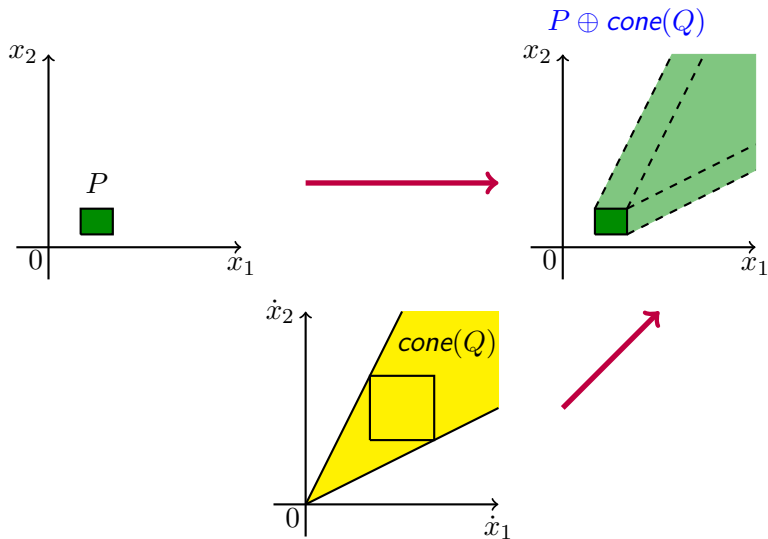


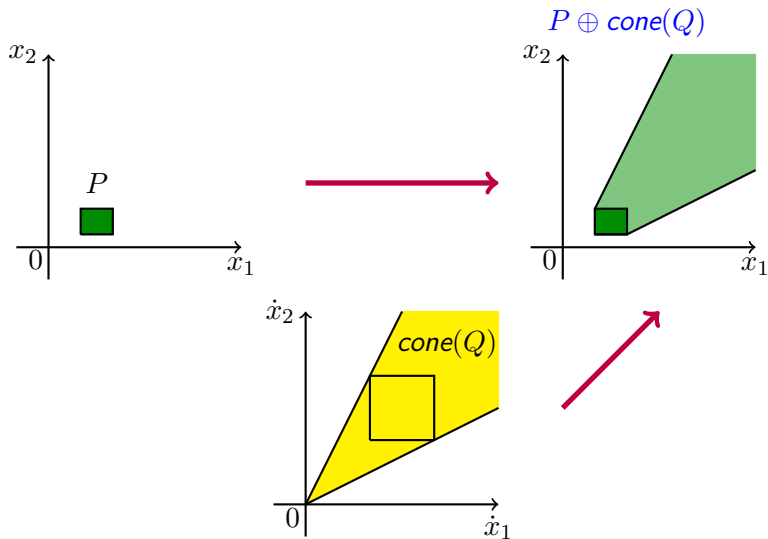


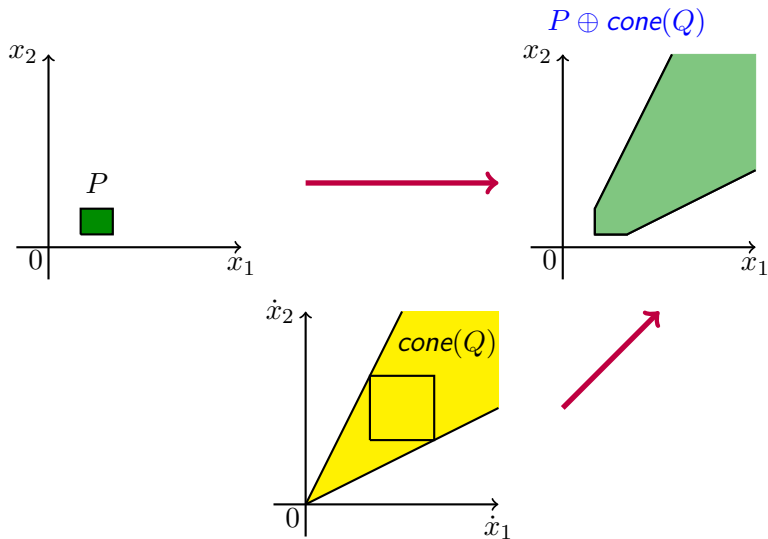


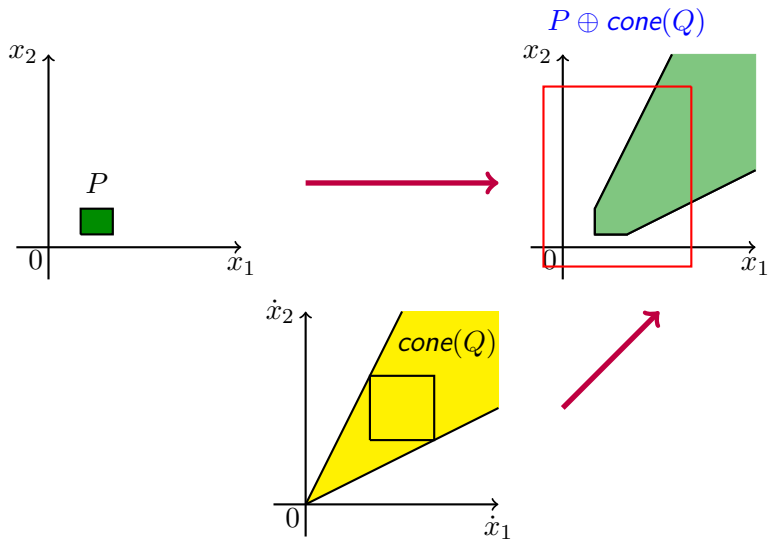


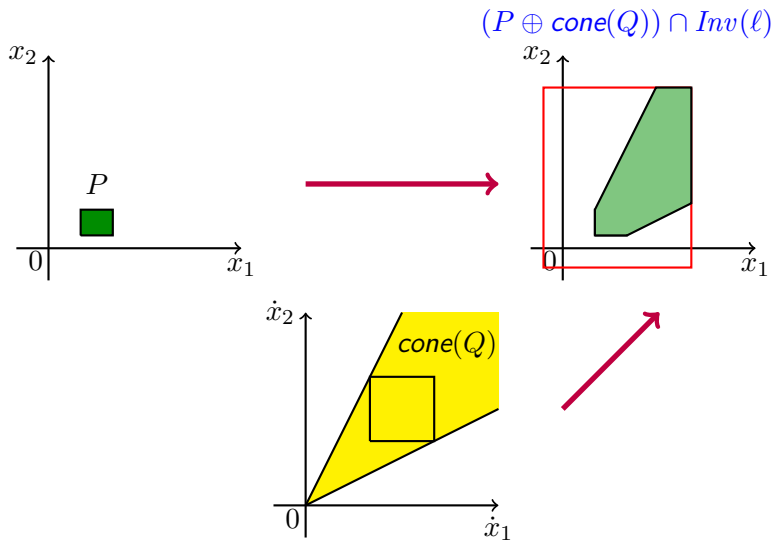




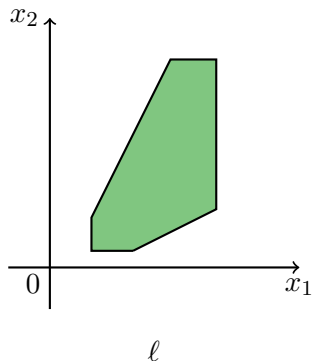


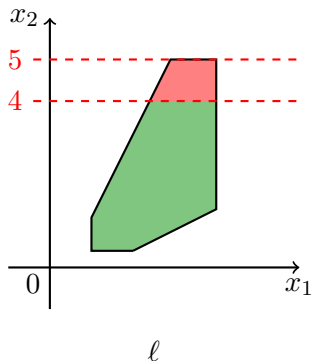


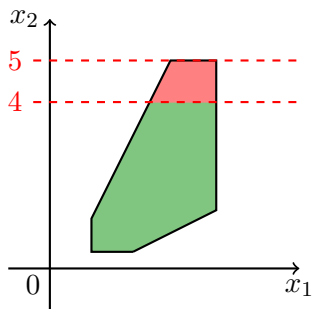




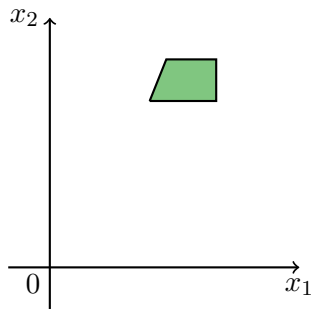




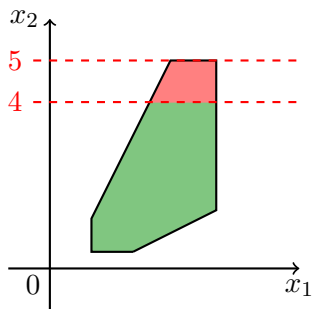




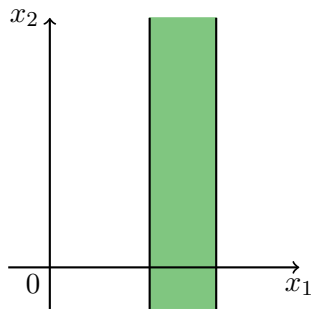
$l$



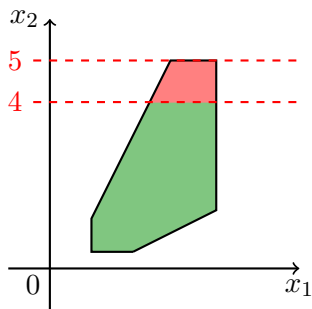
$l'$



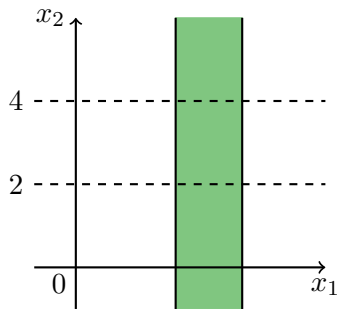
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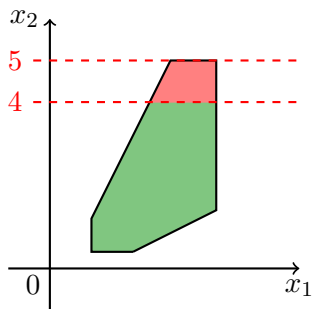
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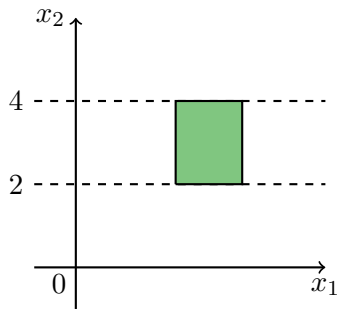
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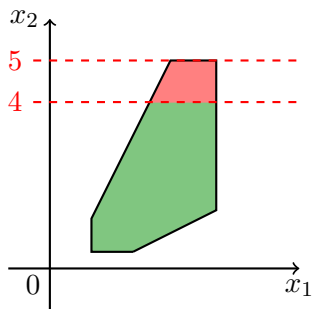
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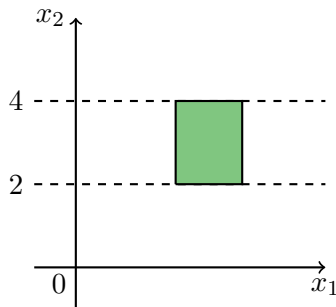
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- Computed via [projection](#) and [Minkowski sum](#).