Modeling and Analysis of Hybrid Systems

Introduction

Prof. Dr. Erika Ábrahám

Informatik 2 - Theory of Hybrid Systems
RWTH Aachen University

SS 2015

Organizational

Lecture:

- Monday 15:15-16:00 in AH 3
- Tuesday 12:15-13:45 in 5056

Exercise:

- Monday 16:00-16:45 in AH 3

Exam dates:

- 1st: 27.07.2015 14:15-16:45
- 2nd: 16.09.2015 15:45-18:15

Learning materials and contact persons

Learning materials available in L2P:

- Slides
- Lecture notes
- Some research publications
- Exercise sheets, solutions

Lecture:

Erika Ábrahám
room: 4229 (E1, 2nd floor), phone: 0241/80-21242
email: abraham@informatik.rwth-aachen.de

Exercise:

Stefan Schupp
room: 4228 (E1, 2nd floor), phone: 0241/80-21243
email: stefan.schupp@informatik.rwth-aachen.de

Further information (topic, evaluations etc.):

`http://ths.rwth-aachen.de/teaching/ss15/lecture-modelling-and-analysis-of-hybrid-systems/`

YABS Backchannelsystem

access: `lanzarote.informatik.rwth-aachen.de:8080`

username: `tim-username`

password: `tim-password`

Contents

Contents

“Hybrid”

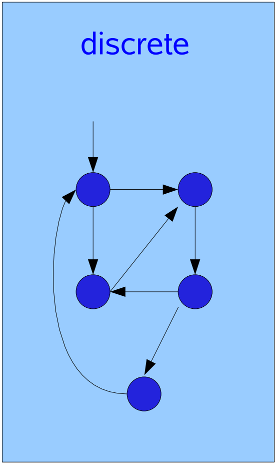
Wikipedia:

> “A hybrid is the combination of two or more different things, aimed at achieving a particular objective or goal.”

A hybrid rose

A hybrid car

Hybrid in computer science



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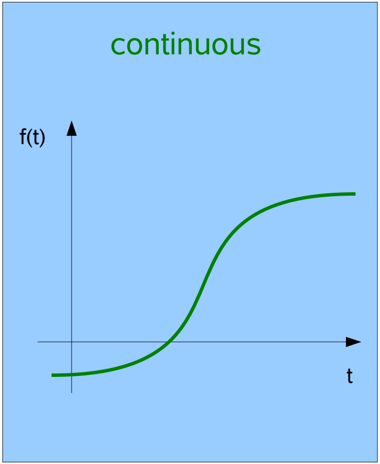
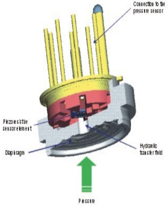
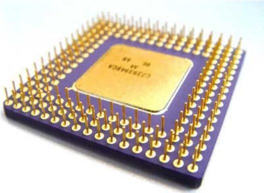


The discrete part





Combined with the continuous part

Example: Thermostat

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- Temperature x is controlled by switching a heater on and off
- x is regulated by a thermostat:
  - $17° \leq x \leq 18° \rightsquigarrow$ "heater on"
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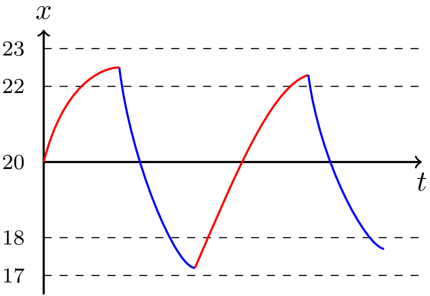
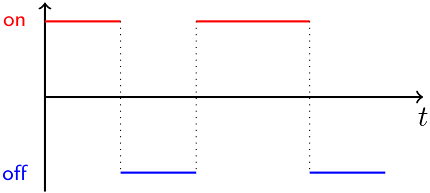
Discrete: switching





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- A model never exactly corresponds to the modeled real system.
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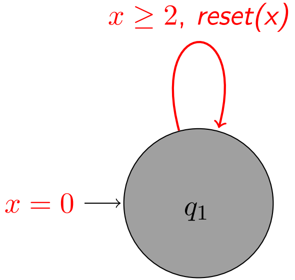
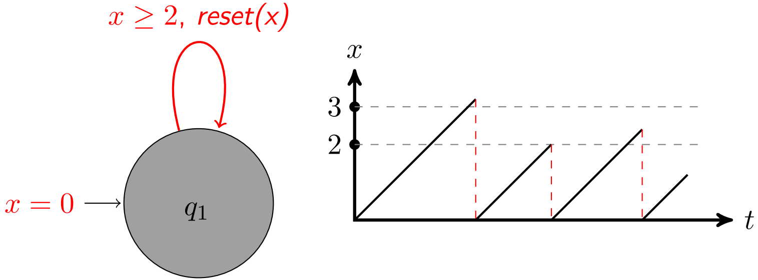
- What you probably also know: Transition systems
- What you perhaps know: Timed automata

Example: Timed automaton



Example: Timed automaton



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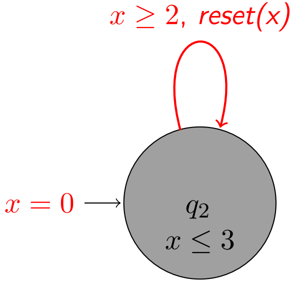

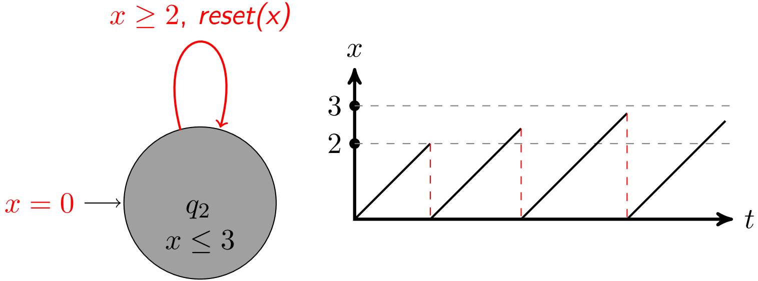
$x \geq 2$, reset(x)
$x = 0$
q_2
$x \leq 3$
x
3
2
t

Modeling general hybrid systems: Hybrid automata

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Let's take again the thermostat as an example.

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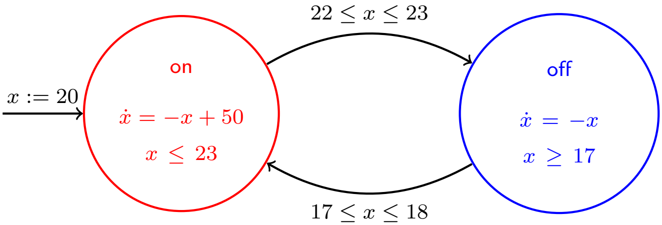

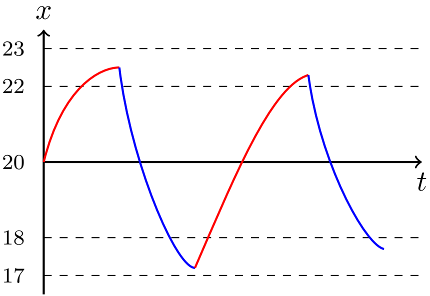

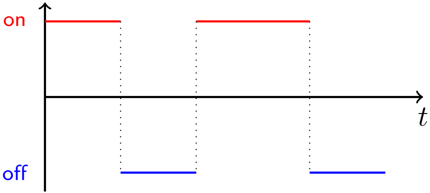

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- Or

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  "The temperature is always below $20°C$."

- Or

  "If the temperature is above $20°C$ it will get below $20°C$ within 5 seconds."

- Or

  "It is always the case that the temperature will somewhen in the future get above $20°C$."

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- ...on the fact if the logic is decidable for the underlying modeling language.
- We will see for which classes of hybrid automata the reachability question is decidable.
- We will deal with
  - (unbounded) reachability for timed automata.
  - (unbounded) reachability for initialized rectangular automata.
  - bounded reachability for linear hybrid automata.
  - reachability approximation for general hybrid automata.

Method for timed automata: Finite abstraction

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Constructive proof of decidability via finite abstraction:

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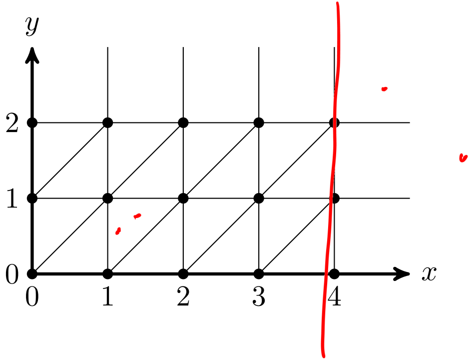

Method for initialized rectangular automata: Transformation

Method for initialized rectangular automata: Transformation

Leading back the proof of decidability to a known problem:

Method for initialized rectangular automata: Transformation

Leading back the proof of decidability to a known problem:

Timed automaton

$\uparrow$

Initialized stopwatch automaton

$\uparrow$

Initialized singular automaton

$\uparrow$

Initialized rectangular automaton

Method for linear hybrid automata: Fixedpoint computation

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Forward reachability computation:

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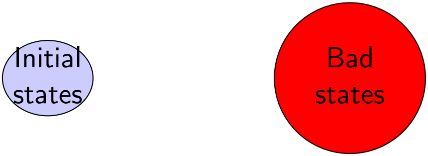
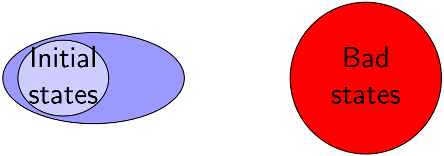


Method for linear hybrid automata: Fixedpoint computation

Forward reachability computation:



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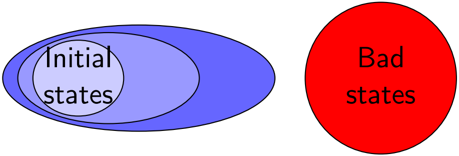

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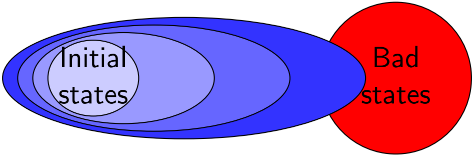

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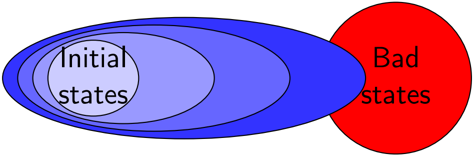

Note: the method is incomplete

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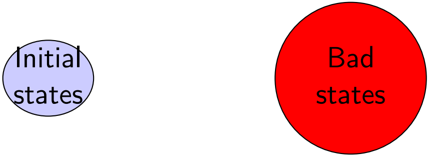

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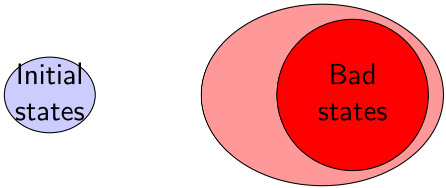

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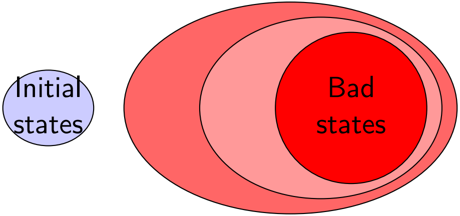

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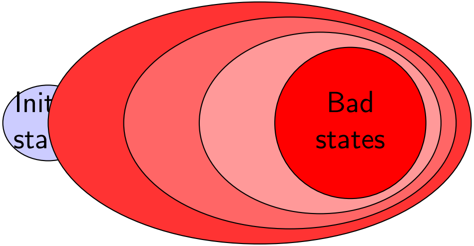

Method for linear hybrid automata: Fixedpoint computation

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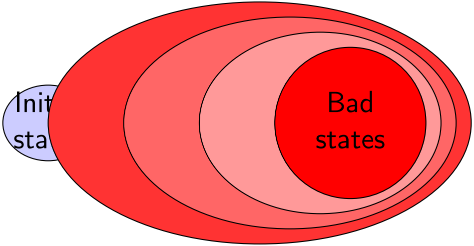

Note: also the backward method is incomplete

Method for hybrid automata: Approximation

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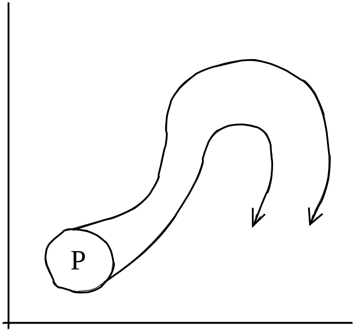
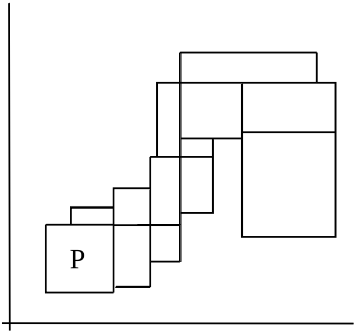
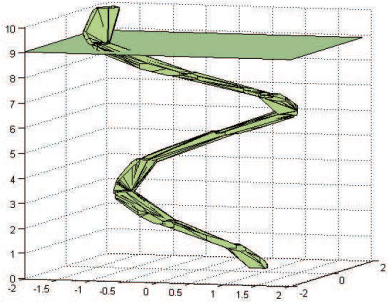
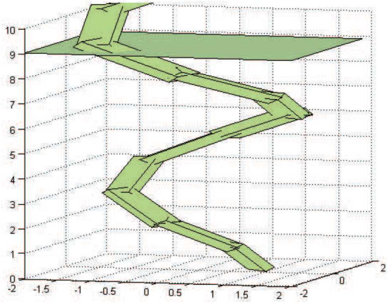


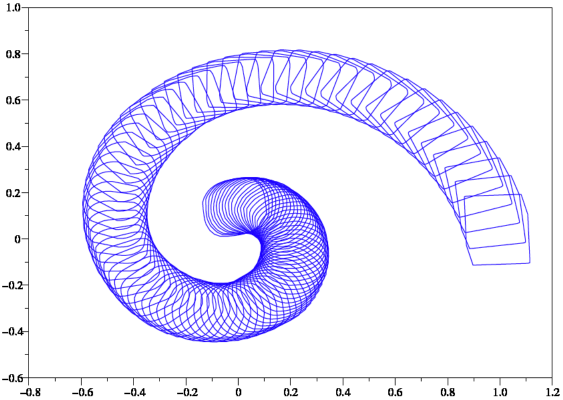


Polyhedra (left) and oriented rectangular hulls (right) in reachability computation





Zonotopes in reachability computation



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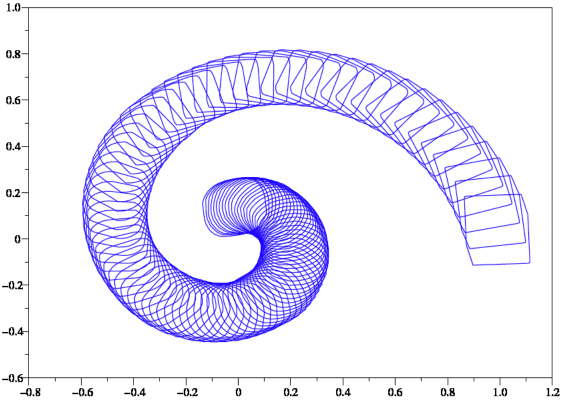

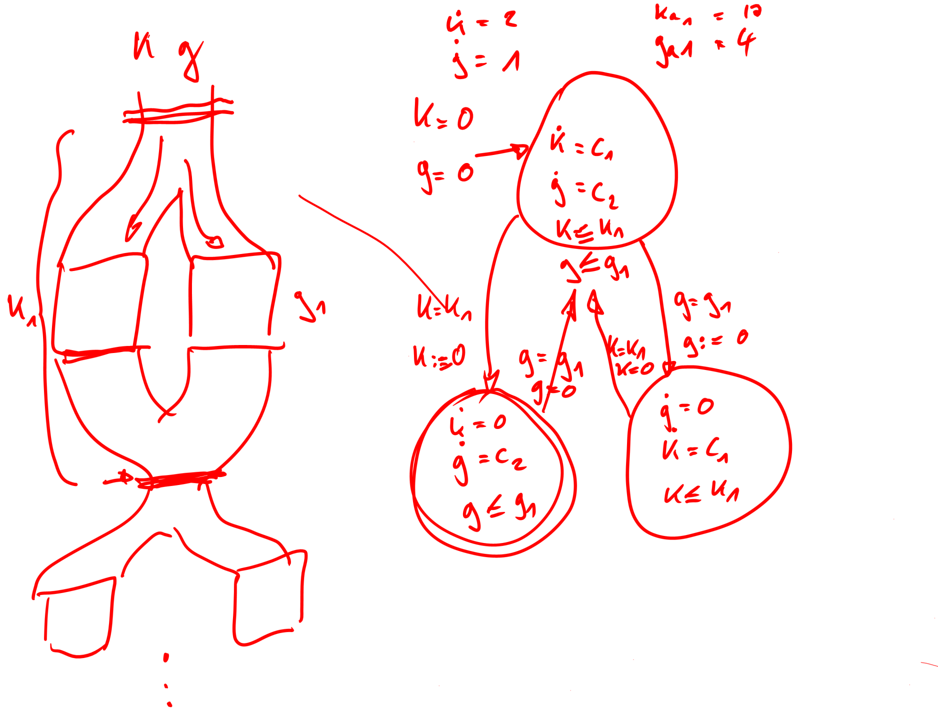

k g
k1
g1
i = 2
j = 1
ka1 = 12
ga1 = 4
k = 0
g = 0
k = c1
g = c2
k ≤ k1
g ≤ g1
k = k1
k := 0
g = g1
g := 0
g = g1
g := 0
k = k1
k := 0
g = g1
g := 0
i = 0
g = c2
g ≤ g1
g = 0
k = c1
k ≤ k1